Large H/A mixing in CP-violating MSSM[1]

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Abstract

In supersymmetric models the two heavy neutral Higgs bosons are nearly degenerate in the decoupling limit. If the theory is CP-noninvariant, the mixing between the states can strongly be affected by the decay widths. The formalism describing this situation is described, and some interesting experimental consequences at a $\gamma\gamma$ collider are discussed.

1 Introduction

The Minimal Supersymmetric extension of the Standard Model (MSSM) is a specific realization of general scenarios which include two doublets in the Higgs sector. After three fields are absorbed by electroweak gauge bosons, the remaining five fields give rise to physical particles. In CP-invariant theories, besides the charged states, two of the three neutral states are CP-even, while the third is CP-odd. However, the MSSM offers new sources of CP violation, which affect the Higgs potential at the loop-level. In such CP-noninvariant theories the three neutral states mix to form a triplet with even and odd components in the wave–functions under CP transformations [1]-[5]. The mixing can become very large if the states are nearly mass-degenerate. This situation is naturally realized for supersymmetric theories in the decoupling limit in which two of the neutral states are heavy.

First, the H/A mixing is discussed in a simple quantum mechanical formalism stressing the importance of absorptive loop-induced off-diagonal terms. Since a photon collider offers unique capabilities to study CP violation, some experimental signatures of CP-mixing in Higgs production and decay processes at $\gamma\gamma$ collisions are discussed at the end.

2 H/A Mixing Formalism

The most general form of the self-interaction of two Higgs doublets in a CP–noninvariant theory is described by the potential [6]

$$\begin{aligned}\mathcal{V} \;=\;& m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}]\\ &+\tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\ &+\left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\}\end{aligned} \tag{1}$$

where $\Phi_{1,2}$ denote two complex $Y = 1$, $\mathrm{SU}(2)_L$ iso-doublet scalar fields. The coefficients are in general all non–zero, and in a CP-violating scenario the parameter m_{12}^2 and (loop-induced in the MSSM) $\lambda_{5,6,7}$ can be complex:

$$m_{12}^2 = m_{12}^{2R} + im_{12}^{2I}, \quad \lambda_{5,6,7} = \lambda_{5,6,7}^R + i\lambda_{5,6,7}^I \tag{2}$$

The scalar fields are assumed to develop non-zero vacuum expectation values

$$\langle\Phi_1\rangle = \frac{v_1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{v_2}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} \tag{3}$$

where, without loss of generality, the two vacuum expectation values v_i $[i = 1, 2]$ can be chosen real and positive. As usual, $v = (v_1^2 + v_2^2)^{-1/2} = 2^{-1/4}G_F^{-1/2} \approx 246$ GeV.

It is useful to rotate the Higgs fields $\Phi_{1,2}$ to the $\Phi_{a,b}$ basis,

$$\begin{aligned}\Phi_a &= \cos\beta\,\Phi_1 + \sin\beta\,\Phi_2\\ \Phi_b &= -\sin\beta\,\Phi_1 + \cos\beta\,\Phi_2\end{aligned} \tag{4}$$

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where $\tan\beta = v_2/v_1$, and the abbreviations $t_\beta = \tan\beta$, $c_\beta = \cos\beta$, $s_{2\beta} = \sin 2\beta$ etc, will be used from now on. In this basis only the field Φ_a develops a non-zero vacuum expectation value

$$\Phi_a = \left(\begin{array}{c} G^+ \\ \frac{1}{\sqrt{2}}\left(v + H_a + iG^0\right) \end{array} \right), \quad \Phi_b = \left(\begin{array}{c} H^+ \\ \frac{1}{\sqrt{2}}\left(H_b + iA\right) \end{array} \right) \tag{5}$$

and the three fields $G^{\pm,0}$ are identified as the would-be Goldstone bosons, while $H^\pm, H_{a,b}$ and A give rise to physical Higgs bosons. The charged Higgs mass $M_{H^\pm}$ and the real mass matrix $\mathcal{M}_0^2$ of neutral Higgs fields in this basis can easily be derived from the potential (1):

$$M_{H^\pm}^2 = M_A^2 + \tfrac{1}{2}\, v^2(\lambda_5^R - \lambda_4) \tag{6}$$

$$\mathcal{M}_0^2 = v^2 \left(\begin{array}{ccc} \lambda & -\hat{\lambda} & -\hat{\lambda}_p \\ -\hat{\lambda} & \lambda - \lambda_A + M_A^2/v^2 & -\lambda_p \\ -\hat{\lambda}_p & -\lambda_p & M_A^2/v^2 \end{array} \right) \tag{7}$$

after eliminating m_{11}^2, m_{22}^2, m_{12}^{2I} from the minimization conditions, and exchanging the real part of m_{12}^2 for the *auxiliary* parameter M_A^2. It is defined by the relation

$$m_{12}^{2R} = \tfrac{1}{2}(M_A^2 s_{2\beta} + v^2(\lambda_5^R s_{2\beta} + \lambda_6^R c_\beta^2 + \lambda_7^R s_\beta^2)) \tag{8}$$

and it will be one of the key parameters in the system. The λ, $\hat{\lambda}$ and λ_A parameters are functions of the real parts, while λ_p and $\hat{\lambda}_p$ are functions of the imaginary parts of the λ_i couplings in (1); their explicit form can be found in [7].

In a CP–invariant theory all λ_i couplings are real and the off-diagonal elements $\lambda_p, \hat{\lambda}_p$ vanish. Thus the neutral mass matrix breaks into the CP-even 2×2 part, and the [stand–alone] CP-odd part. The 2×2 part gives rise to two CP-even neutral mass eigenstates h and H, while the parameter M_A is identified as the mass of the CP-odd Higgs boson A. In the CP-violating case, however, all three H_a, H_b and A states mix leading to H_1, H_2 and H_3 mass eigenstates with indefinite CP-properties.

If the mass differences become small, the mixing of the states is strongly affected by the widths of the states. This is a well known phenomenon for resonance mixing [8] and has recently also been recognized for the Higgs sector [9]. The hermitian mass matrix (7) has therefore to be supplemented by the anti-hermitian part $-iM\Gamma$ incorporating the decay matrix [10]

$$\mathcal{M}^2 = \mathcal{M}_0^2 - iM\Gamma \tag{9}$$

This matrix includes the widths of the Higgs states in the diagonal elements as well as the transition elements within any combination of pairs (ij). They are particularly important in the case of nearly mass-degenerate states. All these elements $(M\Gamma)_{ij}^{AB}$ are built up by loops of the fields (AB) in the self-energy matrix (ij) of the Higgs fields.

In general, the Higgs bosons, the fermions and electroweak gauge bosons, as well as gauginos, higgsinos and scalar states in supersymmetric theories, contribute to the loops in the 3×3 Higgs propagator matrix. In Ref.[7] a simple quantum mechanical formalism has been developed for treating this configuration. In the physically interesting case of decoupling, the mixing structure simplifies considerably allowing for a very simple and transparent approach. Alternative approach requires a full coupled-channel analysis [11].

2.1 Decoupling limit

The decoupling limit [6] is defined by the inequality $M_A^2 \gg |\lambda_i|\, v^2$, with $|\lambda_i| \lesssim O(1)$. In this limit the H_a state becomes the CP-even light Higgs boson h and decouples from H_a and A. The heavy states $H{=}H_b$ and A, as well as $H^\pm$, are nearly mass degenerate, which turns to be crucial for large mixing effects between A and H, analyzed here. It is therefore enough to consider a lower–right 2×2 submatrix of (7) for the heavy H/A states which we write as follows

$$\mathcal{M}_{HA}^2 = \left(\begin{array}{cc} M_H^2 - iM_H\Gamma_H & (M\Gamma)_{HA} \\ (M\Gamma)_{HA} & M_A^2 - iM_A\Gamma_A \end{array} \right) \tag{10}$$

Moreover, the couplings of the heavy Higgs bosons to gauge bosons and their supersymmetric partners are suppressed. In the case of all supersymmetric particle contributions to be suppressed either by couplings or by phase space in $M\Gamma$, it is sufficient to consider only loops built up by the light Higgs boson and the top quark (loops from other (s)particles could be treated in the same way of course) for which we find

$$(M\Gamma)^{hh}_{HH/AA} = \frac{\beta_h}{32\pi} g^2_{Hhh/Ahh} \qquad (M\Gamma)^{hh}_{HA/AH} = \frac{\beta_h}{32\pi} g_{Hhh} g_{Ahh} \tag{11}$$

$$(M\Gamma)^{tt}_{HH/AA} = \frac{3M^2_{H/A}}{8\pi}\,\beta_t\, g^{tt}_{HH/AA} \qquad (M\Gamma)^{tt}_{HA/AH} = \frac{3M^2_{H/A}}{8\pi}\,\beta_t\, g^{tt}_{HA/AH} \tag{12}$$

where β_h (β_t) denotes the velocity of the light Higgs boson h (top quark) in the decays $H/A \to hh$ ($H/A \to t\bar{t}$) [with the heavy Higgs bosons assumed to be mass–degenerate]. The CP–conserving Hhh and CP–violating Ahh couplings in this limit, and for moderately large $\tan\beta$, take particularly simple form

$$g_{Hhh} = -3v\,\mathrm{Re}\lambda_7 \qquad g_{Ahh} = +3v\,\mathrm{Im}\lambda_7 \tag{13}$$

For the top quark loop

$$\begin{aligned} g^{tt}_{HH} &= \beta_t^2 s_H^2 + p_H^2 \qquad\qquad g^{tt}_{AA} = \beta_t^2 s_A^2 + p_A^2 \\ g^{tt}_{HA} &= g^{tt}_{AH} = \beta_t^2 s_H s_A + p_H p_A \end{aligned} \tag{14}$$

where the Htt and Att terms are defined by the Lagrangian $\mathcal{L}_t = H\bar{t}(s_H + i\gamma_5 p_H)t + A\bar{t}(s_A + i\gamma_5 p_A)t$ with the CP–conserving couplings[2] s_H, p_A and the CP–violating couplings p_H, s_A.The loops also contribute to the real part of the mass matrix, either renormalizing the λ parameters of the Higgs potential or generating such parameters if not present yet at the tree level.

2.2 Physical masses and states

The symmetric complex mass-squared matrix $\mathcal{M}^2$ in (10) can be diagonalized through a *rotation by a complex mixing angle*:

$$\mathcal{M}^2_{H_iH_j} \equiv \begin{pmatrix} M^2_{H_2} - iM_{H_2}\Gamma_{H_2} & 0 \\ 0 & M^2_{H_3} - iM_{H_3}\Gamma_{H_3} \end{pmatrix} = C\mathcal{M}^2_{HA}C^{-1} \tag{15}$$

where the mixing matrix and the mixing angle are given as

$$C = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad X = \frac{1}{2}\tan 2\theta = \frac{(M\Gamma)_{HA}}{M^2_H - M^2_A - i\,[M_H\Gamma_H - M_A\Gamma_A]} \tag{16}$$

A non–vanishing (complex) mixing parameter $X \neq 0$ requires CP–violating transitions between H and A either in the real mass matrix, $\lambda_p \neq 0$, or in the decay mass matrix, $\Gamma_{HA} \neq 0$, [or both]. Note however, that even for nearly degenerate masses of M_H and M_A, the mixing could be suppressed if the widths were significantly different. As a result, the mixing phenomena are strongly affected by the form of the decay matrix $M\Gamma$. Since the difference of the widths enters through the denominator in X, the modulus $|X|$ becomes large for small differences, and small widths in general.

The mixing shifts the Higgs masses and widths in a characteristic pattern [8]. The two complex mass values after and before diagonalization are related by the complex mixing angle θ:

$$M^2_{H_3} - M^2_{H_2} - i(M_{H_3}\Gamma_{H_3} - M_{H_2}\Gamma_{H_2}) = [M^2_A - M^2_H - i(M_A\Gamma_A - M_H\Gamma_H)] \times \sqrt{1+4X^2} \tag{17}$$

Since the eigenstates of the complex, non–hermitian matrix $\mathcal{M}^2_{HA}$ in (10) are no longer orthogonal, the `ket` and `bra` states have to defined instead:

$$\begin{aligned} |H_2\rangle &= \cos\theta\,|H\rangle + \sin\theta\,|A\rangle, \qquad \langle\widetilde{H}_2| = \cos\theta\,\langle H| + \sin\theta\,\langle A| \\ |H_3\rangle &= -\sin\theta\,|H\rangle + \cos\theta\,|A\rangle, \qquad \langle\widetilde{H}_3| = -\sin\theta\,\langle H| + \cos\theta\,\langle A| \end{aligned} \tag{18}$$

[2] In the type–II class of CP–conserving 2–Higgs doublet models, to which the MSSM belongs, the two couplings s_H, p_A approach equal values, $\simeq \cot\beta\, m_t/v$, for large Higgs masses.

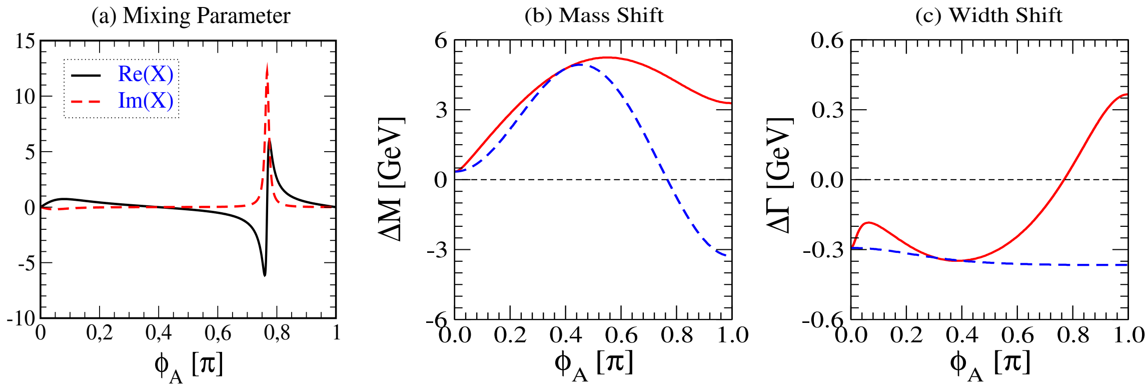

Figure 1: *The ϕ_A dependence of (a) the mixing parameter X and of the shifts of (b) masses and (c) widths in a SUSY model with the CP–violating phase ϕ_A evolving from 0 to π for $\tan\beta = 5$, $M_A = 0.5$ TeV and couplings as specified in the text; in (b,c) the mass and width differences without mixing are shown by the broken lines. $\Re\text{e}/\Im\text{m}X(2\pi - \phi_A) = +\Re\text{e}/-\Im\text{m}X(\phi_A)$ for angles above π.*

For example, the final state F in heavy Higgs formation from the initial state I is generated with the amplitude[3]

$$\langle F|H|I\rangle = \sum_{i=2,3} \langle F|H_i\rangle \, \frac{1}{s - M_{H_i}^2 + iM_{H_i}\Gamma_{H_i}} \, \langle \widetilde{H}_i|I\rangle \tag{19}$$

where the sum runs only over diagonal transitions.

3 Experimental Signatures

To illustrate these general quantum mechanical results in a realistic example, we adopt a specific MSSM scenario with the source of CP–violation localized in the complex trilinear coupling A_t of the soft supersymmetry breaking part of the potential involving the top squark.[4] All other interactions are assumed to be CP–conserving. For $\phi_A \neq 0$, π, the stop–loop corrections induce the CP–violation in the effective Higgs potential (1). The effective λ_i parameters have been calculated in Ref.[3] to two–loop accuracy; to illustrate the crucial points we take the dominant one–loop $t/\tilde{t}$ contributions.

More specifically, we take a typical set of parameters from Refs.[4, 13],

$$M_S = 0.5 \text{ TeV}, \quad |A_t| = 1.0 \text{ TeV}, \quad \mu = 1.0 \text{ TeV}; \quad \tan\beta = 5 \tag{20}$$

and change the phase ϕ_A of the trilinear parameter A_t. With $\phi_A = 0$ we find the following values of the light and heavy Higgs masses and decay widths, and the stop masses:

$$\begin{aligned} M_h &= 129.6 \text{ GeV}, \quad & M_H &= 500.3 \text{ GeV}, \quad & M_A &= 500.0 \text{ GeV} \\ & & \Gamma_H &= 1.2 \text{ GeV}, & \Gamma_A &= 1.5 \text{ GeV} \\ m_{\tilde{t}_1} &= 372 \text{ GeV}, & m_{\tilde{t}_2} &= 647 \text{ GeV} & & \end{aligned} \tag{21}$$

Clearly, with the mass splitting of 0.3 GeV, the heavy Higgs states are not distinguishable. When the phase ϕ_A is turned on,[5] the CP composition, the masses and the decay widths of heavy states are strongly affected, as shown in Figs. 1(a), (b) and (c), while the mass of the light Higgs boson h is not. The heavy two–state system shows a very sharp resonant CP–violating mixing, purely imaginary a little above

[3]Small off-resonance transitions of heavy Higgs bosons H and A to the light Higgs boson h (and to the neutral would-be Goldstone G^0) can be neglected in the decoupling limit to a good approximation.

[4]This assignment is compatible with the bounds on CP–violating SUSY phases derived from experiments on electric dipole moments [12].

[5]With one phase ϕ_A, the complex mixing parameter X obeys the relation $X(2\pi - \phi_A) = X^*(\phi_A)$, implying all CP–even quantities symmetric and all CP–odd quantities anti–symmetric about π.

$\phi_A = 3\pi/4$, Fig. 1(a). The mass shift is enhanced by more than an order of magnitude if the CP–violating phase rises to non-zero values, reaching a maximal value of ~ 5.3 GeV; the width shift changes between -0.3 and $+0.4$ GeV. As a result, the two mass–eigenstates should become clearly distinguishable at future colliders, in particular at a photon collider [14]. Moreover, both states have significant admixtures of CP–even and CP–odd components in the wave–functions. Since $\gamma\gamma$ colliders offer unique conditions for probing the CP–mixing [15, 16, 17], we discuss two experimental examples: Higgs formation in polarised $\gamma\gamma$ collisions and polarisation of top quarks in Higgs decays, where spectacular signatures of resonant mixing can be expected.

3.1 Higgs formation $\gamma\gamma \to H_{2,3}$ with polarized beams

The amplitude of the reaction $\gamma\gamma \to H_i \to F$ is a superposition of H_2 and H_3 exchanges. For equal helicities $\lambda = \pm 1$ of the two photons, the amplitude reads

$$\mathcal{M}^F_\lambda = \sum_{i=2,3} \langle F|H_i\rangle \frac{1}{s - M^2_{H_i} + iM_{H_i}\Gamma_{H_i}} \left[S^\gamma_i(s) + i\lambda P^\gamma_i(s)\right] \tag{22}$$

where $\sqrt{s}$ is the $\gamma\gamma$ energy and the loop–induced $\gamma\gamma H_i$ scalar and pseudoscalar form factors, $S^\gamma_i(s)$ and $P^\gamma_i(s)$, are related to the well–known conventional $\gamma\gamma H/A$ form factors, $S^\gamma_{H,A}$ and $P^\gamma_{H,A}$, by

$$\begin{aligned} S^\gamma_2 &= \cos\theta\, S^\gamma_H + \sin\theta\, S^\gamma_A \qquad & S^\gamma_3 &= -\sin\theta\, S^\gamma_H + \cos\theta\, S^\gamma_A \\ P^\gamma_2 &= \cos\theta\, P^\gamma_H + \sin\theta\, P^\gamma_A \qquad & P^\gamma_3 &= -\sin\theta\, P^\gamma_H + \cos\theta\, P^\gamma_A \end{aligned} \tag{23}$$

Explicit forms of the loop functions $S^\gamma_{H,A}$ and $P^\gamma_{H,A}$ can be found, for example, in Ref.[13]. In our scenario the Higgs–tt couplings are assumed to be CP–conserving, implying negligible top-loop contributions to P^γ_H and S^γ_A since the gluino mass is sufficiently heavy compared with the stop masses [5], while the $\tilde{t}_1$ loop generates a non-negligible CP-violating amplitude S^γ_A. In the region of strong mixing on which we focus, however, the CP-violating vertex corrections have only a small effect on the experimental asymmetries compared with the large impact of CP-violating Higgs-boson mixing.

Polarised photons provide a very powerful tool to investigate the CP properties of Higgs bosons. With linearly polarized photons one can project out the CP–even and CP–odd components of the H_i wave–functions by arranging the photon polarization vectors to be parallel or perpendicular. On the other hand, circular polarisation provides us with a direct insight into the CP-violating nature of Higgs bosons. Two asymmetries are of interest

$$\mathcal{A}_{lin} = \frac{\sigma_\parallel - \sigma_\perp}{\sigma_\parallel + \sigma_\perp} \qquad \mathcal{A}_{hel} = \frac{\sigma_{++} - \sigma_{--}}{\sigma_{++} + \sigma_{--}} \tag{24}$$

where $\sigma_\parallel$, $\sigma_\perp$ and σ_{++}, σ_{--} are the corresponding total $\gamma\gamma$ fusion cross sections for linear and circular polarisations, respectively. Though CP–even, the asymmetry $\mathcal{A}_{lin}$ can serve as a powerful tool nevertheless to probe CP–violating admixtures to the Higgs states since $|\mathcal{A}_{lin}| < 1$ requires both S^γ_i and P^γ_i non-zero couplings. A more direct probe of CP–violation due to H/A mixing is provided by the CP–odd (and also CP$\tilde{\mathrm{T}}$–odd) asymmetry $\mathcal{A}_{lin}$. The CP-properties of the asymmetries $\mathcal{A}_{lin}$ and $\mathcal{A}_{hel}$ are clearly seen from the expressions on top of the $H_i[i = 2, 3]$ mass–poles in the narrow width approximation:

$$\mathcal{A}_{lin}[H_i] \approx \frac{|S^\gamma_i|^2 - |P^\gamma_i|^2}{|S^\gamma_i|^2 + |P^\gamma_i|^2} \qquad \mathcal{A}_{hel}[H_i] \approx \frac{2\,\Im\mathrm{m}(S^\gamma_i P^{\gamma *}_i)}{|S^\gamma_i|^2 + |P^\gamma_i|^2} \tag{25}$$

where they are written directly in terms of the form factors.

Fig.2 show the ϕ_A dependence of the asymmetries $\mathcal{A}_{lin}$ and $\mathcal{A}_{hel}$ at the poles of H_2 and of H_3, respectively, for the same parameter set as in Fig.1 and with the common SUSY scale $M_{\tilde{Q}_3} = M_{\tilde{t}_R} = M_S = 0.5$ TeV for the soft SUSY breaking top squark mass parameters. By varying the $\gamma\gamma$ energy from below M_{H_3} to above M_{H_2}, the asymmetries, $\mathcal{A}_{lin}$ (blue solid line) and $\mathcal{A}_{hel}$ (red dashed line), vary from -0.39 to 0.34 and from -0.29 to 0.59, respectively, as demonstrated on the right panel of Fig.2 with $\phi_A = 3\pi/4$, a phase value close to resonant CP–mixing.

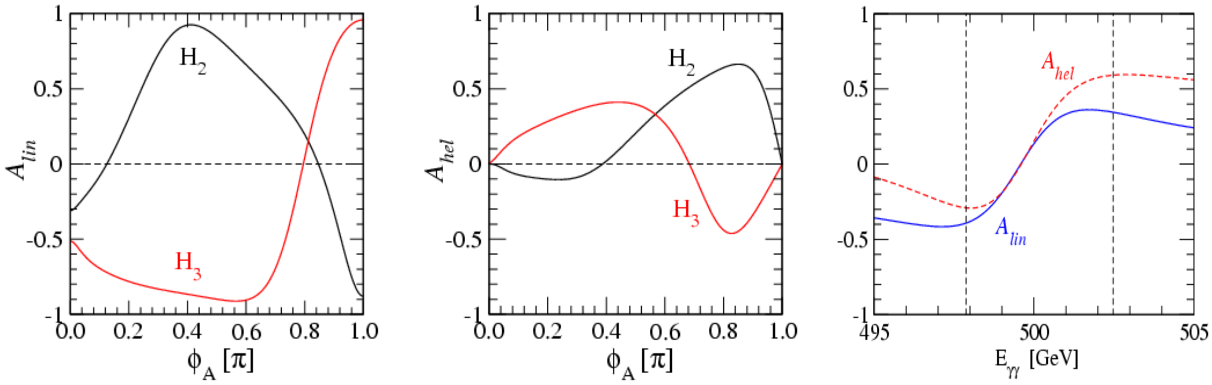

Figure 2: *The ϕ_A dependence of the CP–even and CP–odd correlators, $\mathcal{A}_{lin}$ (left panel) and $\mathcal{A}_{hel}$ (center panel), at the poles of H_2 and H_3, respectively, and the $\gamma\gamma$ energy dependence (right panel) of the correlators, $\mathcal{A}_{lin,hel}$, for $\phi_A = 3\pi/4$ in the production process $\gamma\gamma \to H_i$ in the limit in which H/A mixing is the dominant CP–violating effect. The same parameter set as in Fig.1 is employed. Numerically, $M_{H_2} = 502.5$ GeV, $M_{H_3} = 497.9$ GeV, $\Gamma_{H_2} = 1.28$ GeV and $\Gamma_{H_3} = 1.31$ GeV. The vertical lines on the right panel represent the two mass eigenvalues, M_{H_3} and M_{H_2}.*

3.2 Top quark polarisation in $H_{2,3} \to t\bar{t}$ decay

A second observable of interest is the polarisation of the top quarks in H_i decays produced by $\gamma\gamma$ fusion or elsewhere in various production processes at an e^+e^- linear collider and LHC $H_{2,3} \to t\bar{t}$. Even if the H/Att couplings are [approximately] CP–conserving, the complex rotation matrix C may mix the CP–even H and CP–odd A states leading to CP–violation. In the production–decay process $\gamma\gamma \to H_i \to t\bar{t}$, two correlators between the transverse t and $\bar{t}$ polarization vectors $s_\perp, \bar{s}_\perp$

$$\mathcal{C}_\parallel = \langle s_\perp \cdot \bar{s}_\perp \rangle \qquad \text{and} \qquad \mathcal{C}_\perp = \langle \hat{p}_t \cdot (s_\perp \times \bar{s}_\perp) \rangle \tag{26}$$

can be extracted from the correlation between the two decay planes $t \to bW^+$ and $\bar{t} \to \bar{b}W^-$:

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\phi^*} = \frac{1}{2\pi}\left[1 - \frac{\pi^2}{16}\left(\frac{1-2m_W^2/m_t^2}{1+2m_W^2/m_t^2}\right)^2 \left(\mathcal{C}_\parallel \cos\phi^* + \mathcal{C}_\perp \sin\phi^*\right)\right] \tag{27}$$

where ϕ^* denotes the azimuthal angle between two decay planes [15]. In terms of the helicity amplitudes $\langle\sigma,\lambda\rangle$ for the process $\gamma\gamma \to H_i \to t\bar{t}$, where $\lambda = \pm 1$ denotes the helicities of both photons and $\sigma = \pm 1/2$ the helicities of both top quarks, the asymmetries are given as

$$\mathcal{C}_\parallel = -\frac{2\,\Re\mathrm{e}\sum\langle +,\lambda\rangle\langle -,\lambda\rangle^*}{\sum\left(|\langle +,\lambda\rangle|^2 + |\langle -,\lambda\rangle|^2\right)} \qquad \mathcal{C}_\perp = +\frac{2\,\Im\mathrm{m}\sum\langle +,\lambda\rangle\langle -,\lambda\rangle^*}{\sum\left(|\langle +,\lambda\rangle|^2 + |\langle -,\lambda\rangle|^2\right)} \tag{28}$$

with the sums running over the two photon helicities. Clearly, $\mathcal{C}_\parallel$ is CP–even and $\mathcal{C}_\perp$ is CP–odd.

Fig.3 shows the ϕ_A dependence of the CP–even and CP–odd asymmetries, $\mathcal{C}_\parallel$ and $\mathcal{C}_\perp$, at the poles of H_2 and of H_3, left and center panels respectively. If the invariant $t\bar{t}$ energy is varied throughout the resonance region, the correlators $\mathcal{C}_\parallel$ (blue solid line) and $\mathcal{C}_\perp$ (red dashed line) vary characteristically from -0.43 to -0.27 [non–uniformly] and from 0.84 to -0.94, respectively, as shown in the right panel of Fig.3.

4 CONCLUSIONS

Exciting mixing effects can occur in the supersymmetric Higgs sector if CP–noninvariant interactions are present. In the decoupling regime these effects can become very large, leading to interesting experimental consequences. Higgs formation in $\gamma\gamma$ collisions with polarised beams proves particularly interesting for observing such effects. However, exciting experimental effects are also predicted in such scenarios for $t\bar{t}$ final–state analyses in decays of the heavy Higgs bosons at LHC and in the e^+e^- mode of linear colliders.

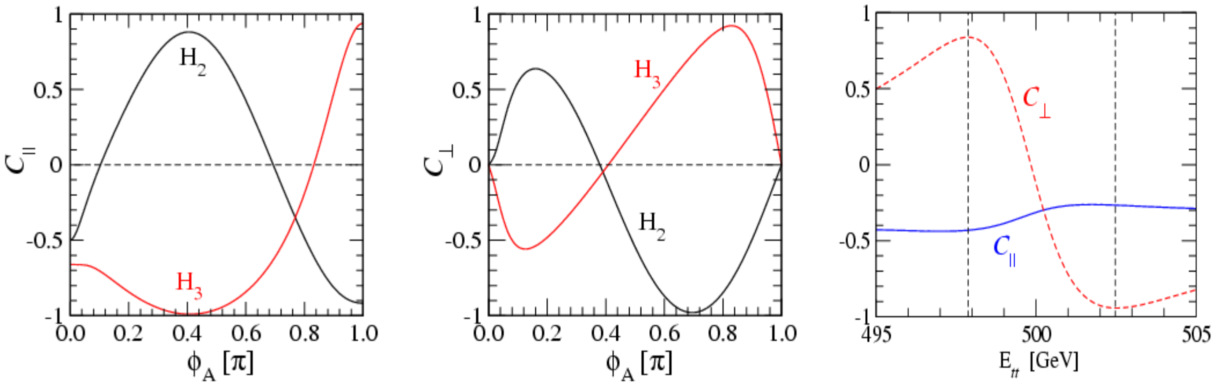

Figure 3: *The ϕ_A dependence of the CP–even and CP–odd correlators, $\mathcal{C}_\parallel$ (left panel) and $\mathcal{C}_\perp$ (center panel), at the pole of H_2 and H_3 and the invariant $t\bar{t}$ energy dependence (right panel) of the correlators $\mathcal{C}_{\parallel,\perp}$ for $\phi_A = 3\pi/4$ in the production–decay chain $\gamma\gamma \to H_i \to t\bar{t}$. [Same SUSY parameter set as in Fig.2.]*

Detailed experimental simulations would be needed to estimate the accuracy with which the asymmetries presented here can be measured. Though not easy to measure, the large magnitude and the rapid, significant variation of the CP–even and CP–odd asymmetries through the resonance region with respect to both the phase ϕ_A and the $\gamma\gamma$ energy would be a very interesting effect to observe in any case.

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