Interacting Rarita-Schwinger field and its spin-parity content

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Abstract

We obtain in analytical form the dressed propagator of the massive Rarita-Schwinger field taking into account all spin components. The nearest analogy for dressing in the $s = 1/2$ Rarita-Schwinger sector is the dressing of two fermions of opposite parity with presence of the mutual transitions. The calculation of the self-energy contributions confirms that besides $s = 3/2$ component the Rarita-Schwinger field contains also two $s = 1/2$ components of opposite parity.

1 Introduction

The covariant description of the spin-3/2 particles is usually based on the Rarita-Schwinger formalism [1] where the main object is the spin-vector field Ψ^μ. However in addition to spin-3/2 this field contains extra spin-1/2 components and it generates the main difficulties in its description [2, 3]. The problem has a long history, we mention here only relatively recent works [4, 5, 6, 7, 8, 9] which contain discussion of the problem and historical review.

The expression for the free propagator $G_0^{\mu\nu}$ is well known (see e.g. [10, 11, 12]), thus we do not present it here. As concerned for the dressed propagator, its construction is a more complicated issue and its total expression is unknown up to now. Thus a practical use of $G^{\mu\nu}$ (in particular for the case of $\Delta(1232)$ production) needs some approximation in its description. The standard approximation [13, 14] consist in a dressing the spin-3/2 components only while the rest components of $G^{\mu\nu}$ can be neglected or considered as bare. Another way to take into account the spin-1/2 components is a numerical solution of the appearing system of equations[4, 9]. In Ref. [13] it was noticed that the spin-1/2 components are necessary to reproduce the experimental data on the $\Delta(1232)$ production in Compton scattering. So the correct account of the extra spin-1/2 component in the Ψ^μ field has also a practical meaning.

Here we derive an analytical expression for the interacting Rarita-Schwinger field's propagator with accounting all spin components and discuss its properties. It turned out that the spin-1/2 part of the dressed propagator has rather compact form, and a crucial point for its deriving is the choosing of a suitable basis. Short variant of this paper was published in [15].

2 Dressed propagator of the Rarita-Schwinger field

The Dyson-Schwinger equation for the propagator of the Rarita-Schwinger field has the following form

$$G^{\mu\nu} = G_0^{\mu\nu} + G^{\mu\alpha} J^{\alpha\beta} G_0^{\beta\nu}. \tag{1}$$

Here $G_0^{\mu\nu}$ and $G^{\mu\nu}$ are the free and full propagators respectively, $J^{\mu\nu}$ is a self-energy contribution. The equation may be rewritten for inverse propagators as

$$(G^{-1})^{\mu\nu} = (G_0^{-1})^{\mu\nu} - J^{\mu\nu}. \tag{2}$$

If we consider the self-energy $J^{\mu\nu}$ as a known value [1], than the problem is reduced to reversing of relation (2). It is useful to have a basis for both propagators and self-energy for this procedure.

1) The most natural basis for the spin-tensor $S^{\mu\nu}(p)$ decomposition is the γ-matrix one:

$$\begin{aligned} S^{\mu\nu}(p) =& g^{\mu\nu} \cdot s_1 + p^\mu p^\nu \cdot s_2 + +\hat{p} p^\mu p^\nu \cdot s_3 + \hat{p} g^{\mu\nu} \cdot s_4 + p^\mu \gamma^\nu \cdot s_5 + \\ & \gamma^\mu p^\nu \cdot s_6 + +\sigma^{\mu\nu} \cdot s_7 + \sigma^{\mu\lambda} p^\lambda p^\nu \cdot s_8 + \sigma^{\nu\lambda} p^\lambda p^\mu \cdot s_9 + \gamma^\lambda \gamma^5 \imath \varepsilon^{\lambda\mu\nu\rho} p^\rho \cdot s_{10}. \end{aligned} \tag{3}$$

Here $S^{\mu\nu}$ is an arbitrary spin-tensor depending on the momentum p, $s_i(p^2)$ are the Lorentz invariant coefficients, and $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$. Altogether there are ten independent components in the decomposition

[1] That is the widely used in the resonance physics "rainbow approximation", see *e.g.* recent review[16]

of $S^{\mu\nu}(p)$. It is known that the γ-matrix decomposition is complete and the coefficients s_i are free of kinematical singularities and constraints. However this basis is inconvenient at multiplication and reversing of the spin-tensor $S^{\mu\nu}(p)$ because the basis elements are not orthogonal to each other. As a result the reversing of the spin-tensor $S^{\mu\nu}(p)$ leads to a system of 10 equations for the coefficients.
2) There is another basis used in consideration of the dressed propagator [4, 9, 13] $G^{\mu\nu}$. It is constructed using the following set of operators [12, 13, 17]

$$\begin{aligned}
(\mathcal{P}^{3/2})^{\mu\nu} =& g^{\mu\nu} - \frac{2}{3}\frac{p^\mu p^\nu}{p^2} - \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{1}{3p^2}(\gamma^\mu p^\nu - \gamma^\nu p^\mu)\hat{p},\\
(\mathcal{P}^{1/2}_{11})^{\mu\nu} =& \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{1}{3}\frac{p^\mu p^\nu}{p^2} - \frac{1}{3p^2}(\gamma^\mu p^\nu - \gamma^\nu p^\mu)\hat{p},\\
(\mathcal{P}^{1/2}_{22})^{\mu\nu} =& \frac{p^\mu p^\nu}{p^2},\\
(\mathcal{P}^{1/2}_{21})^{\mu\nu} =& \sqrt{\frac{3}{p^2}}\cdot\frac{1}{3p^2}(-p^\mu + \gamma^\mu\hat{p})\hat{p}p^\nu,\\
(\mathcal{P}^{1/2}_{12})^{\mu\nu} =& \sqrt{\frac{3}{p^2}}\cdot\frac{1}{3p^2}p^\mu(-p^\nu + \gamma^\nu\hat{p})\hat{p}.
\end{aligned} \tag{4}$$

Here $\mathcal{P}^{3/2}$,$\mathcal{P}^{1/2}_{11}$,$\mathcal{P}^{1/2}_{22}$ are the projection operators while $\mathcal{P}^{1/2}_{21}$,$\mathcal{P}^{1/2}_{12}$ are nilpotent. As for their physical meaning, it is clear that $\mathcal{P}^{3/2}$ corresponds to spin-3/2. The remaining operators should describe two spin-1/2 representations and transitions between them.

The set of operators (4) can be used to decompose the considered spin-tensor as following [4, 9]:

$$\begin{aligned}
S^{\mu\nu}(p) \quad = \quad & (S_1 + S_2\hat{p})(\mathcal{P}^{3/2})^{\mu\nu} + (S_3 + S_4\hat{p})(\mathcal{P}^{1/2}_{11})^{\mu\nu} + (S_5 + S_6\hat{p})(\mathcal{P}^{1/2}_{22})^{\mu\nu} +\\
& (S_7 + S_8\hat{p})(\mathcal{P}^{1/2}_{21})^{\mu\nu} + (S_9 + S_{10}\hat{p})(\mathcal{P}^{1/2}_{12})^{\mu\nu}.
\end{aligned} \tag{5}$$

Let us call this basis as $\hat{p}$-basis. It is more convenient at multiplication since the spin-3/2 components $\mathcal{P}^{3/2}$ have been separated from spin-1/2 ones. However, the spin-1/2 components as before are not orthogonal between themselves and we come to a system of 8 equations when inverting the (2). Another feature of decomposition (5) is existence of the poles $1/p^2$ in different terms. So to avoid this unphysical singularity, we should impose some constraints on the coefficients at zero point.
3) Let us construct the basis which is the most convenient at multiplication of spin-tensors. This basis is built from the operators (4) and the projection operators $\Lambda^\pm$

$$\Lambda^\pm = \frac{\sqrt{p^2} \pm \hat{p}}{2\sqrt{p^2}}, \tag{6}$$

where we assume $\sqrt{p^2}$ to be the first branch of analytical function. Ten elements of this basis look as

$$\begin{aligned}
&\mathcal{P}_1 = \Lambda^+\mathcal{P}^{3/2}, && \mathcal{P}_3 = \Lambda^+\mathcal{P}^{1/2}_{11}, && \mathcal{P}_5 = \Lambda^+\mathcal{P}^{1/2}_{22}, && \mathcal{P}_7 = \Lambda^+\mathcal{P}^{1/2}_{21}, && \mathcal{P}_9 = \Lambda^+\mathcal{P}^{1/2}_{12},\\
&\mathcal{P}_2 = \Lambda^-\mathcal{P}^{3/2}, && \mathcal{P}_4 = \Lambda^-\mathcal{P}^{1/2}_{11}, && \mathcal{P}_6 = \Lambda^-\mathcal{P}^{1/2}_{22}, && \mathcal{P}_8 = \Lambda^-\mathcal{P}^{1/2}_{21}, && \mathcal{P}_{10} = \Lambda^-\mathcal{P}^{1/2}_{12},
\end{aligned} \tag{7}$$

where tensor indices are omitted. We will call (7) as the Λ-basis.

Decomposition of a spin-tensor in this basis has the following form:

$$S^{\mu\nu}(p) = \sum_{i=1}^{10} \mathcal{P}_i^{\mu\nu}\bar{S}_i(p^2). \tag{8}$$

The Λ-basis has very simple multiplicative properties which are represented in the Table 1.

Now we can return to the Dyson-Schwinger equation (2). Let us denote the inverse dressed propagator $(G^{-1})^{\mu\nu}$ and free one $(G_0^{-1})^{\mu\nu}$ by $S^{\mu\nu}$ and $S_0^{\mu\nu}$ respectively. Decomposing the $S^{\mu\nu}$, $S_0^{\mu\nu}$ and $J^{\mu\nu}$ in Λ-basis according to (8) we reduce the equation (2) to set of equations for the scalar coefficients

$$\bar{S}_i(p^2) = \bar{S}_{0i}(p^2) + \bar{J}_i(p^2), \qquad i = 1\ldots10.$$

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$	$\mathcal{P}_6$	$\mathcal{P}_7$	$\mathcal{P}_8$	$\mathcal{P}_9$	$\mathcal{P}_{10}$
$\mathcal{P}_1$	$\mathcal{P}_1$	0	0	0	0	0	0	0	0	0
$\mathcal{P}_2$	0	$\mathcal{P}_2$	0	0	0	0	0	0	0	0
$\mathcal{P}_3$	0	0	$\mathcal{P}_3$	0	0	0	$\mathcal{P}_7$	0	0	0
$\mathcal{P}_4$	0	0	0	$\mathcal{P}_4$	0	0	0	$\mathcal{P}_8$	0	0
$\mathcal{P}_5$	0	0	0	0	$\mathcal{P}_5$	0	0	0	$\mathcal{P}_9$	0
$\mathcal{P}_6$	0	0	0	0	0	$\mathcal{P}_6$	0	0	0	$\mathcal{P}_{10}$
$\mathcal{P}_7$	0	0	0	0	0	$\mathcal{P}_7$	0	0	0	$\mathcal{P}_3$
$\mathcal{P}_8$	0	0	0	0	$\mathcal{P}_8$	0	0	0	$\mathcal{P}_4$	0
$\mathcal{P}_9$	0	0	0	$\mathcal{P}_9$	0	0	0	$\mathcal{P}_5$	0	0
$\mathcal{P}_{10}$	0	0	$\mathcal{P}_{10}$	0	0	0	$\mathcal{P}_6$	0	0	0

Table 1: Properties of the Λ-basis at multiplication.

So the values $\bar{S}_i$ are defined by the bare propagator and the self-energy and may be considered as known.

After it the reversing of the $S_0^{\mu\nu}$ leads to equations for the coefficients $\bar{G}_i$:

$$\Big(\sum_{i=1}^{10}\mathcal{P}_i^{\mu\nu}\cdot\bar{G}_i(p^2)\Big)\cdot\Big(\sum_{k=1}^{10}\mathcal{P}_k^{\mu\nu}\cdot\bar{S}_k(p^2)\Big)=\sum_{i=1}^{6}\mathcal{P}_i^{\mu\nu}, \tag{9}$$

which are easy to solve due to simple multiplicative properties of $\mathcal{P}_i^{\mu\nu}$:

$$\begin{aligned}
&\bar{G}_1=1/\bar{S}_1,\quad \bar{G}_2=1/\bar{S}_2,\\
&\bar{G}_3=\bar{S}_6/\Delta_1,\quad \bar{G}_4=\bar{S}_5/\Delta_2,\quad \bar{G}_5=\bar{S}_4/\Delta_2,\quad \bar{G}_6=\bar{S}_3/\Delta_1,\\
&\bar{G}_7=-\bar{S}_7/\Delta_1,\quad \bar{G}_8=-\bar{S}_8/\Delta_2,\quad \bar{G}_9=-\bar{S}_9/\Delta_2,\quad \bar{G}_{10}=-\bar{S}_{10}/\Delta_1,
\end{aligned} \tag{10}$$

where $\Delta_1=\bar{S}_3\bar{S}_6-\bar{S}_7\bar{S}_{10}$, $\Delta_2=\bar{S}_4\bar{S}_5-\bar{S}_8\bar{S}_9$.

The $\bar{G}_1$, $\bar{G}_2$ terms which describe the spin-3/2 have the usual resonance form and could be obtained from (5) as well. As for $\bar{G}_3-\bar{G}_{10}$ coefficients which describe the spin-1/2 contributions, they have a more complicated structure.

3 Dressing the Dirac fermions

The obtained dressed propagator of the Rarita-Schwinger field has rather unusual structure, so we would like to clarify its physical meaning before renormalization. First of all it's useful to consider the dressing of Dirac fermions with aim to find the close analogy for Rarita-Schwinger field case.

3.1 Dressing the single fermion

The dressed fermion G(p) propagator is solution of the Dyson-Schwinger equation

$$G(p)=G_0+G\Sigma G_0, \tag{11}$$

where G_0 is the bare propagator and Σ is the self-energy contribution.

Decomposition of any matrix 4×4, depending on one momentum p, has the form:

$$S(p)=\sum_{M=1}^{2}\mathcal{P}_M\bar{S}^M,\qquad \mathcal{P}_1=\Lambda^+,\qquad \mathcal{P}_2=\Lambda^-, \tag{12}$$

where we introduced the new notations to stress an analogy with Rarita-Schwinger field.

Dyson-Schwinger equation in this basis takes the form:

$$\bar{G}^M=\bar{G}_0^M+\bar{G}^M\bar{\Sigma}^M\bar{G}_0^M,\qquad M=1,2, \tag{13}$$

and its solution is:

$$(\bar{G}^M)^{-1} = (\bar{G}_0^M)^{-1} - \bar{\Sigma}^M. \tag{14}$$

Let us look at the self-energy contribution $\Sigma(p)$. As an example we shall consider the dressing of baryon resonance N' ($J^P = 1/2^{\pm}$) due to interaction with πN system. Interaction lagrangian is of the form

$$L_{int} = g\overline{\Psi}\,'(x)\gamma^5\Psi(x)\cdot\phi(x) + h.c. \quad \text{for} \quad N' = 1/2^+ \tag{15}$$

and

$$L_{int} = g\overline{\Psi}\,'(x)\Psi(x)\cdot\phi(x) + h.c. \quad \text{for} \quad N' = 1/2^-. \tag{16}$$

Isotopical indexes are omitted since they are irrelevant here.

Positive parity baryon resonance

$$\Sigma(p) = ig^2 \int \frac{d^4k}{(2\pi)^4}\,\gamma^5 \frac{1}{\hat{p}+\hat{k}-m_N}\gamma^5 \frac{1}{k^2-m_\pi^2} = I\cdot A(p^2) + \hat{p}B(p^2) \tag{17}$$

Let us calculate the loop discontinuity through the Landau-Cutkosky rule: [2]

$$\Delta A = -\frac{ig^2 m_N}{(2\pi)^2} I_0, \qquad \Delta B = \frac{ig^2}{(2\pi)^2} I_0 \frac{p^2+m_N^2-m_\pi^2}{2p^2}. \tag{18}$$

Here I_0 is the base integral

$$I_0 = \int d^4k\,\delta\big(k^2-m_\pi^2\big)\delta\big((p+k)^2-m_N^2\big) = \theta\big(p^2-(m_N+m_\pi)^2\big)\frac{\pi}{2}\sqrt{\frac{\lambda\big(p^2,m_N^2,m_\pi^2\big)}{\big(p^2\big)^2}}, \tag{19}$$

where $\lambda\big(a,b,c\big) = \big(a-b-c\big)^2 - 4bc$.

From the parity conservation one can see that in the transition $N'(1/2^+) \to N(1/2^+) + \pi(0^-)$ the πN pair has the orbital momentum $l = 1$. But according to threshold quantum-mechanical theorems [19], the imaginary part of a loop should behave as q^{2l+1} at $q \to 0$ [3], which does not correspond to (18).

Let us calculate the imaginary part of Σ^M component according to (18)

$$\begin{aligned} Im\ \Sigma^1 &= Im\ \big(A+\sqrt{p^2}B\big) = \frac{g^2 I_0}{4\sqrt{p^2}(2\pi)^2}\Big(\sqrt{p^2}-m_N-m_\pi\Big)\Big(\sqrt{p^2}-m_N+m_\pi\Big) \sim q^3, \\ Im\ \Sigma^2 &= Im\ \big(A-\sqrt{p^2}B\big) = -\frac{g^2 I_0}{4\sqrt{p^2}(2\pi)^2}\Big[\big(\sqrt{p^2}+m_N\big)^2 - m_\pi^2\Big] \sim q^1. \end{aligned} \tag{20}$$

One can see that the Σ^1 component demonstrates the proper threshold behavior.

Negative parity baryon resonance

$$\begin{aligned} \Sigma(p) &= ig^2 \int \frac{d^4k}{(2\pi)^4}\,\frac{1}{\hat{k}+\hat{p}-m_N}\cdot\frac{1}{k^2-m_\pi^2} = IA(p^2) + \hat{p}B(p^2), \\ \Delta A &= -i\frac{g^2 m_N}{(2\pi)^2} I_0, \qquad \Delta B = \frac{-ig^2}{(2\pi)^2} I_0 \frac{p^2+m_N^2-m_\pi^2}{2p^2} \end{aligned} \tag{21}$$

Imaginary parts of $\Sigma^{1,2}$ now demonstrate $l = 0$ behavior

$$\begin{aligned} Im\ \Sigma^1 &= -\frac{g^2 I_0}{4\sqrt{p^2}(2\pi)^2}\Big[\big(\sqrt{p^2}+m_N\big)^2 - m_\pi^2\Big] \sim q^1, \\ Im\ \Sigma^2 &= \frac{g^2 I_0}{4\sqrt{p^2}(2\pi)^2}\big(\sqrt{p^2}-m_N-m_\pi\big)\big(\sqrt{p^2}-m_N+m_\pi\big) \sim q^3. \end{aligned} \tag{22}$$

The considered examples show that only an „alive“ component Σ^1, which has the pole $1/\big(\sqrt{p^2}-m\big)$ demonstrates the proper threshold behavior (i.e. the proper parity). Another component Σ^2, which has pole of the form $1/\big(-\sqrt{p^2}-m\big)$ demonstrates the opposite parity.

[2] This is way to avoid the unphysical singularities: to renormalize the A, B components and then to calculate $\Sigma^{\pm}$.

[3] q is the spatial momentum of πN pair in CMS.

3.2 Joint dressing the two fermions of opposite parity

Let us consider the nearest analogy to the Rarita-Schwinger field: the joint dressing of two fermions of opposite parity $1/2^{\pm}$. We will suppose that interaction conserves the parity.

Now the Dyson-Schwinger equation has the matrix form

$$G_{ij} = \big(G_0\big)_{ij} + G_{ik}\Sigma_{kl}\big(G_0\big)_{lj}, \qquad i,j,k,l = 1,2. \tag{23}$$

Every element in this equation has γ-matrix indexes which are not shown.

Basis will contain four operators:

$$\mathcal{P}_1 = \Lambda^+, \quad \mathcal{P}_2 = \Lambda^-, \quad \mathcal{P}_3 = \Lambda^+\gamma^5, \quad \mathcal{P}_4 = \Lambda^-\gamma^5, \tag{24}$$

where $\mathcal{P}_{1,2}$ are projection operators and $\mathcal{P}_{3,4}$ are the nilpotent ones. Decomposition of any γ-matrix, depending on p, now is of the form (compare with (12))

$$S(p) = \sum_{M=1}^{4} \mathcal{P}_M \bar{S}^M. \tag{25}$$

This set of operators has very simple multiplicative properties (see Table 2).

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$
$\mathcal{P}_1$	$\mathcal{P}_1$	0	$\mathcal{P}_3$	0
$\mathcal{P}_2$	0	$\mathcal{P}_2$	0	$\mathcal{P}_4$
$\mathcal{P}_3$	0	$\mathcal{P}_3$	0	$\mathcal{P}_1$
$\mathcal{P}_4$	$\mathcal{P}_4$	0	$\mathcal{P}_2$	0

Table 2: Multiplicative properties of basis (24)

The Dyson-Schwinger equation (23) reduces for equation on the coefficients $\bar{G}^M$:

$$\Big(\sum_{M=1}^{4} \mathcal{P}_M \bar{G}^M\Big)\Big(\sum_{L=1}^{4} \mathcal{P}_L \bar{S}^L\Big) = \mathcal{P}_1 + \mathcal{P}_2, \tag{26}$$

where G_M, S_L are the matrixes 2×2.

It leads to matrix equations:

$$\begin{aligned} &G_1S_1 + G_3S_4 = E_2, \qquad && G_2S_2 + G_4S_3 = E_2,\\ &G_1S_3 + G_3S_2 = 0, && G_4S_1 + G_2S_4 = 0, \end{aligned} \tag{27}$$

where E_2 is the unit matrix 2×2. Solutions:

$$\begin{aligned} G_1 &= \Big[S_1 - S_3\big(S_2\big)^{-1}S_4\Big]^{-1}, & G_2 &= \Big[S_2 - S_4\big(S_1\big)^{-1}S_3\Big]^{-1},\\ G_3 &= -\Big[S_1 - S_3\big(S_2\big)^{-1}S_4\Big]^{-1}S_3\big(S_2\big)^{-1}, & G_4 &= -\Big[S_2 - S_4\big(S_1\big)^{-1}S_3\Big]^{-1}S_4\big(S_1\big)^{-1}. \end{aligned} \tag{28}$$

Now let us concretize these general formulae. Suppose that we have two fermions of different parity, but there is no parity violation in lagrangian. It means that the diagonal loops contain only the I and $\hat{p}$ matrixes while the non-diagonal loops should have γ^5. After it solution (28) takes the form

$$\begin{aligned} G =\,&\Lambda^+ \begin{pmatrix} \dfrac{-m_2 - E - \Sigma^2_{22}}{\Delta_1} & 0 \\ 0 & \dfrac{-m_1 - E - \Sigma^2_{11}}{\Delta_2} \end{pmatrix} + \Lambda^- \begin{pmatrix} \dfrac{-m_2 + E - \Sigma^1_{22}}{\Delta_2} & 0 \\ 0 & \dfrac{-m_1 + E - \Sigma^1_{11}}{\Delta_1} \end{pmatrix} + \\ &+\Lambda^+\gamma^5 \begin{pmatrix} 0 & -\dfrac{\Sigma^3_{12}}{\Delta_1} \\ -\dfrac{\Sigma^3_{21}}{\Delta_2} & 0 \end{pmatrix} + \Lambda^-\gamma^5 \begin{pmatrix} 0 & -\dfrac{\Sigma^4_{12}}{\Delta_2} \\ -\dfrac{\Sigma^4_{21}}{\Delta_1} & 0 \end{pmatrix}. \end{aligned} \tag{29}$$

Here
$$\Delta_1 = \big(-m_1 + E - \Sigma_{11}^2\big)\big(-m_2 - E - \Sigma_{22}^2\big) - \Sigma_{12}^3\Sigma_{21}^4,$$
$$\Delta_2 = \big(-m_1 - E - \Sigma_{11}^1\big)\big(-m_2 + E - \Sigma_{22}^1\big) - \Sigma_{12}^4\Sigma_{21}^3 = \Delta_1\big(E \to -E\big),$$

$E = \sqrt{p^2}$, and $i, j = 1, 2$ numerate the dressing fermion states. The appearance of nilpotent operators in decomposition (29) is an indication for transitions between states of different parity. They are absent in case of mixing of the same parity states.

Let us summarize our consideration of the dressing of Dirac fermions.

1) Besides simple multiplicative properties the projection operators $\Lambda^{\pm}$ are very useful in another aspect: its coefficients exibit the definite parity. But as one can see from the loop calculations (20), (21) the components $\Lambda^{\pm}$ have different parity. There is such correspondence: the parity of the field Ψ is the parity of „alive“ component Λ^{+}, which has the pole $1/(E-m)$. Another component Λ^{-} which has the pole $1/(-E-m)$ demonstrates the opposite parity.

2) In contrast to boson case, even if the interactions conserve the parity, the loop transitions between different parity states are not zero: they are proportional to nilpotent operator $\Lambda^{\pm}\gamma^5$.

4 Spin-parity of the Rarita-Schwinger field

Comparing Tables 1 and 2, one can conclude that presence of the nilpotent operators $\mathcal{P}_7$—$\mathcal{P}_{10}$ in decomposition (9) is an indication for the transitions between components of different parity $1/2^{\pm}$. To make sure in this conclusion, we can calculate loop contributions in propagator. As an example we will take the interaction lagrangian $\pi N\Delta$

$$L_{int} = g_{\pi N\Delta}\ \overline{\Psi}^{\ \mu}(x)(g^{\mu\nu} + a\gamma^{\mu}\gamma^{\nu})\Psi(x)\cdot\partial_{\nu}\phi(x) + h.c.\ . \tag{30}$$

Here a is some arbitrary parameter.

The one-loop self-energy contribution is

$$J^{\mu\nu}(p) = -ig^2_{\pi N\Delta}\int\frac{d^4k}{(2\pi)^4}(g^{\mu\rho} + a\gamma^{\mu}\gamma^{\rho})k^{\rho}\frac{1}{\hat{p}+\hat{k}-m_N}k^{\lambda}(g^{\lambda\nu} + a\gamma^{\lambda}\gamma^{\nu})\frac{1}{k^2-m_{\pi}^2}. \tag{31}$$

Let us calculate the discontinuity of loop contribution in $\hat{p}$ basis (5).

$$\begin{aligned}
\Delta J_1 &= -ig^2 I_0\frac{m_N}{12s}\lambda(s, m_N^2, m_\pi^2),\\
\Delta J_2 &= -ig^2 I_0\frac{1}{24s^2}(s + m_N^2 - m_\pi^2)\lambda,\\
\Delta J_3 &= -ig^2 I_0\frac{m_N}{12s}(\lambda + 6a\lambda - 36a^2 m_\pi^2 s),\\
\Delta J_4 &= -ig^2 I_0\frac{1}{24s^2}[(s + m_N^2 - m_\pi^2)\lambda + 12as\lambda + 36a^2 s(s^2 - m_\pi^2 s - 2m_N^2 s - m_\pi^2 m_N^2 + m_N^4)],\\
\Delta J_5 &= ig^2 I_0\frac{m_N}{4s}[(s - m_N^2 + m_\pi^2)^2 + 2a(s - m_N^2 + m_\pi^2)^2 + 4a^2 m_\pi^2 s],\\
\Delta J_6 &= ig^2 I_0\frac{1}{8s^2}[(s + m_N^2 - m_\pi^2)(s - m_N^2 + m_\pi^2)^2 + 4as(s - m_N^2 + m_\pi^2)(s - m_N^2 - m_\pi^2) +\\
&\quad 4a^2 s(s^2 - m_\pi^2 s - 2m_N^2 s - m_\pi^2 m_N^2 + m_N^4)],\\
\Delta J_7 &= ig^2 I_0\sqrt{\frac{3}{s}}\cdot\frac{1}{24s}[(s - m_N^2 + m_\pi^2)\lambda + 4as(2s^2 - m_\pi^2 s - 4m_N^2 s + 2m_N^4 - m_N^2 m_\pi^2 - m_\pi^4) +\\
&\quad 12a^2 s(s^2 - m_\pi^2 s - 2m_N^2 s - m_N^2 m_\pi^2 + m_N^4)],\\
\Delta J_8 &= -ig^2 I_0\sqrt{\frac{3}{s}}\cdot\frac{am_N}{6s}[(s^2 + 4m_\pi^2 s - 2m_N^2 s + m^4 - 2m^2 m_\pi^2 + m_\pi^4) + 6asm_\pi^2],\\
\Delta J_9 &= \Delta J_7, \qquad\quad \Delta J_{10} = -\Delta J_8.
\end{aligned} \tag{32}$$

Here I_0 is the base integral(19), arguments of λ are shown in the first expression.

We saw that in case of Dirac fermions the propagator decomposition in basis of projection operators demonstrates the definite parity. We can expect the similar property for Rarita-Schwinger field in Λ-basis. Let us verify it by calculating the threshold behavior of imaginary part. Using (32) we have

$$\begin{aligned} &\Delta\bar{J}_1 = \Delta J_1 + E\Delta J_2 \sim q^3, \quad &\Delta\bar{J}_2 = \Delta J_1 - E\Delta J_2 \sim q^5, \quad &\Delta\bar{J}_3 = \Delta J_3 + E\Delta J_4 \sim q^3, \\ &\Delta\bar{J}_4 = \Delta J_3 - E\Delta J_4 \sim q, \quad &\Delta\bar{J}_5 = \Delta J_5 + E\Delta J_6 \sim q, \quad &\Delta\bar{J}_6 = \Delta J_5 - E\Delta J_6 \sim q^3. \end{aligned} \tag{33}$$

Such behavior indicates that the components $\bar{J}_1, \bar{J}_2$ exhibit the spin-parity $3/2^+$, while the pairs of coefficient $\bar{J}_3, \bar{J}_4$ and $\bar{J}_5, \bar{J}_6$ correspond to $1/2^+$ $1/2^-$ contributions respectively.

5 Conclusions

Thus we obtained the simple analytical expression (10) for the interacting Rarita-Schwinger field propagator which accounts for all spin components. To derive it we introduced the spin-tensor basis (7) with very simple multiplicative properties.

The obtained dressed propagator (10) solves an algebraic part of the problem, the following step is renormalization. Note that the investigation of dressed propagator is the alternative for more conventional method based on equations of motion (see, *i.e.* Ref. [18] and references therein). The natural requirement for the renormalization is that the spin-1/2 components should remain unphysical after dressing. In other words the denominators Δ_1, Δ_2 should not acquire of zero in the complex energy plane. However a problem of the renormalization needs a more careful consideration.

We found that the nearest analogy for dressing of the $s = 1/2$ sector is the joint dressing of two Dirac fermions of different parity. Some hint for such spin-parity content may be seen from algebraical properties of Λ basis (7) with presence on nilpotent operetors. Caculation of the self-energy contributions in case of Δ isobar confirms it: in the Rarita-Schwinger field besides the leading $s = 3/2$ contribution there are also two $s = 1/2$ components of different parity.

We thank Organizers of Workshop for warm hospitality.

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