Conical defects in a brane world: gravitational lensing

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Abstract

We investigate gravitational field outside conical defects in the Randall-Sundrum infinite brane world and show that, in contrast to their analogous in the four-dimensional Einstein's gravity, brane defects have an attractive gravitational potential and a radially dependent deficit angle. These properties alter some cosmological effects, such as the effect of gravitational lensing.

1 Introduction

Almost all present-day approaches to the fundamental theory are based on the idea that our Universe is a four-dimensional subspace of some higher dimensional space-time. In the original theories of Kaluza-Klein type spacetime is assumed to be the direct product of a four-dimensional space-time and a compact manifold of internal coordinates. To obtain an appropriate four-dimensional gravity, it was supposed that the size of extra dimensions was of the Plank scale. So, for energies accessible in the earth laboratories the modes of matter fields with a momentum in the direction of extra dimensions can not be exited thus shielding the existence of extra dimensions.

At present time the emphasis in the development of multidimensional theories is shifted towards the brane world picture [1]. One of the most attractive features of this picture is that this scenario enables gravity to become strong at energies of a TeV scale. So, the effects of imbedding can, at least in principle, be observed in the coming experiments.

In our paper we consider the Randall-Sundrum model with one infinite extra dimension and a single positive tension brane. Our investigation is devoted to the effects of imbedding on the formation of images by gravitational lenses localized on the brane. We restrict our consideration by the matter distributions associated with a straight local string and a global monopole – topological defects with angular deficit. The reason is threefold. First, it is known that these two types of topological defects are most likely to be really created in the early universe. Secondary, in the $4D$ Einstein's gravity the structure of associated spacetimes is simple enough to allow analytic calculation. Third, both objects produce no Newtonian gravitational potential in four space-time dimensions, and the main effects outside the defects are produced by the deficit angle.

2 Conical defects in nonbrane cosmology

Even though no scalar particles have been found so far, the role of scalars in modern field theoretical models is very important as these fields provide a natural mechanism of spontaneous symmetry breaking. Coupling of these scalar fields with gauge fields gives rise to different types of topological defects (domain walls, local and global strings and monopoles, textures), the type of defect being dependent on the topology of vacuum manifold. But it was shown that most types of defects are inconsistent with the standard cosmology as they lead to a quick domination of defect energy in the universe, and that local strings and global monopoles are the best candidates to be observed (for details see Refs. [2, 3]).

In the case of a local string vacuum has the topology S^1, and there is a closed loop that can not be shrank to a point without leaving the vacuum manifold. This leads to the existence of a line in the physical space which is associated with a very high linear density of the trapped energy. For cosmological purposes the transverse diameter a vertex line can be ignored, and it can be considered as a Nambu-Goto string. If the string is stretched along z axis corresponding energy-momentum tensor may be approximated by the expression

$$t_{tt} = -t_{zz} = \mu\delta(x_1)\delta(x_2) \ , \tag{1}$$

where μ is the energy per unit length. It is proportional to the square of the symmetry breaking energy scale η.

Coupling of the matter distribution (1) with gravity gives rise of a cylindrically symmetric solution which is the direct product of a two-dimensional Minkowski space and a cone with the deficit angle $8\pi G\mu$ (see Fig.1). If one uses conformally Cartesian coordinates on the plane perpendicular to the string, corresponding solution reads

$$ds^2 = -dt^2 + dz^2 + e^{-2(1-b)\log r}\delta_{ik}x_i x_k \ , \quad r = (x_i x_i)^{1/2} \ , \quad i,k = 1,2 \ , \quad b = 1 - 4G_4\mu \ . \tag{2}$$

One can consider the generalization of the metric (2) for a spherically symmetric case, when any plane containing some fixed point and dividing the space into two equal parts is a cone (see Fig.1). In this case

$$ds^2 = -dt^2 + e^{-2(1-\alpha)\log r}\delta_{ik}x_i x_k \ , \quad i,k = 1,2,3 \ , \quad \alpha^2 = 1 - 8\pi G_4\eta^2 \ . \tag{3}$$

Far from the core corresponding energy-momentum tensor reads approximately

$$t_{tt} = \frac{\eta^2}{r^2} \ , \quad t_{ik} = -\eta^2\frac{x_i x_k}{r^4} \ , \quad i,k = 1,2,3 \ . \tag{4}$$

This defect is known as a global monopole. It was shown that it may be formed during phase transitions in the early universe if the vacuum manifold of the underlying field theoretical model has unshrinkable spheres.

For both solutions (2) and (3) the space-time has no Newtonian potential, and their gravitational properties are determined by the deficit angle. But, in contrast to the string space-time, the space-time of a global monopole is not locally flat, and its gravitational field provides a tidal acceleration proportional to $1/r^2$.

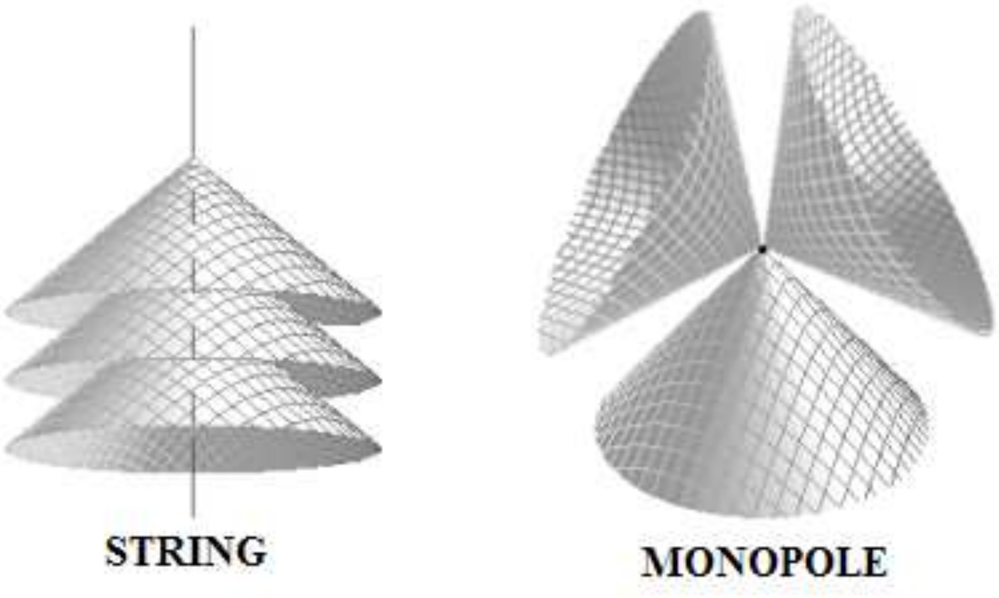

Figure 1: Cosmic string affects surrounding space-time by creating a conical geometry on the section $t = const, z = const$. Space-time of a global monopole is spherically symmetric and any plane dividing the space into two equal parts is a cone.

3 Brane defects: bulk induced gravitational potentials and deficit angles

When matter is present the metric of the RS2 five-dimensional brane world [4] can be written as (for more details see [5] and references therein)

$$ds^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu + 2N_\mu dx^\mu dy + (1+\phi)dy^2 \ ,$$

where

$$\hat{g}_{\mu\nu} = e^{-2k|y|}(\eta_{\mu\nu} + \gamma_{\mu\nu}) \ ,$$

x^μ, $\mu, \nu = 0, 1, 2, 3$, are the coordinates on the brane, and the fifth coordinate y is chosen so that the brane is kept straight and localized at $y = 0$. $\hat{g}_{\mu\nu}$ gives the induced metric on the hypersurfaces $y =$ const. The length k^{-1} is related to the five-dimensional bulk cosmological constant by $\Lambda = -6k^2$ and gives the characteristic length scale of the bulk corrections to the standard Einstein's gravity.

In the linear approximation $\gamma_{\mu\nu}$, N_μ and ϕ are taken to be small, and gauge conditions for the brane to remain at $y = 0$ read [5]

$$N_\mu = -\frac{\operatorname{sgn} y}{2nk}\gamma_{,\mu} \ , \quad \phi = -\frac{\operatorname{sgn} y}{nk}\gamma_{,y} \ , \quad \tilde{\gamma}_{\mu\nu}{}^{,\mu} = 0 \ ,$$

where $\gamma = \eta^{\mu\nu}\gamma_{\mu\nu}$ and $\tilde{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - (1/4)\eta_{\mu\nu}\gamma$ is the traceless part of $\gamma_{\mu\nu}$.

The energy momentum tensor is

$$T^{\mu\nu} = -\frac{3k}{4\pi}\sqrt{1-\phi}\,\hat{g}^{\mu\nu}\delta(y) + t^{\mu\nu}\delta(y) \ , \quad T^{5\mu} = T^{55} = 0 \ .$$

The first term is the background from the brane, and $t^{\mu\nu}$ is the matter perturbation on the brane.

Now we are in a position to consider particular cases of conical defects. General expression for the linearized metric induced on the brane [5] with the use of Eqs. (1) and (4) enables one to show, that at large ($r >> 1/k$) distances the linearized metric of a straight brane string (after appropriate redefinition of the radial coordinate) takes the form

$$ds^2 = (-dt^2 + dz^2)\left[1 - \frac{4G_4\mu}{3k^2r^2}\right] + dr^2 + \left[1 - 8G_4\mu + \frac{16G_4\mu}{3k^2r^2}\right] r^2 d\theta^2 \ , \tag{5}$$

and for a monopole we get

$$d\hat{s}^2 = -dt^2\left[1 - \frac{8\pi G_4\eta^2}{3k^2r^2}\log(2kr)\right] + dr^2 + \\ + \left[1 - 8\pi G_4\eta^2\left(1 - \frac{1}{6k^2r^2}\left(2\log(2kr) + 1\right)\right)\right] r^2 d\Omega_2 \ . \tag{6}$$

We see, that possibility of gravity to propagate in extra dimension changes the geometry near defects. Corresponding spaces are approximately conical, but brane corrections give conical defects a small attractive potential and deform the cones in such a way that the deficit angles become radially dependent.

4 Lensing

In the Randall-Sundrum model matter fields are supposed to be confined to the brane. So, calculating the geodesic equations we must use the intrinsic metrics (5) and (6) and corresponding intrinsic connection coefficients, instead of the full 5-dimensional equivalents.

Rewriting the metric for a string in the form

$$ds^2 = \left(-dt^2 + dz^2\right)A(r) + dr^2 + B(r)d\varphi^2 \ ,$$

$$A(r) = 1 - \frac{4G_4\mu}{3k^2r^2} \ , \quad B(r) = r^2\left[1 - 8G_4\mu\left(1 - \frac{2}{3k^2r^2}\right)\right] \ ,$$

and for a monopole as

$$ds^2 = -dt^2A(r) + dr^2 + B(r)d\Omega_2 \ ,$$

$$A(r) = 1 - \frac{8G_4\pi\eta^2\log(2kr)}{3k^2r^2} \ , \quad B(r) = r^2\left[1 - 8G_4\pi\eta^2\left(1 - \frac{1}{6k^2r^2}\left[2\log(2kr) + 1\right]\right)\right] \ ,$$

we can calculate all nonzero connection coefficients. They are

$$\Gamma^r_{tt} = \frac{1}{2}A' \ , \ \Gamma^t_{rt} = \frac{1}{2}\frac{A'}{A} \ , \ \Gamma^r_{\varphi\varphi} = -\frac{1}{2}B' \ , \ \Gamma^\varphi_{\varphi r} = \frac{1}{2}\frac{B'}{B} \ , \ \Gamma^r_{zz} = -\frac{1}{2}A', \ \Gamma^z_{rz} = \frac{1}{2}\frac{A'}{A}$$

for a brane string, and

$$\Gamma^r_{tt} = \frac{1}{2}A' \ , \ \Gamma^t_{rt} = \frac{1}{2}\frac{A'}{A} \ , \ \Gamma^r_{\theta\theta} = -\frac{1}{2}B' \ , \ \Gamma^\theta_{\theta r} = \frac{1}{2}\frac{B'}{B} \ ,$$

$$\Gamma^r_{\varphi\varphi} = -\frac{1}{2}B'\sin^2\theta, \ \Gamma^\varphi_{\varphi r} = \frac{1}{2}\frac{B'}{B} \ , \ \Gamma^\theta_{\varphi\varphi} = -\sin\theta\cos\theta \ , \ \Gamma^\varphi_{\varphi\theta} = \cot\theta$$

for a monopole.

For the string space-time particle paths are given by

$$\frac{dt}{dp}A = 1 \ , \quad \frac{d\varphi}{dp}B = J \ , \quad \frac{dz}{dp}A = P_z \ , \quad \left(\frac{dr}{dp}\right)^2 - \frac{1}{A} + \frac{J^2}{B} + \frac{P_z^2}{A} = -E \ . \tag{7}$$

As for monopole, we get

$$\frac{dt}{dp}A = 1 \ , \quad \frac{d\varphi}{dp}B = J \ , \quad \left(\frac{dr}{dp}\right)^2 - \frac{1}{A} + \frac{J^2}{B} = -E \ . \tag{8}$$

In the last equation we take into account, that monopole space-time is spherically symmetric. So, particle paths lay in its equatorial plane and we can put $\theta = \pi/2$.

From Eqs. (7) and (8) it follows that $ds^2 = -Edp^2$. Thus $E > 0$ for massive particles and $E = 0$ for photons. It may be shown, that in the case of unbound trajectories $1 - E = P_z^2 + P_T^2$, were J, P_z, P_T are respectively angular, z, and transverse momentum per unit energy of a particle at infinity.

Combining these integrals produces the equations of trajectory. They are

$$\left(\frac{dr}{d\varphi}\right)^2 = -B + \frac{B^2}{AJ^2}(1 - P_z^2) - \frac{B^2 E}{J^2}$$

in the case of a string, and

$$\left(\frac{dr}{d\varphi}\right)^2 = -B + \frac{B^2}{AJ^2} - \frac{B^2 E}{J^2}$$

in the case of a monopole.

Keeping the leading order in $G_4\mu$ (or $G_4\eta^2$) terms we get for a string and a monopole correspondingly

$$\varphi^{str}_{r\to\infty} = \pm\frac{\pi}{2} \pm \frac{1}{2}\frac{\partial}{\partial J}\frac{1}{J}\int\limits_0^{\pi/2} d\varphi \left\{ 8G_4\mu J^2 - \frac{4G_4\mu}{3k^2}\left[1 - P_z^2 + \right.\right.$$
$$\left.\left. +4\left(1 - P_z^2 - E\right)\cos^2\varphi \right]\right\}$$

and

$$\varphi^{mon}_{r\to\infty} = \pm\frac{\pi}{2} \pm \frac{1}{2}\frac{\partial}{\partial J}\frac{1}{J}\int\limits_0^{\pi/2} d\varphi \left\{ 8\pi G_4\eta^2 J^2 - \frac{4\pi G_4\eta^2}{3k^2}\left[(1 - E)\cos^2\varphi + \right.\right.$$
$$\left.\left. +2\left(1 + (1 - E)\cos^2\varphi\right)\ln\left(\frac{2kJ}{\sqrt{1 - E}\cos\varphi}\right)\right]\right\} .$$

Performing the integrations we get

$$\varphi^{str(mon)}_{r\to\infty} = \pm\frac{\pi}{2}\left(1 + \varepsilon^{str(mon)}\right) \ ,$$

with ε being equal to

$$\varepsilon_{str} = 4\mu G_4 + \frac{2}{3}\mu G_4 \frac{3 - 3P_z^2 - 2E}{J^2 k^2}$$

or

$$\varepsilon_{mon} = 4\pi\eta^2 G_4 + \frac{2\pi\eta^2 G_4}{3k^2}\frac{(3-E)}{J^2}\left[\ln\left(\frac{4kJ}{\sqrt{1-E}}\right) - 1\right].$$

These solutions represent a scattering state when particle trajectory is deflected by an angle $\pi\varepsilon$.

Let us consider in more detail the case of a photon scattering. Let d be the distance between an observer and the top of a cone, and L – from the top of a cone and a source. Then, in the simplest case when the defect is positioned between the source and the observer, the visible image will be a circle with the angular magnitude equal to

$$\psi_{mon} = 8\pi^2 G_4\eta^2\frac{L}{L+d} + \frac{L+d}{4\pi^2 G_4\eta^2 k^2 d^2 L}\left[\ln\left(\frac{16\pi^2 G_4\eta^2 kdL}{L+d}\right) - 1\right].$$

And as in the four-dimensional case, if the source, monopole and the observer are misaligned, we will see two images separated by the same angle.

In the case of a string, if the line of sight makes an angle $\theta = \pi/2$ with the string, then the observer will see two images with the angular separation

$$\psi_{str} = 8\pi\mu G_4\frac{L}{L+d} + \frac{L+d}{4Ld^2\pi\mu G_4 k^2}\ .$$

In both cases the first term corresponds to the standard result of the nonbrane cosmology [6, 7] , while the second one depends on the parameter k and gives the brane corrections.

5 Conclusion

We examined the linearized gravitational field around conical defects in the Randall-Sundrum brane world scenario. We showed that brane corrections give defects an attractive potential, in contrast to standard cosmic strings and monopoles. At astronomical distances associated gravitational force is very weak. So significant effects may occur near the defect core. Like their four-dimensional counterparts both spaces are conical at large radii. However possibility of gravity to propagate in extra dimension deforms the cones. This property slightly modifies the effect of gravitational lensing. The closer the light rays pass to the defect, the more noticeable the difference.

Acknowledgments

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