Interactions of spin zero particles with electromagnetic and gravitational fields

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Abstract

The Hamiltonian for relativistic spin 0 particles in an electromagnetic field can be derived by the successive Feshbash-Villars and Foldy-Wouthuysen transformations. As compared with the classical Hamiltonian, the Hamiltonian in the Foldy-Wouthuysen representation contains additional terms describing the momentum-dependent interaction with a nonuniform electric field and the electric and mixed polarizabilities of point particles. The mixed polarizability characterizes the magnetic moment induced by the external electric field and the electric moment induced by the external magnetic field. All additional terms vanish for immobile particles. The generalized Duffin-Kemmer equation for spin 0 particles in a gravitational field is deduced. The relativistic Hamiltonian in the Foldy-Wouthuysen representation is calculated. This Hamiltonian contains the term describing the contact (Darwin) interaction. For the case of a weak spherically symmetric field, the semiclassical equation of motion reproduces the well-known classical result for the angle of particle deflection.

1 Introduction

Spin zero particles do not possess any intrinsic moments. Due to this circumstance, an investigation of these particles provides the best possibility of comparison of results obtained in classical and quantum theories. Additional terms in the quantum Hamiltonian describe interactions of quantum origin.

The Hamiltonian for relativistic spin 0 particles in an electromagnetic field can be derived by the method proposed in Ref. [1]. This method is based on the successive Feshbash-Villars [2] and Foldy-Wouthuysen (FW) [3] transformations. In the present work, the method is generalized and transformations are performed for point particles.

To describe spin 0 particles in a gravitational field, the generalized Duffin-Kemmer (DK) equation is deduced. If the first-order DK equation is used instead of the second-order Klein-Gordon (KG) one, the derivation of the FW Hamiltonian becomes unambiguous. It is important to find the semiclassical equation of particle motion in a weak spherically symmetric field because the results obtained in Refs. [4, 5, 6] do not agree with the well-known classical formula for the angle of particle deflection [7, 8].

The system of units $\hbar = c = 1$ is used.

2 The Hamiltonian in the Foldy-Wouthuysen representation

The connection between second-order and first-order wave equations has been found in Ref. [1]. In the present work, we fulfil a more general transformation.

For spin zero particles in an electromagnetic field, the initial Klein-Gordon equation has the form:

$$\left[\left(i\frac{\partial}{\partial t} - e\varphi\right) - (\boldsymbol{p} - e\boldsymbol{A})^2 - m^2\right]\psi = 0, \tag{1}$$

where φ and $\boldsymbol{A}$ are the scalar and vector potentials of electromagnetic field, $\boldsymbol{p} = -i\nabla$, and ψ is the scalar wave function.

We can introduce the wave functions ϕ and χ satisfying the conditions

$$\psi = \phi + \chi, \tag{2}$$

$$\left(i\frac{\partial}{\partial t} - e\varphi\right)\psi = N(\phi - \chi), \tag{3}$$

where N is an arbitrary positive constant. Eqs. (2),(3) define the generalized Feshbash-Villars transformation, because N can be nonequal to the particle mass, m. In particular, the transformation can be used for massless particles.

Multiplying Eq. (3) by the factor $i\frac{\partial}{\partial t} - e\varphi$ leads to the equation

$$\left(i\frac{\partial}{\partial t} - e\varphi\right)(\phi - \chi) = \frac{\boldsymbol{\pi}^2 + m^2}{N}(\phi - \chi), \tag{4}$$

where $\boldsymbol{\pi} = -i\nabla - e\boldsymbol{A}$ is the kinetic momentum operator.

The matrix form of Eqs. (2),(4) is given by

$$i\frac{\partial \Psi}{\partial t} = \mathcal{H}\Psi, \quad \mathcal{H} = \rho_3\frac{\boldsymbol{\pi}^2 + m^2 + N^2}{2N} + e\varphi + i\rho_2\frac{\boldsymbol{\pi}^2 + m^2 - N^2}{2N}, \tag{5}$$

where $\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}$ is the two-component wave function (bispinor), ρ_i $(i = 1, 2, 3)$ are the Pauli matrices:

$$\rho_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \rho_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \rho_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Components of these matrices act on the spinors.

The Hamiltonian can be split into operators commuting and noncommuting with the operator ρ_3:

$$\mathcal{H} = \rho_3\mathcal{M} + \mathcal{E} + \mathcal{O}, \quad \rho_3\mathcal{M} = \mathcal{M}\rho_3, \quad \rho_3\mathcal{E} = \mathcal{E}\rho_3, \quad \rho_3\mathcal{O} = -\mathcal{O}\rho_3, \tag{6}$$

where

$$\mathcal{M} = \frac{\boldsymbol{\pi}^2 + m^2 + N^2}{2N}, \quad \mathcal{E} = e\varphi, \quad \mathcal{O} = i\rho_2\frac{\boldsymbol{\pi}^2 + m^2 - N^2}{2N}. \tag{7}$$

Now we perform the FW transformation by the method proposed in Ref. [9]. We suppose the interaction energy to be small as compared with the total energy including the rest one. The transformation operator U is given by

$$U = \frac{\epsilon + \mathcal{M} + \rho_3\mathcal{O}}{\sqrt{2\epsilon(\epsilon + \mathcal{M})}}, \quad U^{-1} = \frac{\epsilon + \mathcal{M} - \rho_3\mathcal{O}_1}{\sqrt{2\epsilon(\epsilon + \mathcal{M})}}, \quad \epsilon = \sqrt{\mathcal{M}^2 + \mathcal{O}^2} = \sqrt{\boldsymbol{\pi}^2 + m^2}. \tag{8}$$

This operator brings Hamiltonian (6) to the form

$$\mathcal{H}' = \rho_3\epsilon + \mathcal{E}' + \mathcal{O}', \quad \rho_3\mathcal{E}' = \mathcal{E}'\rho_3, \quad \rho_3\mathcal{O}' = -\mathcal{O}'\rho_3. \tag{9}$$

The odd operator $\mathcal{O}'$ is small compared to both ϵ and the initial Hamiltonian $\mathcal{H}$. This circumstance allows us to apply the usual scheme of the nonrelativistic FW transformation [3, 9, 10]. Such a transformation should be performed with the following operator:

$$U' = \exp(iS'), \quad S' = -\frac{i}{4}\rho_3\left\{\mathcal{O}', \frac{1}{\epsilon}\right\} = -\frac{i}{4}\left[\frac{\rho_3}{\epsilon}, \mathcal{O}'\right], \tag{10}$$

where $[\ldots, \ldots]$ is a commutator and $\{\ldots, \ldots\}$ is an anticommutator. Further calculations are similar to those fulfilled for spin 1/2 particles [9, 10]. As compared with Ref. [10], the particle mass should be replaced by the operator ϵ noncommuting with the operators $\mathcal{E}'$ and $\mathcal{O}'$. If only major corrections are taken into account, the transformed Hamiltonian takes the form

$$\mathcal{H}'' = \rho_3\epsilon + \mathcal{E}' + \frac{\rho_3}{4}\left\{\frac{1}{\epsilon}, \mathcal{O}'^2\right\}. \tag{11}$$

If we use the designations

$$F = \left[\boldsymbol{\pi}^2, e\varphi - i\frac{\partial}{\partial t}\right] = 2ie\boldsymbol{\pi}\cdot\boldsymbol{E},$$
$$G = \left[\boldsymbol{\pi}^2, \left[\boldsymbol{\pi}^2, e\varphi - i\frac{\partial}{\partial t}\right]\right] = 4e(\boldsymbol{\pi}\cdot\nabla)(\boldsymbol{\pi}\cdot\boldsymbol{E}) - 4e^2\boldsymbol{\pi}\cdot(\boldsymbol{E}\times\boldsymbol{H}),$$
$$(\boldsymbol{\pi}\cdot\nabla)(\boldsymbol{\pi}\cdot\boldsymbol{E}) \equiv \pi_i\pi_j\frac{\partial E_j}{\partial x_i},$$

the even and odd terms are given by

$$\mathcal{E}' = e\varphi + \frac{1}{64}\left\{\frac{1}{\epsilon^4}, G\right\}, \quad \mathcal{O}' = \frac{\rho_1}{8}\left\{\frac{1}{\epsilon^2}, F\right\}. \tag{12}$$

The final FW Hamiltonian for point particles in an electromagnetic field is

$$\begin{aligned}\mathcal{H}'' &= \rho_3\epsilon + e\varphi + \frac{1}{64}\left\{\frac{1}{\epsilon}, G\right\} + \frac{\rho_3}{64}\left\{\frac{1}{\epsilon^5}, F^2\right\} \\ &= \rho_3\epsilon + e\varphi + \frac{e}{8\epsilon^4}(\boldsymbol{\pi}\cdot\nabla)(\boldsymbol{\pi}\cdot\boldsymbol{E}) - \frac{e^2}{8\epsilon^4}\boldsymbol{\pi}\cdot(\boldsymbol{E}\times\boldsymbol{H}) - \rho_3\frac{e^2}{8\epsilon^5}(\boldsymbol{\pi}\cdot\boldsymbol{E})^2.\end{aligned} \tag{13}$$

3 Comparison of particle properties in classical and quantum physics

We can perform a semiclassical transition similarly to [9]. The obtained Hamilton operator takes the form

$$\mathcal{H}'' = \epsilon + e\varphi + \frac{e}{8\epsilon^4}(\boldsymbol{\pi}\cdot\nabla)(\boldsymbol{\pi}\cdot\boldsymbol{E}) - \frac{e^2}{8\epsilon^4}\boldsymbol{\pi}\cdot(\boldsymbol{E}\times\boldsymbol{H}) - \frac{e^2}{8\epsilon^5}(\boldsymbol{\pi}\cdot\boldsymbol{E})^2. \tag{14}$$

The corresponding classical Hamiltonian contains only two first terms:

$$\mathcal{H}'' = \epsilon + e\varphi. \tag{15}$$

In semiclassical Hamilton operator (14), last three terms describe properties attributed to extended particles. However, operator (14) does not contain the Darwin term characterizing the contact interaction of immovable particles. Moreover, all the quantum corrections vanish for immovable particles.

Moving particles have some nonclassical properties. The third term in Eq. (14) describes the quadrupole interaction with an electric field. The similar (Blount) term enters the Hamiltonian for spin 1/2 particles [9, 11]. The last term characterizes the electric polarizability of moving particles and the previous term determines the mixed polarizability. The induced electric dipole moment is equal to

$$\boldsymbol{d} = \frac{\partial\mathcal{H}''}{\partial\boldsymbol{E}} = \frac{e^2}{8\epsilon^4}\boldsymbol{\pi}\times\boldsymbol{H} - \frac{e^2}{4\epsilon^5}\boldsymbol{\pi}(\boldsymbol{\pi}\cdot\boldsymbol{E}). \tag{16}$$

The induced magnetic dipole moment is given by

$$\boldsymbol{\mu} = \frac{\partial\mathcal{H}''}{\partial\boldsymbol{H}} = -\frac{e^2}{8\epsilon^4}\boldsymbol{\pi}\times\boldsymbol{E}. \tag{17}$$

Thus, the induced electric dipole moment is proportional to the magnetic field and the induced magnetic dipole moment is proportional to the electric field. For point particles, these properties are of purely quantum origin.

4 Duffin-Kemmer equation for particles in a gravitational field

Spin zero particles in a gravitational field are described by the KG equation

$$(g^{\mu\nu}\nabla_\mu\nabla_\nu + m^2)\psi = 0 \tag{18}$$

or

$$\left(\frac{1}{\sqrt{-g}}\partial_\mu g^{\mu\nu}\sqrt{-g}\partial_\nu + m^2\right)\psi = 0, \tag{19}$$

where ∇_μ is a covariant derivative. The generalization of this equation is given by

$$\left(\frac{1}{\sqrt{-g}}\partial_\mu g^{\mu\nu}\sqrt{-g}\partial_\nu + m^2 + \lambda R\right)\psi = 0, \tag{20}$$

where R is the curvature. When $\lambda = 1/6$, Eq. (20) for the massless scalar field is conformally invariant [12].

In Refs. [13, 14] this equation has been brought to the Hamilton form by a nonunitary transformation. However, for the second-order KG equation it is difficult to prove that such a procedure cannot be performed in an another way. One more method of solution of this problem is the reduction of the KG equation to a first-order form.

It is convenient to use the generalized DK equation. For free particles, it has the form

$$(\beta^\mu \partial_\mu - m)\Phi = 0,$$

where

$$\beta^0 = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \beta^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix},$$
$$\beta^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad \beta^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \tag{21}$$

The covariant generalization of this equation cannot be given by

$$(\beta^\mu \nabla_\mu - m)\Phi = 0,$$

because usual form (21) of matrices β^μ is valid only for the flat spacetime. The transformation to the local flat spacetime can be fulfilled by the tetrad method. The following relation takes place:

$$g^{\mu\nu} = \eta^{ab} e^\mu_a e^\nu_b, \tag{22}$$

where e^μ_a are tetrad vectors and the Latin indexes designate tetrad coordinates. The tensor η^{ab} defines the Minkowski metric and has the signature $+ - - -$.

Let us establish a correspondence between Eqs. (18),(20) and the following equation:

$$\left(\frac{1}{4}\eta^{ab}\left\{e^\mu_a, \nabla_\mu\right\}\left\{e^\nu_b, \nabla_\nu\right\} + m^2\right)\psi = 0. \tag{23}$$

In the general case, covariant derivatives of metric tensor are equal to zero $[\nabla_\mu(g^{\nu\rho}) \equiv g^{\nu\rho}_{;\mu} = 0]$ and covariant derivatives of tetrad vectors are given by

$$\nabla_\mu(e_{a\nu}) \equiv e_{a\nu;\mu} = \gamma_{abc} e^b_\nu e^c_\mu,$$

where γ_{abc} are the Ricci rotation coefficients. Eq. (23) can be transformed to the form

$$\left[g^{\mu\nu}\nabla_\mu\nabla_\nu + m^2 + \frac{1}{4}\eta^{ab}\left(e^\mu_{a;\mu} e^\nu_{b;\nu} + 2e^\mu_{a;\mu;\nu} e^\nu_b\right)\right]\psi = 0. \tag{24}$$

Eq. (24) differs from Eqs. (18),(20) in the small terms proportional to $e^\mu_{a;\mu;\nu}e^\nu_b$ and $e^\mu_{a;\mu}e^\nu_{b;\nu}$. These terms contain second-order and squared first-order derivatives of the metric tensor, respectively. For the Schwarzschild metric, corresponding corrections to the "interaction energy" are of order of $\left(\frac{\hbar c}{\epsilon r}\right)^2$ and $\frac{\hbar^2 c^2 r_g}{\epsilon^2 r^3}$, where ϵ is the total particle energy and $r_g = \frac{2GM}{c^2}$ is the gravitational radius. As a rule, the above terms can be neglected. In this approximation, Eqs. (18),(20), and (24) may be considered as equivalent equations.

Eqs. (23),(24) can be represented as

$$(\mathcal{D}^a \mathcal{D}_a + m^2)\Phi = 0, \tag{25}$$

where $\mathcal{D}_a = \frac{1}{2}\{e^\mu_a, \nabla_\mu\}$. Eq. (25) is equivalent to the DK equation

$$(\beta^a \mathcal{D}_a - m)\Phi = 0. \tag{26}$$

To prove the consistence of these equations, one can multiply Eq. (26) on the left by $\beta_b\mathcal{D}^b + m$:

$$(\beta_b\mathcal{D}^b\beta^a\mathcal{D}_a - m^2)\Phi = 0. \tag{27}$$

Eq. (27) for the fifth component of wave function coincides with Eq. (25).

To use the Hamiltonian approach, covariant DK equation (26) should be expressed in terms of the energy and momentum operators. It is amazing that the transformation can be exactly carried out in the general case. Since

$$\nabla_\mu\Phi = \partial_\mu\Phi, \quad \nabla_\mu e^\mu_a\Phi = (\partial_\mu e^\mu_a + \Gamma^\mu_{\rho\mu}e^\rho_a)\Phi = \left(\partial_\mu e^\mu_a + \frac{\partial \log\sqrt{-g}}{\partial x^\mu}e^\mu_a\right)\Phi,$$

Eq. (26) takes the form

$$\left[\frac{1}{2}\beta^a\left(\{e^\mu_a, \partial_\mu\} + \frac{\partial \log\sqrt{-g}}{\partial x^\mu}e^\mu_a\right) - m\right]\Phi = 0. \tag{28}$$

In these formulae, the operator ∂_μ acts on all quantities from the right and $g = \det g_{\mu\nu}$.

The nonunitary transformation $\Phi' = U\Phi$, where $U = (-g)^{1/4}$, brings Eq. (28) to the form

$$\left(\frac{1}{2}\beta^a\{e^\mu_a, \partial_\mu\} - m\right)\Phi' = 0. \tag{29}$$

This equation contains only partial derivatives.

The nonunitary transformation with the same operator U has been successfully used for spin 1/2 particles [15, 16].

Thus, generalized DK equation (29) exactly follows from initial equation (24). For initial equations (18) and (20), the transformation to Eq. (29) is approximate.

5 Spherically symmetric gravitational field

Let us consider the case of a spherically symmetric gravitational field that is described by the Schwarzschild metric. If we use the isotropic coordinates, the metric tensor is

$$g_{\mu\nu} = \begin{pmatrix} V^2 & 0 & 0 & 0 \\ 0 & -W^2 & 0 & 0 \\ 0 & 0 & -W^2 & 0 \\ 0 & 0 & 0 & -W^2 \end{pmatrix}. \tag{30}$$

For a weak gravitational field,

$$V \approx 1 - \frac{GM}{r}, \quad W \approx 1 + \frac{GM}{r}.$$

Metric (30) also describes the flat Minkowski spacetime in a uniformly accelerated frame and some other cases (see Refs. [17, 18]).

The explicit form of the generalized DK equation equation is given by

$$\left(V^{-1}\beta^0\partial_0 + \frac{1}{2}\{W^{-1}, \boldsymbol{\beta}\cdot\boldsymbol{\partial}\} - m\right)\Phi' = 0$$

or

$$\left(\beta^0\partial_0 + \frac{V}{2}\{W^{-1}, \boldsymbol{\beta}\cdot\boldsymbol{\partial}\} - mV\right)\Phi' = 0. \tag{31}$$

As a result of the nonunitary transformation with the operator $U' = V^{-1/2}$, Eq. (31) becomes Hermitian:

$$\left(\beta^0\partial_0 + \frac{1}{2}\{\mathcal{F}, \boldsymbol{\beta}\cdot\boldsymbol{\partial}\} - mV\right)\Phi'' = 0, \tag{32}$$

where $\Phi'' = U'\Phi'$, $\mathcal{F} = \dfrac{V}{W}$. The relation

$$\frac{1}{4}\left\{V, \{W^{-1}, \boldsymbol{\beta}\cdot\boldsymbol{\partial}\}\right\} = \mathcal{F}^{1/2}\boldsymbol{\beta}\cdot\boldsymbol{\partial}\mathcal{F}^{1/2} = \frac{1}{2}\left\{\mathcal{F}, \boldsymbol{\beta}\cdot\boldsymbol{\partial}\right\}$$

is taken into account.

If we use the explicit form of matrices β^a, we obtain the Hamilton form of Eq. (32):

$$i\frac{\partial\Psi}{\partial t} = \mathcal{H}\Psi, \quad \mathcal{H} = \rho_3\left(m + \frac{\boldsymbol{\Pi}^2}{2m}\right) + i\rho_2\frac{\boldsymbol{\Pi}^2}{2m}, \tag{33}$$

where

$$\boldsymbol{\Pi} = -\frac{i}{2}\{\mathcal{F}, \nabla\} = \frac{1}{2}\{\mathcal{F}, \boldsymbol{p}\}.$$

This equation can be expressed in the form

$$\mathcal{H} = \rho_3\mathcal{M} + \mathcal{O}, \quad \rho_3\mathcal{O} = -\mathcal{O}\rho_3, \quad \mathcal{M} = m + \frac{\boldsymbol{\Pi}^2}{2m}, \quad \mathcal{O} = i\rho_2\frac{\boldsymbol{\Pi}^2}{2m}. \tag{34}$$

Now we perform the exact FW transformation with the operator

$$U = \frac{\epsilon + \mathcal{M} + \rho_3\mathcal{O}}{\sqrt{2\epsilon(\epsilon + \mathcal{M})}}, \quad U^{-1} = \frac{\epsilon + \mathcal{M} - \rho_3\mathcal{O}_1}{\sqrt{2\epsilon(\epsilon + \mathcal{M})}}, \quad \epsilon = \sqrt{\mathcal{M}^2 + \mathcal{O}^2} = \sqrt{\boldsymbol{\Pi}^2 + m^2}. \tag{35}$$

This transformation results in the diagonalized Hamiltonian

$$\begin{aligned}\mathcal{H}'' = \rho_3\sqrt{m^2 + \boldsymbol{\Pi}^2} \approx \rho_3\epsilon' + \frac{1}{2}\left\{\frac{m^2}{\epsilon'}, V - 1\right\} + \frac{1}{2}\left\{\frac{\boldsymbol{p}^2}{\epsilon'}, \mathcal{F} - 1\right\} \\ + \frac{1}{4\epsilon'}\Delta\mathcal{F} - \frac{\epsilon'^2 + m^2}{4\epsilon'^5}(\boldsymbol{p}\cdot\nabla)(\boldsymbol{p}\cdot\nabla)\mathcal{F}, \quad \epsilon' = \sqrt{m^2 + \boldsymbol{p}^2}.\end{aligned} \tag{36}$$

Three first terms agree with corresponding terms calculated in Refs. [13, 14] (for spin 0 particles) and Refs. [17, 18] (for spin 1/2 particles). The third term describing the contact interaction does not coincide with the corresponding term calculated by Accioly and Blas. It is quite natural because in Refs. [13, 14], in contradistinction to the present work, the term proportional to the curvature of space has been taken into account.

6 Deflection of spin zero particles by a gravitational field

The problem of particle deflection by a gravitational field is very important. The statement about an disagreement of the classical and quantum theories [4, 5, 6] is not confirmed in Ref. [19] where the gravitation interaction of spin 1/2 particles has been considered. The semiclassical formula for the deflection of spin 1/2 particles [19] agrees with the corresponding classical expression [7, 8]. Therefore, it is necessary to fulfil an analogous verification for spin 0 particles.

The operator equation of particle motion has the form

$$\frac{d\boldsymbol{p}}{dt} = i[\mathcal{H}'', \boldsymbol{p}] = -\nabla\mathcal{H}''. \tag{37}$$

The semiclassical transition can be performed in the same way as for spin 1/2 particles (see Refs. [9, 19]). As a result, we obtain the equation

$$\frac{d\boldsymbol{p}}{dt} = -\frac{m^2}{\epsilon'}\boldsymbol{\phi} - \frac{\boldsymbol{p}^2}{\epsilon'}\boldsymbol{f}, \quad \boldsymbol{\phi} = \nabla V, \quad \boldsymbol{f} = \nabla\mathcal{F}. \tag{38}$$

Eq. (38) is in accord with the corresponding equation for spin 1/2 particles [19]. Since for a weak spherically symmetric gravitational field

$$\boldsymbol{\phi} = \frac{2GM}{r^3}\boldsymbol{r}, \quad \boldsymbol{f} = \frac{\boldsymbol{\phi}}{2} = \frac{GM}{r^3}\boldsymbol{r},$$

the equation of particle motion is given by

$$\frac{d\boldsymbol{p}}{dt} = -\frac{GM}{r^3}\frac{\epsilon'^2 + m^2}{\epsilon'}\boldsymbol{r}.$$

This equation agrees with the classical gravity. If the angle of the particle deflection is small, it is defined by the formula

$$\theta = \frac{2GM}{\rho}\left(2 + \frac{m^2}{\boldsymbol{p}^2}\right) = \frac{2GM}{\rho \boldsymbol{v}^2}\left(1 + \boldsymbol{v}^2\right), \tag{39}$$

where ρ is the impact parameter. This formula coincides with Eq. (13) of Problem 15.9 from Ref. [7] (see also Ref. [8]). Thus, the results obtained in Refs. [4, 5, 6] are not confirmed.

7 Conclusions

The Hamiltonian for relativistic spin zero particles in an electromagnetic field has been derived by the successive Feshbash-Villars and FW transformations. The Hamiltonian for point particles contains non-classical terms. These terms describe both the quadrupole interaction of moving particles with an electric field and electric and mixed polarizabilities. For point particles, these properties are of purely quantum origin.

The generalized DK equation and the relativistic Hamiltonian in the FW representation for spin zero particles in the gravitational field is obtained. The semiclassical equation of particle motion in the weak spherically symmetric field reproduces the well-known classical result for the angle of particle deflection. Therefore, the statement about an disagreement of the classical and quantum theories [4, 5, 6] is not confirmed.

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References

[1] A.J. Silenko, Yad. Fiz. **64** (2001) 1048 [Phys. Atom. Nucl. **64** (2001) 977]
[2] H. Feshbach and F. Villars, Rev. Mod. Phys. **30** (1958) 24
[3] L.L. Foldy, S.A. Wouthuysen, Phys. Rev. **78** (1950) 29
[4] A. Accioly, Int. J. Mod. Phys. D **11** (2002) 1579
[5] A. Accioly, S. Ragusa, H. Blas, and H. Mikai, Int. J. Theor. Phys. **41** (2002) 1019
[6] A. Accioly and S. Ragusa, Class. Quant. Grav. **19** (2002) 5429
[7] A.P. Lightman, W.H. Press, R.H. Price, and S.A. Teukolsky, *Problem book in relativity and gravitation*, Princeton Univ. Press, Princeton, 1975
[8] D. Fargion, Lett. Nuovo Cim. **31** (1981) 49
[9] A.J. Silenko, J. Math. Phys. **44** (2003) 2952
[10] J.D. Bjorken and S.D. Drell, *Relativistic quantum mechanics*, McGraw-Hill, New York, 1964
[11] E.I. Blount, Phys. Rev. **128** (1962) 2454
[12] N.A. Chernikov and E.A. Tagirov, Ann. Inst. Henri Poincaré **9** (1968) 109
[13] A. Accioly and H. Blas, Phys. Rev. D **66** (2002) 067501; arXiv:gr-qc/0201015
[14] A. Accioly and H. Blas, Mod. Phys. Lett. A **18** (2003) 867
[15] J. Audretsch and G. Shäfer, Gen. Rel. Grav. **9** (1978) 243
[16] K. Varjú and L.H. Ryder, Phys. Rev. D **62** (2000) 024016
[17] Yu.N. Obukhov, Phys. Rev. Lett. **86** (2001) 192; arXiv:gr-qc/0012102
[18] Yu.N. Obukhov, Fortsch. Phys. **50** (2002) 711; arXiv:gr-qc/0112080
[19] A.J. Silenko and O.V. Teryaev, arXiv:gr-qc/0407015