Spin structure function g_1 at $x \ll 1$

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Abstract

Accounting for double-logarithms of x and running QCD coupling leads to expressions for both the non-singlet and singlet components of g_1 quite different compared to the DGLAP-ones. These expressions differ considerably from the DGLAP expressions at small though finite values of x and lead to the Regge asymptotics.

1 Introduction

Deep inelastic scattering (DIS) is one of the basic processes for probing the structure of hadrons. From the theoretical point of view, the inclusive cross section of DIS is a convolution of the leptonic and hadronic tensors, with the information about the structure of the hadrons participating into DIS coming from the hadronic tensor . the forward Compton amplitude, when a deeply off-shell photon with virtuality q^2 scatters off an on-shell hadron with momentum p. The spin-dependent part, $W_{\mu\nu}^{spin}$, of the hadronic tensor is parametrized in terms of two structure functions, g_1 and g_2, as

$$W_{\mu\nu}^{spin} = \imath \epsilon_{\mu\mu\lambda\rho} \frac{q_\lambda m}{pq} \Big[S_\rho g_1 + \big(S_\rho - \frac{(Sq)}{pq} p_\rho \big) g_2 \Big] \approx \imath \epsilon_{\mu\mu\lambda\rho} \frac{q_\lambda m}{pq} \Big[S_\rho^{||} g_1 + S_\rho^{\perp} (g_1 + g_2) \Big], \tag{1}$$

so that g_1 is related to the longitudinal hadron spin-flip scattering, whereas the sum $g_1 + g_2$ is relative to the transverse spin-flips. In Eq. (1), m stands for the hadron mass, $S_\rho^{||}$ and $S_\rho^{\perp}$ are the longitudinal and transverse (with respect to the plane formed by p and q) components of the hadron spin S_ρ. Both g_1 and g_2 depend on $x = -q^2/2pq,\ 0 < x \le 1$ and $Q^2 = -q^2 > 0$. Obviously, small x corresponds to $s = (p+q)^2 \approx 2pq \gg Q^2$. In this case, $S_\rho^{||} \approx p_\rho/m$ and therefore the part of $W_{\mu\nu}^{spin}$ related to g_1 does not depend on m. When $Q^2 \gg m^2$, one can assume the factorization and regard $W_{\mu\nu}^{spin}$ as a convolution of two objects: The first one is the probability Φ ($\Phi = \Phi_q$ and $\Phi = \Phi_g$ to find a polarized parton (a quark or a gluon) within the hadron. The second one is the partonic tensor $\widetilde{W}_{\mu\nu}^{spin}$ defined and parameterized similarly to $W_{\mu\nu}^{spin}$. Whereas the partonic tensor $\widetilde{W}_{\mu\nu}^{spin}$, i.e. the partonic structure functions g_1 and g_2, can be studied within perturbative QCD, $\Phi_{q,g}$ are essentially non-perturbative objects, The lack of knowledge of Φ is usually compensated by introducing initial parton distributions which can be found from phenomenological considerations. In doing so, the non-singlet component g_1^{NS} of g_1 is usually expressed as a convolution of a piece which we denote g_q^{NS} from purely evolution, and the initial polarized quark density Δq:

$$g_1^{NS} = g_q^{NS} \otimes \Delta q \ . \tag{2}$$

Similarly, the singlet structure function g_1^S is expressed in terms of the evolution pieces g_q and g_g and the densities of the polarized quarks and gluons, Δq and Δg:

$$g_1^S = g_q \otimes \Delta q + g_g \otimes \Delta g \ . \tag{3}$$

The subscripts $q,\ g$ in Eqs. (2,3) refer to the kind of the initial partons. We remind that there is no rigorous procedure for calculating Δq and Δg. They have to be found from phenomenological considerations. On the contrary, there are regular perturbative methods for calculating the evolution parts g_q^{NS} and $g_{q,g}$ of g_1. When Q^2 is much greater than the starting point μ^2 of the Q^2-evolution and at the same time $x \ll 1$, it is convenient to rewrite Eqs. (2,3) in the form of the Mellin integral:

$$g_1^{NS} = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/x)^\omega C_{NS}(\omega) e^{\gamma_{NS}(\omega)\ln(Q^2/\mu^2)} \ , \tag{4}$$

$$g_1^S = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/x)^{\omega} \Big[C_q(\omega)\Delta q(\omega) + C_g(\omega)\Delta g(\omega) \Big] e^{\gamma_S(\omega)\ln(Q^2/\mu^2)} ,$$

with $C_{NS}(\omega), C_{q,g}(\omega)$ being the coefficient functions and $\gamma_{NS}(\omega), \gamma_S(\omega)$ the non-singlet and singlet anomalous dimensions respectively. The anomalous dimensions control the Q^2-evolution and the coefficient functions govern the s-evolution which, at fixed Q^2, is equivalent to the x-evolution.

The best known instrument to calculate the DIS structure functions is the DGLAP[1] approach. In the DGLAP framework, both the coefficient functions and the anomalous dimensions are perturbatively known and represented by their one-loop, (or leading order (LO)) [1] and two-loop (next-to-leading order (NLO)) contributions. The relevant Ref.s concerning the NLO calculations can be found in the review [2]. The remaining ingredients to the rhs of Eq. (4), Δq and Δg can be taken, for example, from Ref. [3]. DGLAP provides a quite good description of the experimental data[3]. The extrapolation of DGLAP into the small-x region predicts an asymptotic behavior $\sim \exp(\sqrt{C \ln(1/x) \ln\ln Q^2})$ for all DIS structure functions (with different factors C). However, from a theoretical point of view, such an extrapolation at the small-x is rather doubtful. In particular, it neglects in a systematical way contributions ot the type $\sim (\alpha_s \ln^2(1/x))^k$ which are small when $x \sim 1$ but become relevant when $x \ll 1$. The total resummation of these double-logarithmic (DL) contributions was made in Refs. [4] and Ref. [5] for the non-singlet (g_1^{NS}) and singlet g_1 respectively, and it leads to the Regge (power-like) asymptotics $g_1(g_1^{NS}) \sim (1/x)^{\Delta^{DL}}((1/x)^{\Delta^{DL}_{NS}})$, with $\Delta^{DL}, \Delta^{DL}_{NS}$ being the intercepts calculated ib the double-logarithmic approximation (DLA). The weak point of this resummation in Refs. [4],[5] is the assumption that α_s is kept fixed (at some unknown scale). It leads therefore to the value of the intercepts $\Delta^{DL}, \Delta^{DL}_{NS}$ explicitly depending on this unknown coupling, while α_s is well-known to be running. The results of Refs. [4],[5] had led many authors (see e.g.[6]) to suggest that the DGLAP parametrization $\alpha_s = \alpha_s(Q^2)$ has to be used. However, according to results of Ref. [7], such a parametrization is indeed correct for $x \sim 1$ only and cannot be used for $x \ll 1$. The appropriate dependence of α_s suggested in Ref. [7], has been successfully used to calculate both g_1^{NS} and g_1 singlet at small x in Refs. [8]. In the present talk we review these results.

Instead of a direct study of g_1 like it is done in DGLAP, it is more convenient to consider the forward Compton amplitude A for the photon-parton scattering. As already stated above, we cannot use DGLAP for studying g_1 or A at small x because it does not not include the total resummation of the double- and single-logarithms of x for the anomalous dimensions and coefficient functions, and also the α_s-parametrization used is valid for x not far from 1.

Then, in order to account for the double-logs of both x and Q^2, we need to construct a kind of two-dimensional evolution equations that would combine both the x- and Q^2- evolutions.

Such equations should sum up the contributions of the Feynman graphs involved to all orders in α_s. Some of those graphs have either ultraviolet or infrared (IR) divergences. The ultraviolet divergences are regulated by the usual renormalization procedure. In order to regulate the IR ones, we introduce an IR cut-off μ in the transverse momentum space for the momenta k_i of all virtual quarks and gluons:

$$\mu < k_{i\perp} \tag{5}$$

where $k_{i\perp}$ stands for the transverse (with respect to the plane formed by the external momenta p and q) component of k_i. This technique of regulating the IR divergences was suggested by Lipatov and used first in Ref. [9] for quark-quark scattering. Using this cut-off μ, A acquires a dependence on μ. Then, one can evolve A with respect to μ, constructing the appropriate Infrared Evolution Equations (IREE).

2 Infrared evolution equations for g_1

The system of the infrared evolution equations for g_1 is

$$\begin{aligned}
(\omega + \frac{\partial}{\partial y}) F_q(\omega, y) &= \frac{1}{8\pi^2} \big[F_{qq}(\omega) F_q(\omega, y) + F_{qg}(\omega) F_g(\omega, y) \big] , \\
(\omega + \frac{\partial}{\partial y}) F_g(\omega, y) &= \frac{1}{8\pi^2} \big[F_{gq}(\omega) F_q(\omega, y) + F_{gg}(\omega) F_g(\omega, y) \big] . \qquad (6)
\end{aligned}$$

The amplitudes F_q, F_g are related to A_q, A_g through the Mellin transform. The amplitudes F_{ik}, with $i, k = q, g$, describe the parton-parton forward scattering. They contain DL contributions to all orders in α_s. We can introduce the new anomalous dimensions $H_{ik} = (1/8\pi^2)F_{ik}$. The subscripts "q,g" correspond to the DGLAP-notation. Solving this system leads to

$$g_q(x,Q^2) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath}(1/x)^{\omega}\Big[C_+(\omega)e^{\Omega_+ y} + C_-(\omega)e^{\Omega_- y}\Big] \ , \tag{7}$$

$$g_g(x,Q^2) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath}(1/x)^{\omega}\Big[C_+(\omega)\frac{X+\sqrt{R}}{2H_{qg}}e^{\Omega_+ y} + C_-(\omega)\frac{X-\sqrt{R}}{2H_{qg}}e^{\Omega_- y}\Big]$$

The unknown factors $C_\pm(\omega)$ have to be specified and will be discussed later. All other factors in Eq. (7) can be expressed in therms of H_{ik}:

$$X = H_{gg} - H_{qq}, \ R = (H_{gg} - H_{qq})^2 + 4H_{qg}H_{gq} \tag{8}$$

$$\Omega_\pm = \frac{1}{2}\left[H_{qq} + H_{gg} \pm \sqrt{(H_{qq} - H_{gg})^2 + 4H_{qg}H_{gq}}\right].$$

The anomalous dimension matrix H_{ik} was calculated in Ref. [8]:

$$\begin{aligned} H_{gg} &= \frac{1}{2}\Big(\omega + Y + \frac{b_{qq} - b_{gg}}{Y}\Big), H_{qq} = \frac{1}{2}\Big(\omega + Y - \frac{b_{qq} - b_{gg}}{Y}\Big), \\ H_{gq} &= -\frac{b_{gq}}{Y}, \qquad H_{qg} = -\frac{b_{qg}}{Y} \ . \end{aligned} \tag{9}$$

where

$$Y = -\sqrt{\Big(\omega^2 - 2(b_{qq} + b_{gg}) + \sqrt{[(\omega^2 - 2(b_{qq} + b_{gg}))^2 - 4(b_{qq} - b_{gg})^2 - 16b_{qg}b_{gq}]}\ \Big)/2} \ , \tag{10}$$

$$b_{ik} = a_{ik} + V_{ik}, \tag{11}$$

$$a_{qq} = \frac{A(\omega)C_F}{2\pi}, \ a_{gg} = \frac{2A(\omega)N}{\pi}, \ a_{gq} = -\frac{n_f A'(\omega)}{2\pi}, \ a_{qg} = \frac{A'(\omega)C_F}{\pi} \ , \tag{12}$$

and

$$V_{ik} = \frac{m_{ik}}{\pi^2}D(\omega), \tag{13}$$

with

$$m_{qq} = \frac{C_F}{2N}, \ m_{gg} = -2N^2, \ m_{qg} = n_f\frac{N}{2}, \ m_{gq} = -NC_F. \tag{14}$$

We have used here the notations $C_F = 4/3, N = 3$ and $n_f = 4$. The quantities $A(\omega)$ and $D(\omega)$ account for the running of α_s. They are given by the following expressions:

$$A(\omega) = \frac{1}{b}\Big[\frac{\eta}{\eta^2 + \pi^2} - \int_0^\infty \frac{d\rho e^{-\omega\rho}}{(\rho+\eta)^2 + \pi^2}\Big] \ , \tag{15}$$

$$D(\omega) = \frac{1}{2b^2}\int_0^\infty d\rho e^{-\omega\rho}\ln\big((\rho+\eta)/\eta\big)\Big[\frac{\rho+\eta}{(\rho+\eta)^2 + \pi^2} + \frac{1}{\rho+\eta}\Big] \tag{16}$$

with $\eta = \ln(\mu^2/\Lambda^2_{QCD})$ and $b = (33 - 2n_f)/12\pi$. A' is defined as A with the π^2 term dropped out.

Finally we have to specify the coefficients functions $C_\pm$ appearing in Eq. (7). When $Q^2 = \mu^2$,

$$g_q = \tilde{\Delta}q(x_0), \qquad g_g = \tilde{\Delta}g(x_0) \tag{17}$$

where $\tilde{\Delta}q(x_0)$ and $\tilde{\Delta}g(x_0)$ are the input distributions of the polarized partons at $x_0 = \mu^2/s$. They do not depend on Q^2. Eq. (17) allows us to express $C_\pm(\omega)$ in terms of $\Delta q(\omega)$ and $\Delta g(\omega)$, which are related to $\tilde{\Delta}q(x_0)$ and $\tilde{\Delta}g(x_0)$ through the ordinary Mellin transform. Indeed,

$$g_q(x,Q^2) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath}(1/x)^{\omega}\Big[\Big(A^{(-)}\Delta q + B\Delta g\Big)e^{\Omega_+ y} + \Big(A^{(+)}\Delta q - B\Delta g\Big)e^{\Omega_- y}\Big] \ , \tag{18}$$

$$g_g(x,Q^2) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath}(1/x)^{\omega}\Big[\Big(E\Delta q + A^{(+)}\Delta g\Big)e^{\Omega_+ y} + \Big(-E\Delta q + A^{(-)}\Delta g\Big)e^{\Omega_- y}\Big] \quad (19)$$

with

$$A^{(\pm)} = \Big(\frac{1}{2} \pm \frac{X}{2\sqrt{R}}\Big),\ B = \frac{H_{qg}}{\sqrt{R}},\ E = \frac{H_{gq}}{\sqrt{R}}\ . \quad (20)$$

Eqs. (18, (19) express g_1 in terms of the parton distributions $\Delta q(\omega)$ and $\Delta g(\omega)$, which are related to the distributions $\tilde{\Delta} q(x_0)$ and $\tilde{\Delta} g(x_0)$ at very low x: $x_0 \approx \mu^2/s \ll 1$. Therefore, they hardly can be found from experimental data. It is much more useful to express g_q, g_g in terms of the initial parton densities $\tilde{\delta} q$ and $\tilde{\delta} g$ defined at $x \sim 1$. We can do it, using the evolution of $\tilde{\Delta} q(x_0)$, $\tilde{\Delta} g(x_0)$ with respect to s. Indeed, the s-evolution of $\tilde{\delta} q, \tilde{\delta} q$ from $s \approx \mu^2$ to $s \gg \mu^2$ at fixed Q^2 $(Q^2 = \mu^2)$ is equivalent to their x-evolution from $x \sim 1$ to $x \ll 1$. In the ω-space, the system of IREE for the parton distributions looks quite similar to Eqs. (6). However, the eqs for $\Delta q,\ \Delta g$ are now algebraic because they do not depend on Q^2. Solving them, we obtain:

$$\Delta q = \frac{(< e_q^2 > /2)\big[\omega(\omega - H_{gg})\delta q + \omega H_{qg}\hat{\delta g}\big]}{\big[\omega^2 - \omega(H_{qq} + H_{gg}) + (H_{qq}H_{gg} - H_{qg}H_{gq})\big]}\ , \quad (21)$$

$$\Delta g = \frac{(< e_q^2 > /2)\big[\omega H_{gq}\delta q + \omega(\omega - H_{qq})\hat{\delta} g\big]}{\big[\omega^2 - \omega(H_{qq} + H_{gg}) + (H_{qq}H_{gg} - H_{qg}H_{gq})\big]}\ . \quad (22)$$

Therefore g_1 is expressed in terms of the initial parton densities $\delta q, \delta g$.

When we put $H_{qg} = H_{gq} = H_{gg} = 0$ and do not sum over e_q, we arrive at the expression for the non-singlet structure function g_1^{NS}: Obviously, in this case $A^{(+)} = B = E = \Omega_- = 0,\ A^{(-)} = 1,\ \ \Omega_+ = H_{qq}$. However, the nonsinglet anomalous dimension H_{qq} should be calculated in the limit $b_{gg} = b_{qg} = b_{gq} = 0$. We denote such $H_{qq} \equiv H^{NS}$ and arrive at

$$g_1^{NS} = \frac{e_q^2}{2}\int_{-\imath\infty}^{\imath\infty}\frac{d\omega}{2\pi\imath}\Big(\frac{\omega\delta q}{\omega - H^{NS}}\Big)\Big(1/x\Big)^{\omega}\Big(Q^2/\mu^2\Big)^{H^{NS}}\ . \quad (23)$$

3 Small-x asymptotics for g_1

When $x \to 0$ and $Q^2 \gg \mu^2$, one can neglect contributions with Ω_- in Eqs. (7). As is well known, $g_1 \sim (1/x)^{\omega_0}$ at $x \to 0$, with ω_0 being the position of the leading singularity of the integrand of g_1 . According to Eqs. (9), the leading singularity, ω^{NS} for g_1^{NS} is the rightmost root of the equation

$$\omega^2 - 4b_{qq} = 0 \quad (24)$$

while the leading singularity, ω_0 for g_1 is the rightmost root of

$$\omega^4 - 4(b_{qq} + b_{gg})\omega^2 + 16(b_{qq}b_{gg} - b_{qg}b_{gq}) = 0\ . \quad (25)$$

In our approach, all factors b_{ik} depend on $\eta = \ln(\mu^2/\Lambda_{QCD})$, so the roots of Eqs. (24,25) also depend on η. We call intercepts the maximums of these roots:

$$g_1^{NS} \sim e_q^2\delta q(1/x)^{\Delta_{NS}}(Q^2/\mu^2)^{\Delta_{NS}/2}, \quad g_1 \sim (< e_q^2 > /2)[Z_1\delta q + Z_2\delta_g](1/x)^{\Delta_S}(Q^2/\mu^2)^{\Delta_S/2}, \quad (26)$$

and we find for the intercepts

$$\Delta_{NS} \approx 0.4,\ \Delta_S \approx 0.86 \quad (27)$$

and $Z_1 = -1.2,\ Z_2 = -0.08$. This implies that g_1^{NS} is positive when $x \to 0$ whereas g_1^S can be either positive or negative, depending on the relation between δq and δg. In particular, g_1 is positive when

$$15\delta q + \delta g < 0. \quad (28)$$

otherwise it is negative. In other words, the sign of g_1 at small x can be positive if the initial gluon density is negative and large.

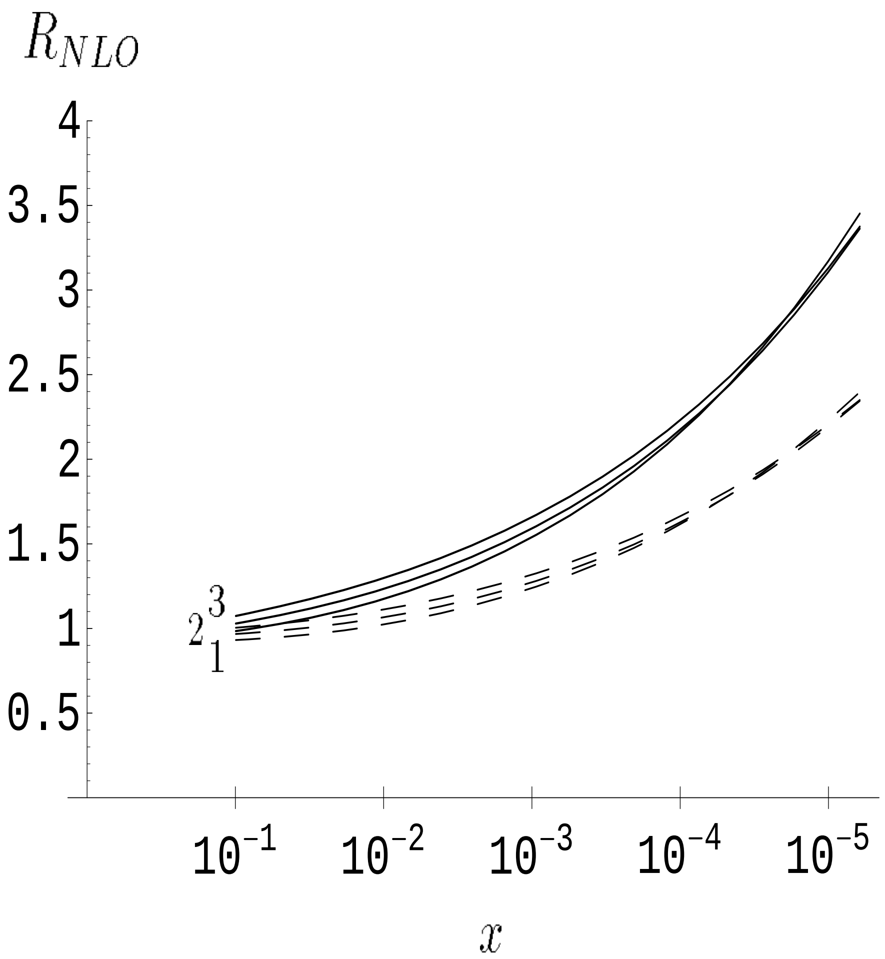

Figure 1: $R_{NLO}(x)$ for f^{NS} (dashed lines 1,2,3) and g_1^{NS} (solid lines 1,2,3). Lines 1,2,3 correspond to $Q^2 = 10$ GeV^2, $Q^2 = 20$ GeV^2, $Q^2 = 40$ GeV^2, respectively.

4 g_1^{NS} at finite values of x

Let us estimate the impact of the total resummation of DL and SL contributions on g_1^{NS}. According to Eq. (23), the value of $g_1(x, Q^2)^{NS}$ depends both on the perturbative terms and on the inputs Δq. The latter can be obtained by fitting the experimental data and it is known (see e.g. [3]) that widely different formulae for Δq can be used. In order to avoid discussing the fitting procedure and as in this paper we present our results only the perturbative part of g_1^{NS}, we can assume

$$\Delta q = \delta(1 - x) \ . \tag{29}$$

Then let us calculate the ratios

$$R_{LO} = g_1^{NS} / \tilde{g_1}_{LO}^{NS} \ , R_{NLO} = g_1^{NS} / \tilde{g_1}_{NLO}^{NS} \ , \tag{30}$$

where $\tilde{g_1}_{LO}^{NS}$ is the LO DGLAP non-singlet g_1^{NS} with the one-loop anomalous dimension and $\tilde{g_1}_{NLO}^{NS}$ is the NLO DGLAP g_1^{NS} with the two-loop anomalous dimension. The results of a numerical calculations for R at $Q^2 = 20\text{GeV}^2$, $\mu = 1.5\text{GeV}$ are shown in Fig. 1.

5 Conclusion

The total resummation of the most singular ($\sim \alpha_s^n/\omega^{2n+1}$) terms in the expressions for the anomalous dimensions and the coefficient functions leads to the expressions for the singlet and the non-singlet structure functions g_1. It guarantees the Regge (power-like) behavior (26) of g_1, g_1^{NS} when $x \to 0$, with the intercepts given by Eq. (27). The intercepts Δ_{NS}, Δ_S are obtained with the running QCD coupling effects taken into account. The value of the non-singlet intercept $\Delta_{NS} \approx 0.4$ is now confirmed by several independent analysis [11] of experimental data and our result $\Delta_S \approx 0.86$ is in a good agreement with the estimate of Ref. [12]: $\Delta_S = 0.88 \pm 0.14$ obtained from analysis of the HERMES data. Eq. (23) states that g_1^{NS} is positive both at $x \sim 1$ and at $x \ll 1$. The situation concerning the singlet g_1 is more involved: being positive at $x \sim 1$, the singlet g_1 can remain positive at $x \ll 1$ only if the initial parton densities obey Eq. (28), otherwise it becomes negative. The ratio of our results versus the DGLAP ones for non-singlet g_1 is given in Fig. 1. It shows explicitly that the impact of high-order DL contributions is small at $x \geq 0.1$ but it grows fast when x is approaching $10^{-3} - 10^{-4}$.

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