Asymptotic properties of the deuteron form factors

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1 Introduction

The deuteron is in focus of theoretical and experimental researches on an extent of several decades. The reasons of attention to this nuclear system are well known. First, this is the unique stable two-particles model. So it can be investigated with a great precision. Second, there is a plenty of the reliable experimental information about this system. That's why the deuteron is the first test of every model of NN interaction. Third, studying the deuteron structure in reactions of ed-scattering at very high momentum trasnfer it is possible to receive the information on a role of quark degrees of freedom in nuclear systems. Fourth, the deuteron-bounding energy is small. So the deuteron is a source of the information about a structure of its components.

The deuteron electromagnetic form factors allow us to describe quantifiable an interaction between the deuteron and electromagnetic field. Form factors can be calculated in terms of information about electron elastic scattering by deuteron.

The electron elastic scattering cross-section by deuteron is calculating by this formula [1], [2], [3]:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left[1+\frac{2E_i}{M_d}\sin^{-2}\frac{\theta}{2}\right]^{-1}[G_C^2+G_Q^2+(1+2(1+\eta_D)\tan^2\frac{\theta}{2})G_M^2], \tag{1}$$

with

$$\eta_D = -\frac{q^2}{4M_d^2}, \quad (\frac{d\sigma}{d\Omega})_{Mott} = \frac{\alpha^2}{4E_i^2}\cos^2\frac{\theta}{2}\csc^4\frac{\theta}{2}. \tag{2}$$

The expression $(\frac{d\sigma}{d\Omega})_{Mott}$ is a scattering cross-section by non-structure spin-free particle derived by Mott. 3 terms (G_C, G_Q, G_M) are the form factors making contribution to the full cross-section. They are due to charge, quadrupole moment and magnetic moment of the deuteron.

In the case of low momentum transfer we can describe an elastic ed-scattering by a nonrelativistic model. But there is a discrepancy between the theory and experiments in the case of high momentum transfer. But the modern development of elastic ed-scattering experiments is due to the advent of exactly high-intensity electron beam. For example, scientists of Jefferson Laboratory carry out experiments of ed-scattering at very high momentum transfer (12 GeV^2).

In the Ref. [4] formulas defining form factors were derived by means of relativistic Hamiltonian dynamics. In this paper we find asymptotic properties of these expressions in the case of high momentum transfer. We also find asymptotics of the nonrelativistic form factors to estimate relativistic corrections. Last, we establish connection between asymptotic form factors behavior and the behavior of wave functions at small distances.

2 Nonrelativistic deuteron form factors at $Q^2 \to \infty$

In the case of nonrelativistic impulse approximation we present the next expressions for standart electromagnetic form factors: charge, quadrupole and magnetic dipole [5]:

$$G_C^{NR}(Q^2) = \sum_{l,l'} \int k^2\, dk\, k'^{\,2}\, dk'\, u^l(k)\, \tilde{g}_{0C}^{ll'}(k\,, Q^2\,, k')\, u^{l'}(k')\,,$$

$$G_Q^{NR}(Q^2) = \frac{2\,M_d^2}{Q^2} \sum_{l,l'} \int k^2\, dk\, k'^{\,2}\, dk'\, u^l(k)\, \tilde{g}_{0Q}^{ll'}(k\,, Q^2\,, k')\, u^{l'}(k')\,, \tag{3}$$

$$G_M^{NR}(Q^2) = -\,M_d \sum_{l,l'} \int k^2\, dk\, k'^{\,2}\, dk'\, u^l(k)\, \tilde{g}_{0M}^{ll'}(k\,, Q^2\,, k')\, u^{l'}(k')\,.$$

In these formulas $\tilde{g}_{0i}^{ll'}(k\,, Q^2\,, k')$, $i = C, Q, M$ are nonrelativistic free two-particle charge, quadrupole and magnetic dipole form factors. They describe electromagnetic properties of the non-interacting two nucleon system. $u_l(k)$ are the wave functions, l , $l' = 0{,}2$ are orbital moments. Formulas for nonrelativistic free two-particle form factors are given in the Ref. [5].

In the modern models of nucleon-nucleon interaction the deuteron wave functions are represented by the next fits:

$$u_0(k) = \sqrt{\frac{2}{\pi}} \sum_j \frac{C_j}{(k^2 + m_j^2)}\,, \quad u_2(k) = \sqrt{\frac{2}{\pi}} \sum_j \frac{D_j}{(k^2 + m_j^2)}\,. \tag{4}$$

C_j and D_j are the numerical coefficients.

Corresponding expressions in coordinate are following:

$$u_0(r) = \sum_j C_j \exp\left(-m_j\, r\right)\,,$$

$$u_2(r) = \sum_j D_j \exp\left(-m_j\, r\right) \left[1 + \frac{3}{m_j\, r} + \frac{3}{(m_j\, r)^2}\right]\,.$$

Coefficients C_j and D_j are fitted for used wave functions to behave right at origin. Conditions for these coefficients are represented by formulas:

$$\sum_j C_j = 0, \quad \sum_j D_j = \sum_j D_j m_j^2 = \sum_j \frac{D_j}{m_j^2} = 0.$$

These conditions gives us the next standard behavior at small distances:

$$u_0(r) \sim r\,, \quad u_2(r) \ \sim r^3\,.$$

Let $F_d(Q^2) = \sqrt{A(Q^2)}$. In this expression $A(Q^2)$ is elastic structure function, which enters the differential cross-section. This function could be expressed in terms of the electromagnetic deuteron form factors: charge, quadrupole and magnetic dipole:

$$A(Q^2) = G_C^2(Q^2) + \frac{8}{9}\,\eta^2\, G_Q^2(Q^2) + \frac{2}{3}\,\eta\, G_M^2(Q^2)\,, \quad \eta = \frac{Q^2}{4M_d^2}\,. \tag{5}$$

We find asymptotic properties of integrals (3) by a modificated Laplas method with Q^2 as a large positive parameter. Calculating these integrals we obtain the next deuteron form factors asymptotics:

$$F_d^{NR}(Q^2) \ \sim \ \frac{1}{(Q^2)^3}. \tag{6}$$

3 Relativistic deuteron form factors at $Q^2 \to \infty$

In the case that $Q^2 > 1$ GeV^2 it is necessary to take into account relativistic corrections in the electromagnetic deuteron structure. Relativistic description of the deuteron is constructed by means of relativistic Hamiltonian dynamics. This approach was elucidate in the Ref. [4]. In this approach we present electromagnetic deuteron form factors by analogy with nonrelativistic case [5]:

$$\begin{aligned}
G_C(Q^2) &= \sum_{l,l'} \int d\sqrt{s} d\sqrt{s'} \varphi^l(s) g_{0C}^{ll'}(s, Q^2, s') \varphi^{l'}(s'), \\
G_Q(Q^2) &= \frac{2M_d^2}{Q^2} \sum_{l,l'} \int d\sqrt{s} d\sqrt{s'} \varphi^l(s) g_{0Q}^{ll'}(s, Q^2, s') \varphi^{l'}(s'), \\
G_M(Q^2) &= -M_d \sum_{l,l'} \int d\sqrt{s} d\sqrt{s'} \varphi^l(s) g_{0M}^{ll'}(s, Q^2, s') \varphi^{l'}(s').
\end{aligned} \tag{7}$$

In these formulas $g_{0i}^{ll'}(s, Q^2, s'), i = C, Q, M$ are relativistic free two-particle charge, quadrupole and magnetic dipole form factors. Formulas for relativistic free two-particles form factors are given in the Ref. [5]. $\varphi^l(s)$ are the deuteron wave functions in sense of Relativistic Hamiltonian Dynamics. These functions are solutions of eigenvalue problem for a mass squared operator for the deuteron:

$$\hat{M}_d^2 \left|\psi\right\rangle = M_d^2 \left|\psi\right\rangle .$$

The mass operator is constructed in the following way:

$$\hat{M}_d^2 = \hat{M}_0^2 + \hat{V} .$$

In this expression M_0 is a mass operator of the two-nucleon system without interaction, and V is an interaction operator.

An eigenvalue problem for a mass squared operator is a nonrelativistic Schrödinger equation within a second order on deuteron binding energy. So the deuteron wave functions in sense of relativistic Hamiltonian dynamics differ from nonrelativistic wave functions in fact by conditions of normalization with relativistic density of states only:

$$\sum_{l=0,2} \int_0^\infty \varphi_l^2(k) \frac{dk}{2\sqrt{k^2 + M^2}} = 1,$$

$$\varphi_l(k) = \sqrt[4]{4(k^2 + M^2)}\, k\, u_l(k) .$$

Calculating integrals (7) we obtain relativistic form factors asymptotics by analogy with (6).

$$F_d(Q^2) \sim \frac{1}{(Q^2)^{13/4}}. \tag{8}$$

We compare obtained function with experimentally measured (see [6]). An area of asymptotic behavior is not achieved in the case of momentum transfer from [6]. Using deuteron $A(Q^2)$ results from JLab we obtain the next behavior of this function in experimentally accessible area:

$$F_d^{exp}(Q^2) \sim \frac{1}{(Q^2)^{3.8}}. \tag{9}$$

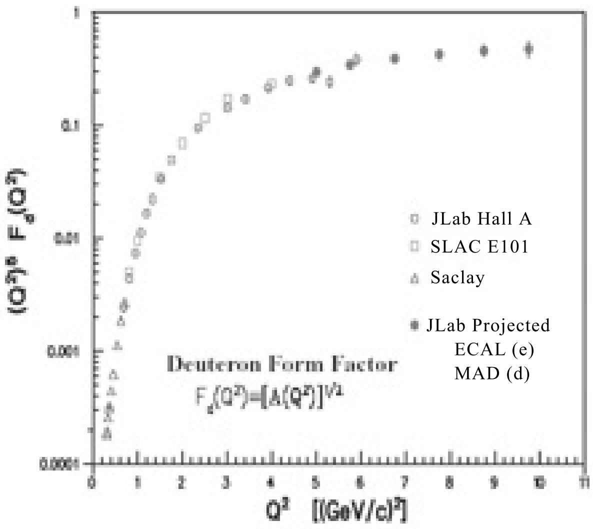

Figure 1: Projected data for the deuteron form factor $F_d(Q^2)$ with an 11 GeV JLab beam. Also shown are existing JLab, SLAC, and Saclay data [9]

4 Quark dynamics

At distances much less than the nucleon size, the underlying quark substructure of the nucleons cannot be ignored. At sufficiently "large" momentum transfers, the few-body form factors are expected to be calculable in terms of only quarks and gluons. The first attempt at a quark-gluon description of the few-body elastic form factors was based on the dimensional-scaling quark model. It was made in Refs. [7], [8]. The underlying dynamical mechanism during elastic scattering is the hard rescattering of the constituent quarks via exchange of hard gluons. In this approach asymptotics of the deuteron form factors should follow the power law:

$$F_h(Q^2) \ \sim\ \frac{1}{(Q^2)^{n_h-1}}\ ,$$

n_h – number of constituent quarks in hadrons. For the deuteron case:

$$F_h(Q^2) \ \sim\ \frac{1}{(Q^2)^5}\ .$$

Fig. 1 shows the recent JLab Hall A and older SLAC and Saclay data on the deuteron form factor, multiplied by $\left(Q^2\right)^5$. This is predicted data, but it is evident that the data tends to asymptotics of quark counting rules.

So an inverse problem can be set. Theoretically, asymptotics of the quark dynamics allow us to obtain information on the wave functions behaviour at origin. The deuteron wave function can be presented in the following way:

$$u_0(k) = \sqrt{\frac{2}{\pi}} \sum_j \frac{C_j}{(k^2+m_j^2)^\alpha}\ .$$

Here α is a some additional parameter. In standard representation (4) $\alpha = 1$ and it doesn't result in quark asymptotics. It was obtained that quark asymptotics is achieved with $\alpha = 11/4$.

In coordinate representation we obtain the next expression for the deuteron wave function:

$$u_0(r) = \frac{32}{21\sqrt{\pi}\Gamma\left(\frac{3}{4}\right)} \sum_j C_j\, m_j \left(\frac{r}{2\, m_j}\right)^{9/4} K_{\frac{5}{4}}(r\, m_j)\ .$$

It has the following behaviour in small and large distances in this representation:

$$r \to 0\,, \quad u_0(r) \sim r\,\frac{2\,\Gamma\left(\frac{1}{4}\right)}{21\,\sqrt{\pi}\Gamma\left(\frac{3}{4}\right)}\sum_j \frac{C_j}{m_j^{5/2}}\,,$$

$$r \to \infty\,, \quad u_0(r) \sim r^{7/4}\frac{2^{9/4}}{21\,\Gamma\left(\frac{3}{4}\right)}\sum_j \frac{C_j}{m_j^{7/4}}\,e^{-r\,m_j}\,.$$

As we can see, it is linear in small distances and the wave function diminishes exponentially in large distances, that is right.

5 Conclusions

In summary we obtained an asymptotics of the deuteron electromagnetic form factors in the nonrelativistic momentum approximation. We also estimated the relativistic corrections to the asymptotics. Finally, we have obtained the expressions for the wave functions which give the asymptotic of the electromagnetic form factors coinciding with quark counting rules.

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