On the Coulomb dissociation of relativistic nuclei and hypernuclei with small binding energies

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Abstract

Using the analogy with the problem of ionization and excitation of atoms at the propagation of relativistic charged particles through matter, the process of Coulomb dissociation of weakly bound relativistic nuclei and hypernuclei is theoretically investigated in the framework of the two-cluster deuteron-like model. Explicit expressions for the total cross-section of Coulomb disintegration of weakly bound systems are derived, taking into account the corrections connected with the finite size of the target nucleus; numerical estimates for the Coulomb dissociation of relativistic hypernuclei $^3H_\Lambda$ and $^6He_\Lambda$ are performed. It is shown that, due to the sharp dependence of the cross-section of Coulomb dissociation upon the binding energy, the experimental measurement of this cross-section in the case of weakly bound relativistic nuclei and hypernuclei allows one to determine the values of binding energy for these systems.

1 Introduction

The process of dissociation of nuclei in the Coulomb field of fast charged particles has been discussed repeatedly (see, for example, [1-3]. In the recent years the interest to this electromagnetic process was revived in connection with the creation of beams of relativistic nuclei and the problem of identification of relativistic hypernuclei [4,5]. In the paper [6] the process of excitation and disintegration of relativistic nuclei and hypernuclei was investigated using the direct analogy with the problem of ionization and excitation of atoms at the propagation of relativistic charged particles through matter [7]. In this work we will discuss the application of the results of [6] to the weakly bound deuteron-like systems consisting of two compact clusters (charged and neutral), the distance between which essentially exceeds the sizes of clusters themselves and the target nucleus radius R. The sharp increase of the cross-section of Coulomb dissociation at the decrease of binding energies allows one to determine experimentally these binding energies in the case of weakly bound nuclei and hypernuclei, studying their disintegration at ultrarelativistic velocities. We will investigate also the role of the finite sizes of the target nucleus.

2 Excitation and disintegration of relativistic nuclei in the Coulomb field of a point-like charge

The total cross-section of excitation and dissociation in the field of a Coulomb centre with the charge Ze can be presented in the form of a relativistic invariant:

$$\sigma = \frac{4\pi(Z\alpha)^2}{(u_c u)^2 - 1} \sum_{n\neq 0} \int \frac{\overline{|\langle n|j|0\rangle u_c|^2}}{|q^2|^2} d(|q^2|. \tag{1}$$

Here $\hbar = c = 1$, $\alpha = e^2 = 1/137$ is the electromagnetic constant, q is the 4-momentum of the virtual photon transferred to the projectile nucleus, $\langle n|j|0\rangle = \{\langle n|\rho|0\rangle, \langle n|\mathbf{j}|0\rangle\}$ is the 4-vector of the current of transition from the ground state $|0\rangle$ of the projectile nucleus to the excited state of the continuous or discrete spectrum $|n\rangle$, u_c and u are the 4-velocities of the Coulomb centre and the projectile nucleus, respectively. The summation in Eq. (1) is performed over all quantum numbers of final states including spin and angular variables, and the upper bar denotes the averaging over polarizations of the initial ground state of the projectile nucleus, which is assumed to be unpolarized.

In the rest frame of the projectile nucleus (i.e. in the *anti-laboratory* frame) we have:

$$q^2 = \epsilon_{n0}^2 - \mathbf{q}^2, \quad (uu_c)^2 - 1 = \gamma_c^2 v_c^2; \tag{2}$$

$$\langle n|j|0\rangle u_c = \gamma_c \langle n|\rho|0\rangle - \langle n|\mathbf{j}|0\rangle \mathbf{v}_c. \tag{3}$$

Here $q_0 = \epsilon_{n0}$ is the excitation energy, $\mathbf{v}_c$ is the velocity of the Coulomb centre, $\gamma_c = (1 - v^2)^{-1/2}$ is the Lorentz-factor; $v_c = |\mathbf{v}_c|$. It follows from the condition of continuity of the electromagnetic current that:

$$\langle n|j|0\rangle q = \langle n|\rho|0\rangle \epsilon_{n0} - \langle n|\mathbf{j}|0\rangle \mathbf{q} = 0, \quad \langle n|\rho|0\rangle = \frac{\langle n|\mathbf{j}|0\rangle \mathbf{q}}{\epsilon_{n0}}. \tag{4}$$

The continuity condition for the current of the Coulomb centre gives:

$$q u_c = \gamma_c (\epsilon_{n0} - \mathbf{q}\mathbf{v}_c) = 0, \quad \epsilon_{n0} = \mathbf{q}\mathbf{v}_c. \tag{5}$$

Taking into account Eqs. (2)-(5), we find that the following relations are valid in the rest frame of the projectile nucleus:

$$\mathbf{q} = \mathbf{q}_\perp + \mathbf{v}_c \frac{\epsilon_{n0}}{v_c^2}, \quad -q^2 = \mathbf{q}_\perp^2 + \frac{\epsilon_{n0}}{\gamma_c^2 v_c^2}; \tag{6}$$

$$\langle n|j|0\rangle u_c = \gamma_c \left[\langle n|\mathbf{j}|0\rangle \left(\frac{\mathbf{q}_\perp}{\epsilon_{n0}} + \frac{\mathbf{v_c}}{\gamma_c^2 v_c^2} \right) \right]. \tag{7}$$

Here $\mathbf{q}_\perp$ is the transverse momentum transferred to the nucleus. Substituting these expressions into Eq. (1), we obtain:

$$\sigma = \frac{4\pi (Z\alpha)^2}{v^2} \int \left(\mathbf{q}_\perp^2 + \frac{\epsilon_{n0}}{\gamma^2 v^2} \right)^{-2} \sum_{n \neq 0} \overline{\left| \langle n|\mathbf{j}|0\rangle \left(\frac{\mathbf{q}_\perp}{\epsilon_{n0}} - \frac{\mathbf{v}}{\gamma^2 v^2} \right) \right|^2} d(\mathbf{q}_\perp^2), \tag{8}$$

where $v = |\mathbf{v}|$ is the velocity of the projectile nucleus in the rest frame of the Coulomb centre (i.e. in the *laboratory* frame; it is evident that $\mathbf{v} = -\mathbf{v}_c$, $v = v_c$, $\gamma = \gamma_c$).

At the first glance, the formulas (1) and (8) for the cross-section of Coulomb dissociation correspond to the one-photon exchange, i.e. to the approximation $Z\alpha \ll 1$. However, the analysis shows that even at large values of Z, when $Z\alpha \sim 1$, the corrections to these formulas still remain small. That is connected with the fact that the considered results can be justified in the framework of the impulse approach with the amplitude of the Coulomb scattering; in doing so, the exact amplitude of the Coulomb scattering in the region of small transferred momenta, where the *main* contribution into σ is provided, differs from the amplitude obtained within the Born approximation only by phase.

At small transferred transverse momenta $|\mathbf{q}_\perp| \ll 1/R_{\mathrm{pr}}$, where R_{pr} is the radius of the projectile nucleus, the transition current is expressed directly through the matrix element of the dipole moment:

$$\langle n|\mathbf{j}|0\rangle = -i\epsilon_{no} \langle n| \sum_p \mathbf{r}_p |0\rangle. \tag{9}$$

In Eq. (9) the summation is performed over the coordinates of all the protons in the projectile nucleus. In accordance with the rule of multiplication of matrices, taking into account the equality $\langle 0| \sum_p \mathbf{r}_p |0\rangle = 0$, that arises due to the space parity conservation, the following relation holds:

$$\sum_{n \neq 0} |\langle n| \sum_p \mathbf{r}_p |0\rangle|^2 = \langle 0| (\sum_p \mathbf{r}_p)^2 |0\rangle. \tag{10}$$

Finally, dividing the integration range into two intervals corresponding to very small and larger $\mathbf{q}_\perp^2$, the following formula for the cross-section of the Coulomb dissociation emerges, after all transformations [6]:

$$\sigma = \frac{4\pi (Z\alpha)^2}{3v^2} \langle 0| \left(\sum_p \mathbf{r}_p \right)^2 |0\rangle \left[\ln \left(\frac{\gamma^2 v^2}{\epsilon_{\mathrm{bin}}^2 \langle 0| \left(\sum_p \mathbf{r}_p \right)^2 |0\rangle} \right) - 2A + B - v^2 \right]. \tag{11}$$

Here ϵ_{bin} is the binding energy of the projectile nucleus, the constant A involves the dependence of the minimal momentum transfer, at the transition to excited states $|n\rangle$, upon the excitation energy ϵ_{n0} (at the disintegration $\epsilon_{n0} > \epsilon_{\rm bin}$), the constant B describes the contribution of *comparatively large* transfers of transverse momentum. Calculations lead to the expressions:

$$A = \frac{\sum_{n\neq 0}\langle n|\sum_p \mathbf{r}_p|0\rangle|^2 \ln(\epsilon_{n0}/\epsilon_{\rm bin})}{\langle 0|\left(\sum_p \mathbf{r}_p\right)^2|0\rangle}, \tag{12}$$

$$B = -3\int_0^\infty \ln y \frac{d}{dy}\left(\frac{G(y)}{y}\right) dy, \tag{13}$$

where

$$y = \mathbf{q}_\perp^2 \langle 0|\left(\sum_p \mathbf{r}_p\right)^2|0\rangle, \tag{14}$$

and, taking into account the completeness condition,

$$G(y) = \sum_{n\neq 0} |\, \langle n|\sum_p \exp(-i\mathbf{q}_\perp \mathbf{r}_p)|0\rangle\,|^2 =$$

$$= \langle 0|\,|\sum_p \exp(-i\mathbf{q}_\perp \mathbf{r}_p)\,|^2\,|0\rangle - |\langle 0|\sum_p \exp(-i\mathbf{q}_\perp \mathbf{r}_p)\,|0\rangle\,|^2. \tag{15}$$

At $y \ll 1$ the function $G(y) \approx y/3$, at $y \gg 1$ the function $G(y) \to z$, where z is the number of protons in the projectile nucleus. For the projectile nucleus with the unity charge we have: $G(y) = 1 - F^2(y)$, where

$$F(y) \equiv F(\mathbf{q}_\perp^2) = \langle 0|\, \exp(-i\mathbf{q}_\perp \mathbf{r}_p)\,|0\rangle$$

is the electromagnetic formfactor of the ground state.

3 Contribution of finite sizes of the target nucleus

Taking into account that the target nucleus is not point-like, one should subtract the correction term ΔB from the constant B in Eq. (11):

$$\Delta B = 3\int_0^\infty \frac{G(y)(1-H(y))}{y^2}\, dy, \tag{16}$$

Here $G(y)$ is determined by Eq.(15) as before, and

$$H(y) = \frac{1}{Z^2}|\, \langle 0'|\sum_p \exp(-i\mathbf{q}_\perp \mathbf{r}'_p)\,|0'\rangle\,|^2 \tag{17}$$

is the square of the electromagnetic formfactor of the ground state of the target nucleus $|0'\rangle$; in doing so, $\mathbf{r}'_p$ is the coordinate of a proton in the target nucleus. For the uniform distribution of protons over the volume of the target nucleus

$$H(y) = 9\left(\frac{\sin x}{x^3} - \frac{\cos x}{x^2}\right)^2, \tag{18}$$

where

$$x = \sqrt{y}\frac{R_{\rm tag}}{\sqrt{\langle 0|(\sum_p \mathbf{r}_p)^2|0\rangle}}, \tag{19}$$

$R_{\rm tag}$ is the radius of the target nucleus. It is clear that $H(0) = 1$.

Let us note that Eq. (16) describes the influence of finite sizes of the target nucleus on the process of Coulomb dissociation of the projectile nucleus *without* the excitation and breakup of the target nucleus.

When the quantum state of the target nucleus is not fixed and transitions into all excited states are taken into account, the function $H(y)$ is replaced by the expression [6]

$$\widetilde{H}(y) = \frac{1}{Z^2}\langle 0'| \, |(\sum_p \exp(-i\mathbf{q}_\perp \mathbf{r}'_p)|^2 \, |0'\rangle. \tag{17a}$$

It is easy to show that at the uniform distribution of the charge Z the functions $\widetilde{H}(y)$ and $H(y)$ are connected by the relation

$$\widetilde{H}(y) = \frac{1}{Z} + H(y)\left(1 - \frac{1}{Z}\right); \tag{17b}$$

thus, in the case of heavy target nuclei the corrections ΔB and $\widetilde{\Delta B} = [1-(1/Z)]\Delta B$ practically coincide.

It follows from Eq. (11) that at ultrarelativistic energies of the projectile nucleus ($v \to 1$, $\gamma \gg 1$) the cross-section of the Coulomb dissociation of this nucleus, irrespective of the relation between radii of the projectile and target nuclei, has the structure:

$$\sigma = a \ln\gamma + b, \quad a = \frac{8\pi(Z\alpha)^2}{3}\langle 0|(\sum_p \mathbf{r}_p)^2|0\rangle. \tag{20}$$

4 Weakly bound systems. Two-cluster model

Now we will consider the Coulomb disintegration of the "friable", deuteron-like nuclei consisting of two clusters (charged and neutral), the average distance between which is significantly larger than the radius of the force action and the sizes of the clusters themselves. In this case the excited bound states are absent, and the normalized wave function of the ground state, corresponding to the zero orbital momentum, has the following form:

$$\phi_0(r) = \frac{1}{\sqrt{2\pi\rho}}\frac{\exp(-r/\rho)}{r}, \tag{21}$$

where $r = |\mathbf{r}|$ is the distance between the clusters,

$$\rho = \left(2\frac{m_1 m_2}{M}\epsilon_{\rm bin}\right)^{-1/2}. \tag{22}$$

Here m_1 and m_2 are the masses of the charged and neutral clusters, respectively, $M = m_1 + m_2$ is the mass of the deuteron-like nucleus, $\epsilon_0 = (-\epsilon_{\rm bin})$ is the energy of its bound state. In the given case the quantity $\langle 0|(\sum_p \mathbf{r}_p)^2|0\rangle$ can be explicitly determined:

$$\langle 0|(\sum_p \mathbf{r}_p)^2|0\rangle \approx 4\pi z^2\left(\frac{m_2}{M}\right)^2\int_0^\infty \phi_0^2(r) r^4 dr =$$

$$= z^2\left(\frac{m_2}{M}\right)^2\frac{\rho^2}{2} = \frac{1}{4}\, z^2\frac{m_2}{m_1 M\epsilon_{\rm bin}}, \tag{23}$$

where z is the number of protons in the charged cluster.

Using Eqs. (15) and (21), we obtain the analytical expression for the function $G(y)$:

$$G(y) = z^2\left(1 - \left|\int \phi_0^2(r)\exp\left(-i\mathbf{q}_\perp \mathbf{r}\frac{m_2}{M}\right)d^3\mathbf{r}\right|^2\right) =$$

$$= z^2\left(1 - \frac{2z^2}{y}\left[\arctan\left(\sqrt{\frac{y}{2z^2}}\right)\right]^2\right). \tag{24}$$

Here, according to Eqs. (14) and (23),

$$y = \frac{1}{4}\,\mathbf{q}_\perp^2 z^2\frac{m_2}{m_1 M\epsilon_{\rm bin}}.$$

Substituting Eq. (24) into Eq. (13), we obtain the following value for the constant B:

$$B = -3z^2 \int_0^\infty \ln y \frac{d}{dy} \left(\frac{1}{y} - \frac{2z^2}{y^2} \left[\arctan \left(\sqrt{\frac{y}{2z^2}} \right) \right]^2 \right) dy = \ln 2z^2 + C, \tag{25}$$

where [6]

$$C = 6 \int_0^\infty \ln u \left(\frac{1}{u^3} + \frac{\arctan u}{u^4(1+u^2)} - \frac{2}{u^5} (\arctan u)^2 \right) \approx 0.316 \tag{26}$$

Meantime, taking into account that at small binding energies all excited states belong to the continuous spectrum, for a two-cluster system with the wave function $\phi_0(r)$ (see Eq. (21)) the constant A, determined by Eq. (12), is given by the integral:

$$A = \frac{16}{\pi} \int_0^\infty \frac{t^{3/2}}{(1+t)^4} \ln(1+t) dt \approx 1.218. \tag{27}$$

Finally, Eq. (11) leads to the following result for the total cross-section of Coulomb disintegration of the *weakly bound two-cluster nuclei*, taking into account also the correction term $\Delta B(Z)$ connected with the finite size of the target nucleus:

$$\sigma = \frac{\pi}{3} (Z\alpha)^2 z^2 \frac{m_2}{v^2 M m_1 \epsilon_{\rm bin}} \left[\ln \left(\frac{8\gamma^2 v^2 M m_1}{m_2 \epsilon_{\rm bin}} \right) - (2A - C) - v^2 - \Delta B(Z) \right]. \tag{28}$$

Here $2A - C \approx 2.12$; the quantity $\Delta B(Z)$ in Eq. (28) is determined according to Eq. (16).

It is well seen from Eq. (28) that in the limit of very small binding energies $\epsilon_{\rm bin}$ the cross-section of Coulomb disintegration increases inversely proportionally to $\epsilon_{\rm bin}$; in so doing, the logaryphmic term in the square brackets essentially exceeds the other ones. Measuring experimentally the Coulomb dissociation cross-section σ for weakly bound nuclei and hypernuclei, one can determine, in principle, the value of the binding energy $\epsilon_{\rm bin}$ for these nuclei.

5 Projectile nuclei with small binding energies: corrections due to the finite radius of the target nucleus

Due to the correction term $\Delta B(Z)$, the cross-section of Coulomb dissociation σ decreases as compared with the case of the point-like target ($\Delta\sigma < 0$). Substituting Eqs. (24) and (18) into the Eq. (16), we obtain the quantity $\Delta B(Z)$ in the case of two-cluster projectile nuclei as the integral

$$\Delta B(Z) = \int_0^\infty \frac{s^2 - (\arctan s)^2}{s^5} \left[1 - 9 \left(\frac{\sin x}{x^3} - \frac{\cos x}{x^2} \right)^2 \right] ds, \tag{29}$$

where, in accordance with Eqs. (19) and (23),

$$s = \frac{1}{z} \sqrt{\frac{y}{2}}, \quad x = \chi s, \quad \chi = 2 \frac{R_{\rm tag}}{\rho} \frac{M}{m_2} = 2\sqrt{2}\, R_{\rm tag} \sqrt{\frac{M m_1}{m_2}\, \epsilon_{\rm bin}}. \tag{30}$$

It should be emphasized that, owing to the dependence of the quantity ΔB upon the radius of the target nucleus ($R_{\rm tag} = 1.1\, A^{1/3}$, where A is the total number of nucleons in the nucleus; for heavy nuclei $R_{\rm tag} \approx 1.5\, Z^{1/3}$) the total cross-section of Coulomb dissociation σ gets a certain deviation from the pure dependence $\sim Z^2$. However, according to our calculations, the dependence of the total cross-section of Coulomb disintegration of weakly bound nuclei and hypernuclei upon Z *cannot be presented* as $\sim Z^{2-\delta}$ with a small *constant* δ, contrary to the statements in the papers [8,9].

In Fig.1 the correction term ΔB is presented as a function of χ. The corresponding correction to the cross-section of Coulomb dissociation is

$$\Delta\sigma = -\frac{\pi}{3} (Z\alpha)^2 z^2 \frac{m_2}{v^2 M m_1 \epsilon_{\rm bin}} \Delta B(Z), \tag{31}$$

As shown by the analysis, the quantity $\Delta B(Z)$ decreases to zero with the decrease of the ratio $R_{\rm tag}/\rho$: at $\chi \to 0$ we have $\Delta B \sim \chi^2 \ln \chi$.

6 Calculations of the cross-section of Coulomb dissociation for the hypernuclei $^3H_\Lambda$ and $^6He_\Lambda$

1. For the process

$$^3H_\Lambda + Z \to d + \Lambda + Z$$

of Coulomb dissociation of the hypernucleus $^3H_\Lambda$ into the deuteron and the Λ-particle ($z = 1$, $M = M(^3H_\Lambda) = 2993.6$ Mev/c^2: $m_1 = m_d = 1878$ MeV/c^2; $m_2 = m_\Lambda = 1115.7$ MeV/c^2) we obtain, taking $\gamma = 6$ and $\epsilon_{\rm bin} \approx 0.08$ MeV:

a) Tin target ($Z = 50$).
$\sigma_0 = 1.88$ barn; $\Delta\sigma = -0.14$ barn; $\sigma = 1.74$ barn.

b) Uranium target ($Z = 92$).
$\sigma_0 = 6.38$ barn; $\Delta\sigma = -0.58$ barn; $\sigma = 5.8$ barn.

Here σ_0 is the cross-section calculated for the point-like Coulomb centre, $\Delta\sigma$ is the correction to the cross-section due to the finite radius of the target nucleus.

2. For the process

$$^6He_\Lambda + Z \to {}^5He_\Lambda + n + Z$$

of Coulomb dissociation of the hypernucleus $^6He_\Lambda$ into $^5He_\Lambda$ and the neutron ($z = 2$, $M = M(^6He_\Lambda) = 5.78$ GeV/c^2, $m_1 = m(^5He_\Lambda) = 4.84$ GeV/c^2, $m_2 = m_n = 939$ Mev/c^2) the results are as follows, taking $\gamma = 6$ and $\epsilon_{\rm bin} \approx 0.15$ MeV :

a) Tin target ($Z = 50$).
$Z = 50$; $\sigma_0 = 0.74$ barn; $\Delta\sigma = -0.14$ barn; $\sigma = 0.6$ barn.

b) Uranium target ($Z = 92$).
$Z = 92$; $\sigma_0 = 2.50$ barn; $\Delta\sigma = -0.54$ barn; $\sigma = 1.96$ barn.

So, for the considered cases (especially – for $^6He_\Lambda$) the correction $\Delta\sigma$ to the Coulomb dissociation cross-section, emerging due to the finite size of the target, proves to be rather essential.

7 Summary

1. The process of Coulomb dissociation of weakly bound relativistic nuclei and hypernuclei has been studied within the two-cluster "deuteron-like" model. Explicit expressions for the total effective cross-section of Coulomb disintegration, taking into account the corrections conditioned by the finite size of the target nucleus, are obtained.

2. It is shown that the experimental measurement of the Coulomb dissociation cross-section for weakly bound nuclei and hypernuclei enables one to determine the value of the binding energy for these systems.

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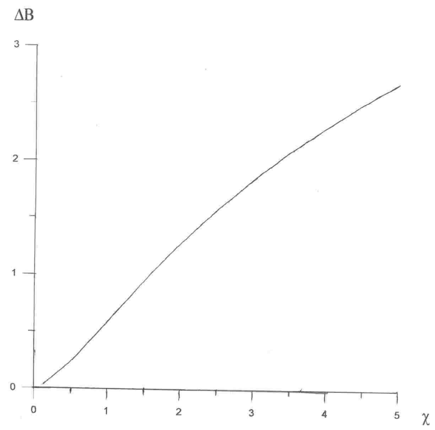

Figure 1: Dependence of the correction term ΔB (29) upon the parameter χ (30)