The decay $H \to hh$ in the MSSM. Complete one-loop analysis

Yu. P. Philippov
Dept. of General and Theoretical Physics, Samara State University
Russia, 443011, Samara, Akad. Pavlova st., 1.

Abstract

The Higgs boson effective self-coupling λ_{hhH} and corresponding decay rate $\Gamma(H \to hh)$ are calculated in the framework of the Minimal Supersymmetric Standard Model (MSSM) for the complete set of one-loop diagrams. The comparison with previous results, where only the leading $t - \tilde{t}$, $b - \tilde{b}$ correction terms were taken into account, is carried out. We analyse the dependencies of the self-coupling and decay rate on $\tan\beta$ and mass of light Higgs boson; it is demonstrated that the tree-level self-coupling (decay rate) could acquire substantial one-loop corrections, which could be phenomenologically important for reconstruction procedure of the Higgs Potential.

Introduction

The Mechanism of the Generation of fundamental particles Masses (MGM) is one of some crucial elements in build-up of a major class of quantum gauge models, for instance, Standard model (SM), and its supersymmetric modifications. Owing to this element it is possible to generate masses of mater fields (accounting Yukawa interaction) and intermediate gauge bosons fields. Thereat the indicated models save set of important properties, such as a gauge invariance and renormalizability.

Traditionally, MGM includes in itself two basic components [1]:
(i) Spontaneous Symmetry Breaking (SSB) [2] and
(ii) the Higgs Mechanism [3].

Each component is characterized by proper system of basic positions. So SSB is banked on introduction of the additional system of scalar fields and supposition about existence nonzero expectation value for the neutral components of given system. The Higgs mechanism is based on the suppositions about selection of the Higgs fields parametrization, existence of Yukawa gauge invariant interaction and fixing of arbitrariness in gauge fields transformations. Today there are only indirect proofs of the given mechanism realization in nature. For instance, today there is no experimental fact which contradict with SM predictions [4].

In order to confirm MGM on experiment directly, it is needed [5]
(1) to detect the Higgs bosons and measure their masses,
(2) to measure the Higgs boson couplings with gauge bosons and fermions and prove that these couplings increase with their masses,
(3) to determine the Higgs boson self-couplings.

The last step is important especially for the case of MSSM, where the self-couplings structure are also determined by the soft supersymmetry breaking principle.

The majority of MSSM Higgs self-couplings can be measured potentially on future high luminosity colliders in next processes.

Higgs self-couplings measuring

$\mathbf{e^+ - e^-}$ **- linear collider**
(1) $e^+ e^- \to Z + H_i H_j \{AA\}$, $i,j = 1,2$
(2) $e^+ e^- \to \nu_e \bar{\nu}_e + H_i H_j \{AA\}$, $i,j = 1,2$
(3) $e^+ e^- \to A + H_i H_j \{AA\}$, $i,j = 1,2$
where $H_1 = h, H_2 = H$

LHC
(1) $q\,\bar{q} \to W/Z + H_i H_j \{AA\}$, $i,j = 1,2$
(2) $q\,q' \to q\,q' + H_i H_j \{AA\}$, $i,j = 1,2$
(3) $q\,\bar{q} \to A + H_i H_j \{AA\}$, $i,j = 1,2$
(4) $g\,g \to H_i H_j \{AA\}$, $i,j = 1,2$

In practice, not all the cross sections can be measured experimentally since some are too small. In the works [6], [7] it was shown that self-couplings λ_{hhh}, λ_{hhH} have maximum chances to be measured on the linear collider. The last coupling is interest especially, because this one characterize decay rate $\Gamma(H \to hh)$. As it was shown in [8] this decay plays a dominant role in part of the model parameter space for moderate values of tanβ and $M_H \in (200, 350)$ GeV. In this range, the branching ratio $\mathrm{Br}(H \to hh)$, derived from the partial width

$$\Gamma(H \to hh) = \frac{|\lambda_{hhH}|^2}{32\pi M_H}\sqrt{1 - \frac{4M_h^2}{M_H^2}} \tag{1}$$

is neither too small nor too close to unity to measured directly. If double Higgs production on $e^+ - e^-$ - linear collider is mediated via the resonant decay of H, then the corresponding total cross section increases by about an order of magnitude [9] and up to 2 orders of magnitude for gluon-gluon fusion at LHC [10].

In the works [11]-[13] the theoretical examination of given self-coupling with leading one-loop corrections was realized. It was demonstrated that one can have sizable corrections, which change substantially the tree-level results. In the works [7], [14], decay rate (1) was investigated taken into account leading $t-\tilde{t}$, $b-\tilde{b}$ one-loop corrections and demonstrated that given corrections increase $\Gamma(H \to hh)$, hence enhance our chances to measure self-coupling λ_{hhH}. However, any estimates of non-leading one-loop contributions in this works are absent. Because *the general goal of this work* is calculation λ_{hhH} and $\Gamma(H \to hh)$ taken into account all one-loop contributions. Our main interest is the analysis of modifications in the Higgs potential by the corrections coming from the unsurveyed Feynman diagrams.

In section 1 the tree level structure of MSSM Higgs sector and the main technical details of calculation are presented. Some numerical results and their comparative analysis are contained in section 2.

1 Calculational basis

1.1 The tree level structure of MSSM Higgs sector

The Higgs sector of the MSSM consists of two scalar doublets

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} (v_1 + \phi_1^0 - i\chi_1^0)/\sqrt{2} \\ -\phi_1^- \end{pmatrix}, H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix} \tag{2}$$

with opposite hypercharge $Y_1 = -Y_2 = -1$ and vacuum expectation values v_1, v_2. The Higgs potential of the MSSM is given by

$$\begin{aligned} V_{\mathrm{Higgs}} &= m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 + m_{12}^2\,\left(\epsilon_{ij}\,H_1^i\,H_2^j + \mathrm{h.c.}\right) \\ &+ \frac{1}{8}(g^2 + g'^2)\,\left(|H_1|^2 - |H_2|^2\right)^2 - \frac{1}{2}\,g^2\,|H_1^\dagger\,H_2|^2 \end{aligned} \tag{3}$$

with soft breaking parameters m_1^2, m_2^2, m_{12}^2 and the gauge couplings g_1, g_2 is diagonalized by the rotations

$$\begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} = D(\alpha)\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}, \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix} = D(\beta)\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} = D(\beta)\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} \tag{4}$$

where

$$D(\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \tag{5}$$

G^0, $G^\pm$ describe the unphysical Goldstone modes. The spectrum of physical MSSM Higgs states consists of (1) two neutral $CP-$ even Higgs fields $\{h^0, H^0\}$, (2) neutral $CP-$ odd Higgs field A^0, (3) charged fields $H^\pm$.

The masses of the gauge bosons and electric charge are determined by

$$M_W^2 = \frac{1}{4}g^2v^2,\ M_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2,\ v^2 = (v_1^2 + v_2^2),\ e = \frac{g^2g'^2}{g^2 + g'^2} \tag{6}$$

Thus, the potential (3) contains two independent free parameters, which can conveniently be chosen as

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta), \tag{7}$$

where M_A is the mass of the A^0 boson. In terms of (6), (7) the masses of other physical states read:

$$m_{h,H}{}^2 = \frac{1}{2}\left(M_A^2 + M_Z^2 \mp \sqrt{(M_A^2+M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\,\right), \qquad m_{H^\pm}^2 = M_A^2 + M_W^2 \tag{8}$$

Connection between mixing angles is represented by

$$\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, \qquad -\frac{\pi}{2} < \alpha < 0. \tag{9}$$

A part of (3) which responds for three- and four-point interactions of Higgs fields has the form

$$\mathcal{L}_{Int}^{Higgs} = \mathcal{L}_{Int}^{(3)} + \mathcal{L}_{Int}^{(4)} \tag{10}$$

where

$$\begin{aligned} \mathcal{L}_{Int}^{(3)} \;=\; & \lambda_{hhh}^0 hhh + \lambda_{hhH}^0 hhH + \lambda_{hHH}^0 hHH + \lambda_{HHH}^0 HHH + \lambda_{hAA}^0 hAA + \\ & \lambda_{HAA}^0 HAA + \lambda_{hH^+H^-}^0 hH^+H^- + \lambda_{HH^+H^-}^0 HH^+H^- \end{aligned} \tag{11}$$

The tree level Higgs boson self-couplings $\{\lambda^0\}$ are represented in (11) as functions of two indicated before free parameters: M_A and $\tan\beta$. For instance,

$$\lambda_{hhH}^0 = \lambda_0[2\sin(2\alpha)\sin(\alpha+\beta) - \cos(2\alpha)\cos(\alpha+\beta)] \tag{12}$$

where $\lambda_0 = \frac{M_z^2}{v}$. However this simple scenario is changed when radiative corrections are taken into account.

1.2 On-shell renormalization of vertex

Main object characterizes triple Higgs boson interaction hhH is the 3-point vertex function (VF). This function in one-loop approximation can be represented by the following generic expression

$$\Gamma_{hhH}^{(1)} = \Gamma_{hhH}^{(0)} + \Delta\hat{\Gamma}_{hhH}^{(1)} \tag{13}$$

where the subscript 0 refers to the tree-level function, which correspond directly to the expression for the hhH Higgs coupling already given in (12). The renormalized one-loop contributions are summarized in $\Delta\hat{\Gamma}_{hhH}^{(1)}$. The last one can be obtained in next form

$$\Delta\hat{\Gamma}_{hhH}^{(1)} = \Delta\Gamma_{hhH}^{(1)} + \delta\Gamma_{hhH}^{(1)} \tag{14}$$

where first term represents one-loop unrenormalized contributions and last term is counterterm for given VF. Its structure is determined both the Higgs potential (3) and common rules of standard renormalization procedure [15] with $m_i^2 \to Z_{H_i}^{-1}(m_i^2 + \delta m_i^2)$, $g \to Z_1^W Z_2^{W\,-\frac{3}{2}} g$, $g' \to Z_1^B Z_2^{B\,-\frac{3}{2}} g'$, $v_i \to Z_{H_i}^{1/2}(v_i - \delta v_i)$, and with field renormalization constants δZ_{H_i}.

In the on-shell scheme, adopted in this work, the counterterms are fixed by imposing the following renormalization conditions [16]:
– the on-shell conditions for $M_{W,Z}$ and the electric charge e as in the minimal SM,
– the on-shell condition for the A^0 boson with the pole mass M_A,
– the tadpole conditions for vanishing renormalized tadpoles, i.e. the sum of the 1-loop tadpole diagrams for H^0, h^0, and the corresponding tadpole counterterm is equal to zero,
– the renormalization of $\tan\beta$ in such a way that the relation $\tan\beta = v_2/v_1$ is valid for the 1-loop Higgs minima. By this set of conditions, the input for the MSSM Higgs sector is fixed by the pole mass M_A and $\tan\beta$, together with the standard gauge-sector input $M_{W,Z}$ and e.

It is also necessary to note other features of our calculation. The author used Feynman gauge. Tensor reduction and algorithms for the scalar one-loop integrals calculation were employed [17]. The experimental data were taken from [18].

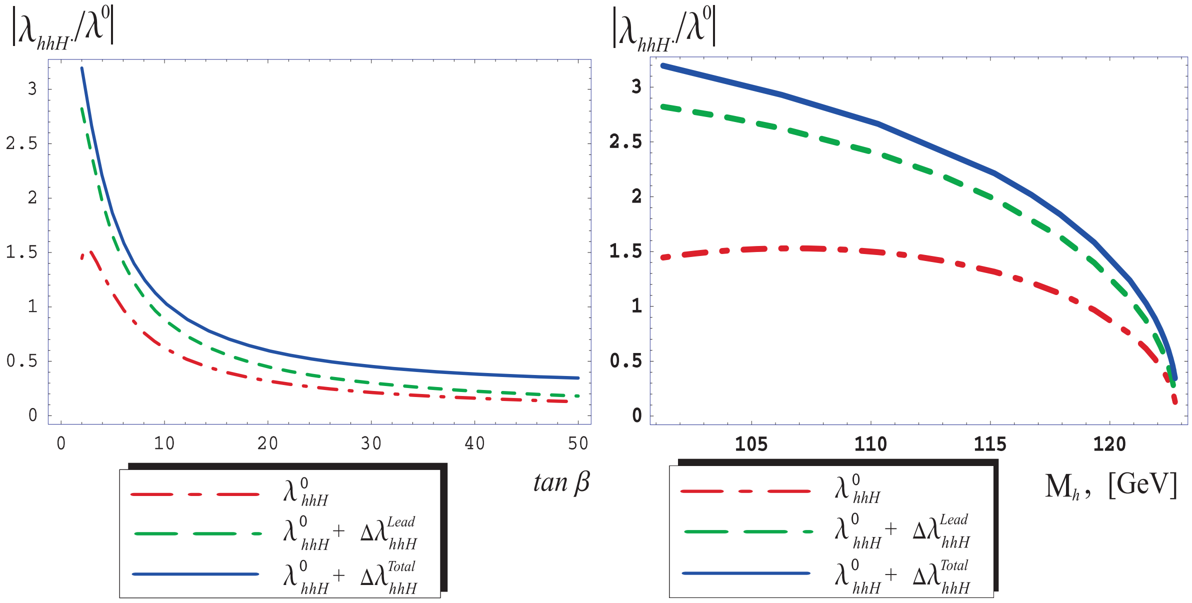

Figure 1: Dependencies of the Higgs self-coupling λ_{hhH} (on the tree and one-loop levels) on $\tan\beta$ and M_h, in units $\lambda_0 = \frac{M_z^2}{v}$, for $M_A = 500$ GeV, $M_3 = \mu = 300$ GeV, $M_{\widetilde{Q}} = M_{\widetilde{U}} = M_{\widetilde{D}} = M_{\widetilde{R}} = M_{\widetilde{L}} = 1$ TeV, $A_f = 200$ GeV.

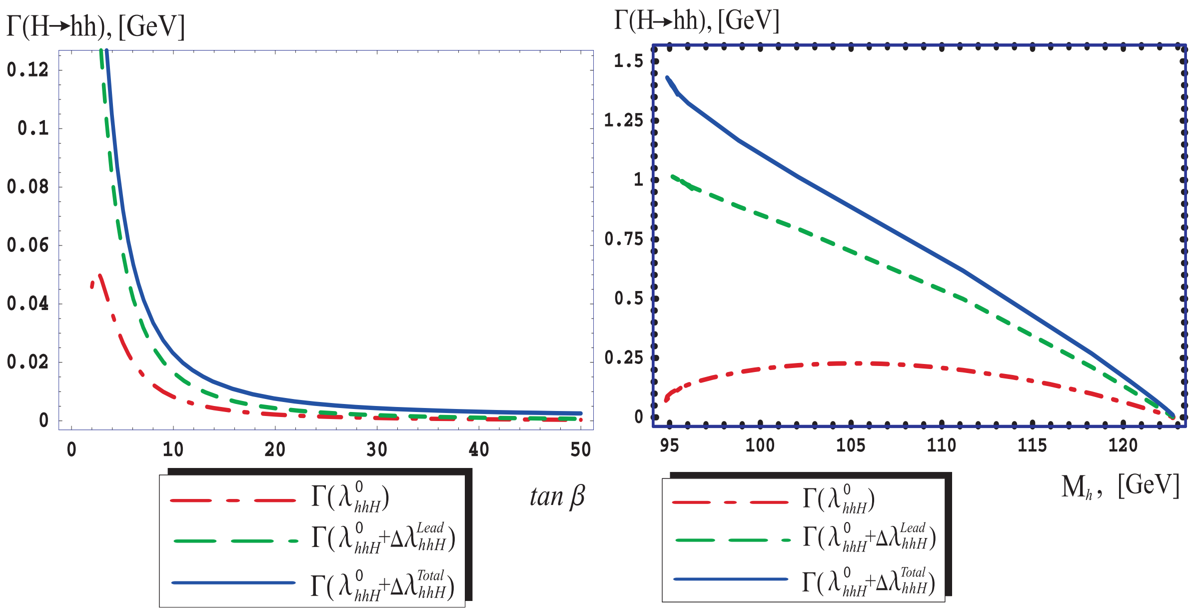

Figure 2: Dependencies of the decay rate $\Gamma(H \to hh)$ (on the tree and one-loop levels) on $\tan\beta$ and M_h, for $M_A = 500$ GeV, $M_3 = \mu = 300$ GeV, $M_{\widetilde{Q}} = M_{\widetilde{U}} = M_{\widetilde{D}} = M_{\widetilde{R}} = M_{\widetilde{L}} = 1$ TeV, $A_f = 200$ GeV.

2 Results and their analysis

Two types of dependencies are represented in this paper: Higgs self-coupling (decay rate) dependencies on $\tan\beta$, M_h. In Fig. 1 three kinds of dependencies on $\tan\beta$ and M_h are represented: tree-level self-coupling (λ^0_{hhH}), self-coupling with leading $t - \tilde{t}$ one-loop correction ($\lambda^0_{hhH} + \Delta\lambda^{Lead}_{hhH}$) and self-coupling with complete one-loop contribution ($\lambda^0_{hhH} + \Delta\lambda^{Comp}_{hhH}$). Analytic expressions for ($\Delta\lambda^{Lead}_{hhH}$) can be found in [11]-[13] in the case of large mass approximation for virtual particles (and also for case SUSY breaking scale $M_s \gg m_t$ see [19]). In calculations it have been used expression with leading one-loop correction for M_h, which was represented in work [6].

Obviously, differences between $(\lambda^0_{hhH} + \Delta\lambda^{Lead}_{hhH})$ and $(\lambda^0_{hhH} + \Delta\lambda^{Comp}_{hhH})$ curves are insignificant for small values $\tan\beta$ and major values of M_h accordantly ($\sim 10\%$ from curve $(\lambda^0_{hhh}+\Delta\lambda^{Lead}_{hhh})$ for $\tan\beta = 5$). While for moderate and major values of $\tan\beta$ this difference is larger, hence the role of the non-leading contributions increase. We have that case when the account of the non-leading contributions becomes essential.

As its well known, the lower boundary on the lightest Higgs mass makes 115 GeV. In future the experiment will only advance on scale of energies up. Therefore it will be more complicated to define self-coupling because its values decrease with growing of mass M_h.

In Fig. 2 the decay rate $\Gamma(H \to hh)$ dependencies on $\tan\beta$ and M_h are shown. One can observe that the decay rate is weekly dependent on $\tan\beta$ for moderate and major values of $\tan\beta$ and gets small values. Herewith decay rapidly enhance with decreasing of $\tan\beta$. Because the greatest chances in rate measuring we have in the region of small $\tan\beta \sim 3 - 5$. It is important to note that dependence $\Gamma_{H\to hh}(M_h)$ (with complete one-loop contribution) is almost linear. However in interest region of M_h values for experimental investigation decay rate decreases with M_h growth. Dependencies $\lambda_{hhH}(M_A)$, $\Gamma_{H\to hh}(M_A)$ is more feeble for $\tan\beta \in \{2, 50\}$, because the author does't consider this case.

The summing error of results is determined both determination error of t - quark mass ($\delta m = 5.1$ GeV) and generality of soft supersymmetry breaking parameters values. They are equal 5 GeV and 8 GeV (for $M_A = 500$ GeV) accordingly.

Conclusion

In this work the neutral Higgs boson self-coupling λ_{hhH} and corresponding decay rate $\Gamma(H \to hh)$ with complete set of the one-loop diagrams are represented and analyzed in the framework of the MSSM. The effective dependencies of ones on $\tan\beta$ and M_h are evaluated with using of on-shell renormalization scheme. The authors have avoided any approximations in one-loop scalar integrals calculations. It has been shown, that the tree-level self-coupling (decay rate) can acquire sizable one-loop corrections. These must be taken into account for detailed comparative analysis of theory and experimental data, which could be phenomenologically important for reconstruction procedure of Higgs potential.

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