Local reality in quantum theory

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Abstract

We describe a scheme for constructing quantum mechanics in which the Hilbert space and linear operators are secondary structures of the theory. The elements of a noncommutative algebra (i.e., the observables) and the nonlinear functionals on this algebra (i.e., the physical states which associate with results of a single measurement) serve as the primary components of the theory. It is shown that in such scheme the mathematical formalism of the standard quantum mechanics does not contradict a hypothesis about existence of an objective local reality and a principle of causality.

Principles of locality and the causality occupy a central place in structure of quantum field theory. So the axiom of locality is one of basic in the Wightman axiomatics [1]. In the Bogoliubov [2] approach the condition of causality is a main constructive element of the theory. The local algebras are the basic component of the algebraic approach [3]. The properties of these algebras are significally determined by the axiom of locality. The large role plays also the Haag principle of primitive causality [4].

The principles of locality and causality are tightly related among themselves. On the physical essence they are very close to the concept of "the objective local theory" which was introduced in works of Bell [5, 6]. The basic supposition of Bell consists in the following.

The properties of a physical system exist objectively, irrespective of measurement and are fulfilled the requirements:
1) every system is characterized by some variables, probably, correlated for two systems(causality);
2) the results of measurement of one system do not depend on whether measurements on other system is made (locality);
3) the characteristics of statistical ensembles depends only on conditions in earlier times, "the retrospective causality" is impossible.

Actually, these requirements are the further development of the concept of "the physical reality", formulated in the famous work of Einstein-Podolsky-Rosen [7]:
a) every element of the physical reality must have a counter-part in the complete physical theory;
b) if, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there is an element of physical reality corresponding to this physical quantity.

All these statements seem quite reasonable. However, they have one essential drawback — they badly agree with the basic conceptions of the standard quantum mechanics (theory of Bohr, Heisenberg, Dirac, von Neumann). At least, so it is considered. On the other hand, the corollaries of the standard quantum mechanics perfectly agree with enormous range of phenomena.

In proposed work I want to show that the statements, formulated by Bell and Einstein-Podolsky-Rosen, and corollaries of the standard quantum mechanics are not in such antagonistic inconsistency, as it seems at the first glance.

We shall accept the algebraic approach to quantum theory: the elements of some algebra corresponds to observable quantities. It is convenient to consider complex combinations of the observables, which further will be referred to as dynamical quantities.

In this context, we adopt

Postulate 1.

Dynamic variables correspond to elements of an involutive associative (in the general case) noncommutative algebra $\mathfrak{A}$, satisfying the following conditions :
1. for any element $\hat{R}\in\mathfrak{A}$ there exists a Hermitian element $\hat{A}$ ($\hat{A}^*=\hat{A}$) such that $\hat{R}^*\hat{R}=\hat{A}^2$;
2. if $\hat{R}^*\hat{R}=0$, then $\hat{R}=0$.

The observable variables correspond to the Hermitian elements of the algebra $\mathfrak{A}$. We let $\mathfrak{A}_+$ denote the set of these elements.

The so-called simultaneously measurable (compatible) observables play a preferential role in quantum mechanics. It is observables, for which there are measuring apparatuses (system of apparatuses), permitting in principle to measure them simultaneously to any desired degree of precision.

We adopt

Postulate 2.

Mutually commuting elements of the set $\mathfrak{A}_+$ *correspond to compatible observables.*

In connection with this postulate commutative subalgebras of the algebra $\mathfrak{A}$ will play essential role in the following. Let us designate a maximal real commutative subalgebra of the algebra $\mathfrak{A}$ by $\mathfrak{Q}_\xi$ ($\mathfrak{Q}_\xi \equiv \{\hat{Q}\}_\xi \in \mathfrak{A}_+$). It is algebra of the simultaneously measurable observables. The subscript ξ distinguishes one such subalgebra from the other.

The Hermitian elements of the algebra $\mathfrak{A}$ are latent form of the observable quantities. The explicit form of an observable should be some number. This means that to determine the explicit form of the observables on the Hermitian elements of the algebra $\mathfrak{A}$ we must define some functional $\varphi(\hat{A}) = A$, where A is a real number. Physically, the latent form of the observable $\hat{A}$ becomes explicit as a result of measurement. This means that the functional $\varphi(\hat{A})$ should determine the value of the observable $\hat{A}$ that may result from *a concrete (individual)* measurement. We call this functional the physical state of the quantum object.

Only mutually commuting elements can be measured in an individual experiment. The sum and the product of the observables should correspond to the sum and the product of the measurement results: $\hat{A}_1 + \hat{A}_2 \to A_1 + A_2$ and $\hat{A}_1\hat{A}_2 \to A_1A_2$. We use the following definition [8].

Let $\mathfrak{B}$ be a complex (real) commutative algebra and φ be a linear functional on $\mathfrak{B}$. If

$$\varphi(\hat{B}_1\hat{B}_2) = \varphi(\hat{B}_1)\varphi(\hat{B}_2) \tag{1}$$

for any $\hat{B}_1, \hat{B}_2 \in \mathfrak{B}$, then the functional φ is called a complex (real) homomorphism on algebra $\mathfrak{B}$.

A functional that satisfies equality (1) also called a multiplicative functional.

We now formulate

Postulate 3.

The physical state of a quantum object, appearing in the individual measurement, is described by a (generally, multi-valued) functional $\varphi(\hat{A})$ ($\hat{A} \in \mathfrak{A}_+$), *for which the restriction* ($\varphi_\xi(\hat{A})$) *on any subalgebra* $\mathfrak{Q}_\xi$ *is single-valued and is a real homomorphism* ($\varphi_\xi(\hat{A}) = A$ *is real number).*

It is possible to show (see [8]) that the functionals, appearing in the third postulate, have the properties:

$$\begin{aligned}
&/1/ \quad \varphi_\xi(0) = 0; \\
&/2/ \quad \varphi_\xi(\hat{I}) = 1; \\
&/3/ \quad \varphi_\xi(\hat{A}^2) \geq 0; \\
&/4/ \quad \text{if } \lambda = \varphi_\xi(\hat{A}), \text{ than } \lambda \in \sigma(\hat{A}); \\
&/5/ \quad \text{if } \lambda \in \sigma(\hat{A}), \text{ than } \lambda = \varphi_\xi(\hat{A}) \text{ for some } \varphi_\xi(\hat{A}).
\end{aligned} \tag{2}$$

Here $\sigma(\hat{A})$ is a spectrum of the element $\hat{A}$ in algebra $\mathfrak{A}$. In the standard quantum mechanics the corresponding properties of individual measurements are postulated, here they are consequences of the third postulate.

The multi-valuedness of the functional φ is caused by the fact that the result of measurement can depend not only on the measured quantum object, but also on the nature of the measuring device.

Let us say that the device, measuring an observable $\hat{A}$, is coordinated with subalgebra of observables $\mathfrak{Q}_\xi \quad (\hat{A} \in \mathfrak{Q}_\xi)$, if for any physical state φ the measurement outcome is $\varphi_\xi(\hat{A})$.

The coordination of the device with this or that subalgebra $\mathfrak{Q}_\xi$ is determined by its classical characteristics, i.e. by the construction, position in space and so forth. Multi-valuedness of the functional $\varphi(\ )$ allows to introduce it in a consistent manner. It is possible to make it by direct construction. Due to multi-valuedness of the functional φ, the conditions of the Kochen-Specker [9] no-go theorem are not fulfilled for it. Thus, the functional $\varphi(\ )$ does not describe the value of an observable $\hat{A}$ in a particular physical state. It describes reaction of the particular type of measuring device to the observable $\hat{A}$. Correspondingly a physical reality is not the value of an observable $\hat{A}$ in the considered physical state, but a reaction of the measuring device to this state.

If the functional $\varphi(\ )$ is single-valued in the point $\hat{A}$, we shall say that the corresponding physical state φ is stable on the observable $\hat{A}$.

Now we shall introduce a construction, which corresponds to a pure state in the standard quantum mechanics. The functional φ maps a set $\mathfrak{Q}_\xi = \{\hat{Q}\}_\xi$ (maximal commutative subalgebra) into the set of real numbers:

$$\{\hat{Q}\}_\xi \xrightarrow{\varphi} \{Q = \varphi(\hat{Q})\}_\xi.$$

For different functionals $\varphi_i(\ )$, $\varphi_j(\ )$ the sets $\{\varphi_i(\hat{Q})\}$, $\{\varphi_j(\hat{Q})\}$ can either differ or coincide. If for all $\hat{Q} \in \{\hat{Q}\}$ is valid $\varphi_i(\hat{Q}) = \varphi_j(\hat{Q}) = Q$, then we shall call physical functionals $\varphi_i(\ )$ and $\varphi_j(\ )$ as $\{Q\}$-equivalent functionals.

Let $\{\varphi\}_Q$ be the set of all $\{Q\}$-equivalent functionals, stable on observables of the subalgebra $\{\hat{Q}\}_\xi$. The set of corresponding physical states we shall call a quantum (pure) state, and we shall designate it by Ψ_Q.

Let us consider the quantum Ψ_Q-ensemble as a general population (in sense of probability theory), and any experiment aimed to measured an observable $\hat{A}$ as a trial. Let the event $\tilde{A}$ be experiment in which the measured value of the observable $\hat{A}$ is no larger then $\tilde{A}$, i.e., $\varphi(\hat{A}) = A \leq \tilde{A}$. This event is not unconditional. By virtue of the second postulate one trial cannot be event for two noncommuting observables. The probability of the event $\tilde{A}$ is determined by structure of quantum ensemble and this condition. Let this probability be equal to $P(\tilde{A})$. We designate $\{\varphi\}_Q^{\hat{A}} \quad (\{\varphi\}_Q^{\hat{A}} \subset \{\varphi\}_Q)$ the set of the physical states, which figure in denumerable sample of mutually independent random trials for measurement of the observable $\hat{A}$.

By definition, the probability of appearance of the event $\tilde{A}$ in each trials is equal to $P(\tilde{A})$. It determines a probability measure $\mu(\varphi)$ $(\varphi(\hat{A}) \leq \tilde{A})$ on any such sample. In its turn, the measure $\mu(\varphi)$ determines distribution of values $A_i = \varphi_i(\hat{A})$ of the observable $\hat{A}$ and expectation $< A >$ in this sample:

$$< A >= \int_{\{\varphi\}_Q^{\hat{A}}} d\mu(\varphi)\, \varphi(\hat{A}).$$

Let for $1 \leq i \leq n$ functionals $\varphi_i \in \{\varphi\}_Q^{\hat{A}}$, then according to the theorem of Hinchin (the law of large numbers, see for example [10]) the aleatory variable $\bar{A}_n = (A_1 + \ldots + A_n)/n$ converges on probability to $< A >$ as $n \to \infty$. Thus,

$$\text{P-}\lim_{n\to\infty} \frac{1}{n}\Big(\varphi_1(\hat{A}) + \ldots + \varphi_n(\hat{A})\Big) =< A >\equiv \Psi_Q(\hat{A}). \tag{3}$$

Formula (3) defines the functional (quantum average) on the set $\mathfrak{A}_+$.

The totality of quantum experiments leads to conclusion that we should accept

Postulate 4.

The functional $\Psi_Q(\ )$ *is linear on the set* $\mathfrak{A}_+$.

This means that

$$\Psi_Q(\hat{A} + \hat{B}) = \Psi_Q(\hat{A}) + \Psi_Q(\hat{B})$$

even in the case where $[\hat{A}, \hat{B}] \neq 0$.

Any element $\hat{R}$ of the algebra $\mathfrak{A}$ can be uniquely expressed in form $\hat{R} = \hat{A} + i\hat{B}$, where $\hat{A}, \hat{B} \in \mathfrak{A}_+$. Therefore, the functional $\Psi_Q(\ )$ can be continued to a linear functional on the algebra $\mathfrak{A}$: $\Psi_Q(\hat{R}) = \Psi_Q(\hat{A}) + i\Psi_Q(\hat{B})$.

The equality $\|\hat{R}\|^2 \equiv \sup_Q \Psi_Q(\hat{R}^*\hat{R})$ defines a norm of an element $\hat{R} \in \mathfrak{A}$. This norm has properties $\|\hat{R}^*\| = \|\hat{R}\|$, $\|\hat{R}^*\hat{R}\| = \|\hat{R}\|^2$. With such norm the algebra $\mathfrak{A}$ is a C^*-algebra.

Therefore, according to the Gelfand-Naumark-Segal construction (see, for example, [3]) the functional $\Psi_Q(\ )$ canonically generates a Hilbert space and the representation of the algebra $\mathfrak{A}$ by linear operators in this Hilbert space. In other words, in the proposed approach it is possible to reproduce the mathematical formalism of the standard quantum mechanics completely.

The physical state is "a physical reality", which Einstein did not see in the standard quantum mechanics, and so he considered that the quantum mechanics is the incomplete theory.

At the same time, the physical states can play a role of the Bell's variables which characterize physical system. Here, however, it is necessary to make an essential remark. Bell implied that these variables are certain numerical parameters (practically hidden parameters). On their basis he has obtained the famous inequality, which contradicts corollaries of the standard quantum mechanics and experiments. In the proposed approach physical states are nonlinear functionals (many-valued). The Bell's inequality does not follow from existence of such specific "hidden parameters" [11].

Besides, for such "hidden parameters" the reasoning of von Neumann [12] on impossibility of existence of hidden parameters is not valid [13]. Here, the basic fact is the nonlinearity of the functional describing the physical state. Actually von Neumann has shown that the linearity of a state is in the conflict with causality and the hypothesis about the hidden parameters.

From this he has made a deduction that causality is absent at the microscopic level, and the causality occurs due to averaging over large number of noncausal events at the macroscopic level.

The approach formulated in the present work allows to solve the same conflict in the opposite way. It is possible to suppose that there is causality at the level of a single microscopic phenomenon, and the linearity is absent. The linearity of the (quantum) state occurs due to averaging over quantum ensemble. The transition from a single phenomenon to the quantum ensemble replaces the initial determinism by probabilistic interpretation.

We now discuss the place of the approach to quantum theory proposed in this article among different possible approaches. In any modern approach to quantum theory, the main structure elements are the observables and the states of the physical system. In standard quantum mechanics, the basic structure element is the Hilbert space. The observable variables are associated with self-adjoint linear operators in this space, while the states are associated with either vectors or statistical operators.

The mathematical formalism constructed within the framework of standard quantum mechanics works very well. The question therefore arises whether it is worthwhile to make efforts towards constructing quantum-theoretical models beyond standard quantum mechanics.

There are good reasons for making such efforts. First is the problem of the physical interpretation. The Hilbert space is a rather specific mathematical concept whose physical interpretation is not straightforward. The consistent formulation of standard quantum mechanics relies on a special "quantum logic" that is also hard to interpret physically. There are problems of causality and locality in quantum measurements. All this is the subject of an everlasting debate on the consistent interpretation of quantum mechanics, a debate that intensified in recent years.

The other reason to attempt to go beyond standard quantum mechanics is the difficulties that the theory encounters in describing systems with an infinite number of degrees of freedom. As shown by von Neumann, all representations of the canonical commutation relation for systems with a finite number of degrees of freedom are equivalent; therefore, standard quantum mechanics can provide a universal description of such systems. However, this statement cannot be made for systems with an infinite number of degrees of freedom. Therefore, a more general construction is needed for obtaining their universal description. Exactly such a construction can be realized in the algebraic approach.

There are two main ingredient in the algebraic approach: the algebra of observables and the set of linear positive functionals on the algebra of observables, interpreted as the set of states of the physical systems.

These functionals have a rather simple physical interpretation. It is postulated that the value of the functional on an element of the algebra coincides with the average value of the observable for physical systems that are in the corresponding state.

On one hand, the framework of the algebraic approach is wider than that of the standard quantum mechanics. Therefore, the algebraic approach holds promise to help circumvent (at least partly) the difficulties that standard quantum mechanics faces. On the other hand, the Gelfand-Neimark-Segal construction allows obtaining the principal components of standard quantum mechanics, i.e., the Hilbert space and linear operators, within the algebraic approach. In the framework of the algebraic approach, these concepts are not primary but secondary. The construction used in standard quantum mechanics is one of the possible nonequivalent representations of the algebra of observables. Hence, the passage from standard quantum mechanics to the algebraic approach is a passage to a deeper level of the quantum theory.

In this sense, the system of postulates proposed in this article can be interpreted as a passage to a level that is yet deeper. The first two of the formulated postulates related to the algebra of the observables are adopted directly from the traditional algebraic approach. Other than this, however, the approach we propose is different. In this approach, the principal role belongs to real commutative subalgebras of observables.

Using these subalgebras, we introduce the essentially new element, i.e., the nonlinear functional φ whose values describe the possible result of an individual measurement (see Postulate 3). This functional is interpreted as the physical state. In contrast, we call the state defined in the traditional algebraic approach (the linear functional) the quantum state.

In the proposed approach, the primary element is the physical state, while the quantum state turns out to be a secondary element, the equivalence class of the physical states. In standard quantum mechanics and the traditional algebraic approach, the result of an individual measurement (the physical reality) does not have any mathematical counter-part. In the proposed approach, such a counter-part is the nonlinear functional φ. The functional φ allows a transparent physical interpretation. In addition, this functional allows bringing the structure of the algebra of observables to the form of the C^*-algebra. This allows using the Gelfand-Neimark-Segal construction and passing to the mathematical scheme of standard quantum mechanics. The traditional algebraic approach normally postulates that the algebra of observables is the C^*-algebra. This is not quite obvious physically.

Standard quantum mechanics and the traditional algebraic approach postulate that the functional associated with the quantum state statistically describes the average value of the corresponding observable. In the proposed approach, this functional is constructed through statistical averaging of the functionals φ. This construction implies that the functional describes the average value of the observable.

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