Should it be possible to reconstruct the equation of state from gravitational radiation of single superdense sources?

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Abstract

This work is done in the framework of CRDF/NFSAT Collaboration on "Gravitational Waves from Neutron Stars with superfluid Component". Calculating Gravitational wave parameters from superdense configurations with different equations of state we found that there is one to one correspondence between equation of state and GW parameters. So we conclude that there should be possible to guess the equation of state from gravitational waves of compact stelar objects.

1 Introduction

We do expect that in the next couple of years a new window to astrophysical objects will be opened: gravitational wave detectors of second generation will start to function with desired accuracy. Along with the main challenge to detect a gravitational wave, if succeeded, detectors may provide valuable information on the type and main parameters of the source, that can not be obtained in other ways. The small interaction of gravitational waves with matter will allow us to look for waves coming directly from cores of superdense compact objects providing direct information about Equation of state, size, type, rotating frequencies of the last. Isolated single compact objects with superfluid interiors undergoing selfsimilar oscillations are going to be promising sources for gravitational waves[1]. We are considering a special regime, for rotating configuration where "deformation energy" is going to feed gravitational waves through quasi-radial oscillations.

2 Quasi-radial Oscillations

Quasi-radial oscillations of rotating compact objects were investigated in the early 1970's [6, 4] where the frequency spectrum of the fundamental oscillation mode for maximally rotating compact objects was determined. These stars are oblate due to their rotation and consequently they have a non-zero quadrupole moment. The oscillations add a time dependance to the quadrupole moment [8]. The oscillation is described by assigning each mass element a time dependent coordinate given by $x_\alpha = x^0_\alpha(1+\eta\sin\omega t)$ where $\eta \ll 1$ and a constant. Thus, the reduced quadrupole moment is given by:

$$Q_{\alpha\beta} = \int \rho \left(x_\alpha x_\beta - 1/3\delta_{\alpha\beta}x^2\right) d^3x \simeq Q^0_{\alpha\beta}\left(1+2\eta\sin\omega t\right) \tag{1}$$

where $Q^0_{\alpha\beta}$ are the components of the quadrupole moment of the rotating oblate compact object in equilibrium and we have neglected terms of order η^2. Taking the axis of rotation to lie along the z-axis, the non-zero components of the quadrupole moment obey:

$$Q^0 = -Q^0_{zz} = 2Q^0_{xx} = 2Q^0_{yy}. \tag{2}$$

The power emitted in gravitational radiation is given by:

$$J = \frac{G}{5c^5}\left|\frac{d^3}{dt^3}Q_{\alpha\beta}\right|^2, \tag{3}$$

and consequently one obtains:

$$J = \frac{6G}{5c^5}\eta^2\omega^6\left|Q^0\right|^2\cos^2\omega t' = J_0\cos^2\omega t' \tag{4}$$

where the retarded time is $t' = t - r/c$ for a source at distance r.

To determine the wave form and the angular distribution of the radiation, we rotate to coordinates in which the wave vector lies along the z-axis and use the transverse-traceless gauge. Consequently,

$$h_+ = 1/2\,(h_{xx} - h_{yy}) = \frac{3GQ^0\eta\omega^2}{c^4 r}\sin^2\theta \sin\omega t' h_\times = h_{xy} = 0, \tag{5}$$

where θ is the angle between the wave vector and the axis of rotation of the compact object. We can express the strain amplitude in terms of the power by combining Eq. 4 with Eq. 5 to obtain:

$$h_+ = \sqrt{\frac{15GJ_0}{2c^3}}\frac{1}{r\omega}\sin^2\theta\sin\omega t'. \tag{6}$$

If an energy source can be found to drive the pulsations, the rate at which power is put into the vibrations can be combined with the lifetime of the energy source to estimate the strain amplitude from an individual white dwarf and thus from the galactic population as a whole. We discuss a possible mechanism in the next section.

3 Quadrupole Moments and Oscillation Frequencies

Equilibrium configurations of neutron stars and quasiradial oscillations were discussed in the papers [6, 4] in connection with integral characteristics and stability conditions.

$\rho_{c(15)}$	M/M_o	$\Omega_{\max}$ (kHz)	$I_{(44)}$	ω (kHz)	$N_{(57)}$	$Q^0_{(43)}$	$W_{r(52)}$	$\alpha - \alpha_0$	$W_{g(53)}$
0.546	0.7815	5.97	4.92	3	1.51	7.68	0.877	0.082	1.79
1.14	1.3737	8.37	9.85	5	1.76	9.73	3.45	0.068	1.46
2.44	1.7127	12.3	11.1	7	2.02	30.7	8.40	0.070	1.30

In the table the results of those calculations are presented for a real baryon gas equation of state. It is necessary to use a known equation of state and central density ρ_c to find the following integral characteristics, mass (M), moment of inertia (I), maximum rotational frequency (Ω_m), and quadrupole moment (Q^0_{zz}). The frequency of adiabatic quasiradial oscillation (ω) is also given. All values are calculated in a general relativistic expansion about the rotation frequency Ω to order Ω^2. Calculations have shown [6, 4] that as the mass increases from 0.1 M_o to 0.5 M_o the oscillation frequency ω quickly rises from 10 Hz up to 2 kHz. For $M > 0.5\ M_o$ the oscillation frequency shows a very slow increase from 2 kHz up to 10 kHz. We note that the oscillation frequencies are calculated for the nonrotating configurations, however calculations in [6, 4] show that there is very little change in ω when going to the maximally rotating configuration. The oblateness of the star and consequently the value of the quadrupole moment Q^0_{zz} do depend on the rotation of the neutron star and the values shown in Table are calculated for the maximally rotating configuration. Since the quaduropole moments are calculated to Ω^2 order we can find the appropriate values for any rotation rate using Table above and scaling by $(\Omega/\Omega_m)^2$.

4 Continuous Energy Sources

To determine the gravitational wave amplitude h_0 for oscillating neutron stars, it is necessary to find a permanent source of energy to sustain undamped oscillations and therefore to estimate the intensity J_0. One possible source of energy is the deformation energy of the neutron star. For rotating neutron stars, the surfaces of constant density are rotating elipsoids. During the spin-down, these surfaces tend towards sphericity. Because the crust is a crystalline solid, the process of spin-down will be accompanied by starquakes which will relieve the stress built up in the core and drive quasiradial oscillations. We propose that part of the deformation energy is converted to gravitational radiation in this process. The deformation energy of the star can be calculated in the following way [7]:

$$W_{def}(\Omega) = (M - M_0)\,c^2 - W_r(\Omega), \tag{7}$$

where M and M_0 are the rotating and non-rotating masses for the same baryon number, respectively. The kinetic energy of rotation is $W_r(\Omega) = I\Omega^2/2$. Using results obtained in [5] for the gravitational coefficient of packing α for rotating and non-rotating configuration, we find:

$$\alpha = 1 - \frac{M}{mN} \tag{8}$$

$$\alpha_0 = 1 - \frac{M_0}{mN} \tag{9}$$

where m is the baryon mass and N is the baryon number. The difference in masses $\Delta M = M - M_0$ is then:

$$\Delta M = (\alpha - \alpha_0)\, mN. \tag{10}$$

In Table above the coefficient of packing as well as all other values are calculated for a maximally rotating configuration. We obtain values for lower rotation rates as above scaling with the factor $(\Omega/\Omega_m)^2$. Therefore the deformation energy for a rotating star with angular velocity Ω is given by:

$$W_{def}(\Omega) = \left(\frac{\Omega}{\Omega_m}\right)^2 W_{def}(\Omega_m). \tag{11}$$

If part of the deformation energy is radiated as gravitational waves during the lifetime of the pulsar, the gravitational wave intensity J_0 can be estimated by:

$$J_0 \approx \beta \frac{W_{def}}{\tau}, \tag{12}$$

where

$$\tau = \frac{\Omega}{2|\dot{\Omega}|} \tag{13}$$

is the lifetime of the pulsar, and $\beta \ll 1$ is the branching ratio indicated the fraction of energy converted to gravitational radiation. Thus, using the previous equations, we obtain:

$$J_0 \approx \frac{2\beta\Omega|\dot{\Omega}|}{\Omega_m^2} W_{def}(\Omega_m). \tag{14}$$

We can now estimate the intensity and amplitude of gravitational radiation from neutron stars if we know the appropriate parameters that appear in Eq. 14. We take for a model neutron star $\rho_c = 1.14\times10^{15}\mathrm{gcm}^{-3}$ corresponding to a mass $M = 1.4\ M_o$, and read the following values from Table above: $\Omega_m = 8.4\times10^3$ Hz and $W_{def}(\Omega_m) = 1.8 \times 10^{53}$ erg. Substituting these values into Eq. 14, we find:

$$J_0 \approx \left(\Omega|\dot{\Omega}|\right) 5.1 \times 10^{45}\ \mathrm{erg\ s}^2. \tag{15}$$

5 Glitches

We now turn to burst signals coming from glitches. It is known that the angular velocity of pulsars is continuously decreasing due to magneto-dipole radiation. On top of this continuous change there are irregular fluctuations and jumps (known as glitches) in the angular velocity on the order of $\Delta\Omega/\Omega \sim 10^{-6} - 10^{-9}$ as well as its time derivative ($\dot{\Omega}$) on the order of $|\Delta\dot{\Omega}/\dot{\Omega}| \sim 10^{-2} - 10^{-4}$ [2]. Henceforth we suggest that glitches are possible sources of gravitational radiation. It is possible that during a glitch some of the rotational energy is transmitted to the crust and excites harmonic oscillations. We then assume that the energy transmitted to the crust is then emitted as gravitational radiation. The energy of spin-up during the glitch is given by:

$$\Delta W = I\Omega\Delta\Omega. \tag{16}$$

The power which is transmitted to the crust is then determined by:

$$\Delta\dot{W} = I\Omega\Delta\dot{\Omega} = I\Omega\dot{\Omega}\frac{\Delta\dot{\Omega}}{\dot{\Omega}}, \tag{17}$$

where we have ignored both $\dot{I}$ and $\dot{\Omega}\Delta\Omega$ as both of these terms are very small. To obtain an upper limit on the strength of the gravitational radiation, we assume that all the power transmitted to the crust is then emitted in gravitational radiation, so that $J_0 = \Delta\dot{W}$. Using Eq. 6, we find the amplitude for the gravitational wave to be:

$$h_0 = \left(2 \times 10^{-9}\ (\mathrm{erg\ s})^{-1/2}\ \mathrm{cm}\right) \frac{\sqrt{\Delta\dot{W}}}{\omega r}. \tag{18}$$

We now estimate an upper limit on the gravitational radiation using the Vela pulsar. Using observational values for the Vela pulsar, we take the following values: $\Omega \approx 70\ \mathrm{s}^{-1}$, $|\dot{\Omega}| \approx 10^{-10}\ \mathrm{s}^{-2}$, and $r \approx 0.3$ kpc. After the glitch, the angular velocity gradually relaxes back to its original value. Usually, after a hundred days from the moment of the glitch the behavior of the angular velocity begins to fluctuate with relative changes in $\dot{\Omega}$ on the order of $|\Delta\dot{\Omega}/\dot{\Omega}| \approx 10^{-4}$ [2]. Using the values for I, ω, and Q^0 from our model above and using the values of Ω, $\dot{\Omega}$, and $\left(\Delta\dot{\Omega}/|\dot{\Omega}|\right)_{max}$ one can find:

$$J_{0max} \approx 7 \times 10^{34}\ \mathrm{erg\ s}^{-1} \tag{19}$$

$$h_{0max} \approx 6.8 \times 10^{-27} \tag{20}$$

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It is clear that after the glitch J_0 and h_0 will decrease at the same rate as given in Eq. **??** for $|\Delta\dot{\Omega}/\dot{\Omega}|$.

For fluctuations of the angular velocity, we have $\Delta\dot{\Omega}/|\dot{\Omega}| \sim 10^{-2}\left(\Delta\dot{\Omega}/|\dot{\Omega}|\right)_{max}$. Therefore we can see that the intensity J_0 is two orders of magnitude less the value in Eq. 19 and the amplitude h_0 one order of magnitude less than the value given in Eq. 20.

We can also estimate the value of η. Substituting the values for J_{0max}, ω, and Q^0 into Eq. 4 we find $\eta \approx 1.6 \times 10^{-4}$.

We will estimate the amplitude of the gravitational radiation for the Crab pulsar as well. We take the same model above and use $\Omega \approx 190\ \mathrm{s}^{-1}$, $|\dot{\Omega}| \approx 2.65 \times 10^{-9}\ \mathrm{s}^{-2}$, $\left(\Delta\dot{\Omega}/|\dot{\Omega}|\right)_{max} \approx 5 \times 10^{-3}$, and $r \approx 2$ kpc [21]. We then find $J_{0max} \approx 2 \times 10^{36}\ \mathrm{erg\ s}^{-1}$, $h_{0max} \approx 10^{-26}$, and $\eta \approx 10^{-3}$.

6 Conclusions

The numbers estimated in the paper are showing that with gravitational wave detectors of new generation we could register gravitational waves from pulsars and white dwarfs. The comparison of gravitational wave amplitude and frequency will give us information on equation of state of the superdense object.

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