On a non-standard matter distribution in the Randall-Sundrum model

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Abstract

In the zero mode approximation we solve exactly the equations of motion for linearized gravity in the Randall-Sundrum model with a non-standard distribution of matter in the neighbourhood of the negative tension brane. It is shown that the form of this distribution can strongly affect the coupling of the radion to matter. We believe that such a situation can arise in models with a realistic mechanisms of matter localization.

1 Introduction

In paper [1] an exact solution for a system of two branes interacting with gravity in a five-dimensional space-time was found. This model is called the Randall-Sundrum model (usually abbreviated as RS1 model), and it is widely discussed in the literature (see Refs. [2, 3] for reviews and references). The Randall-Sundrum solution for the metric is given by

$$ds^2 = \gamma_{MN}dx^M dx^N = \gamma_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{1}$$

where $\gamma_{\mu\nu} = e^{2\sigma(y)}\eta_{\mu\nu}$, $\eta_{\mu\nu}$ is the Minkowski metric and the function $\sigma(y) = -k|y|$ in the interval $-R \le y \le R$. The parameter k is positive and has the dimension of mass. The branes are located at the points $y = 0$ and $y = R$.

We denote $\hat\kappa = \sqrt{16\pi\hat G}$, where $\hat G$ is the five-dimensional gravitational constant, and parameterize the metric g_{MN} as

$$g_{MN} = \gamma_{MN} + \hat\kappa h_{MN}, \tag{2}$$

h_{MN} being the metric fluctuations. It was shown in [6] that with the help of the gauge transformations for h_{MN} we can impose the gauge

$$h_{\mu 4} = 0,\ h_{44} = h_{44}(x) \equiv \phi(x), \tag{3}$$

which will be called the *unitary gauge* (see [4]). We would like to emphasize that the branes remain straight in this gauge, i.e. we *do not* use the bent-brane formulation, which allegedly destroys the structure of the model (this problem was discussed in [5]).

The general form of interaction with matter is standard,

$$\frac{\hat\kappa}{2}\int h^{MN}(x,y)T_{MN}\sqrt{-\gamma}dxdy, \tag{4}$$

where T_{MN} is the energy-momentum tensor of matter. We would like to note that the matter described by T_{MN} does not change the background solution for the metric.

We will examine the $h_{\mu\nu}$-components of the metric fluctuations h_{MN}, since all four-dimensional physical effects can be described in terms of this field. Obviously, the unitary gauge conditions (3) do not fix the gauge of this field. In fact, after imposing these gauge conditions there remain residual gauge transformations which change the longitudinal components of the field $h_{\mu\nu}$ [6]. Nevertheless, it turns out that it is convenient to solve the equations of motion for linearized gravity in the unitary gauge and then to choose an appropriate gauge in our four-dimensional world on the brane. We will use the de Donder gauge for the field $h_{\mu\nu}$ on the brane, which corresponds to the choice of *harmonic coordinates*.

2 Equations of motion

The equations of motion for different components of the metric fluctuations in the unitary gauge take the form (see [4, 6]):

1) $\mu 4$-component,

$$\partial_4(\partial_\mu \tilde{h} - \partial^\nu h_{\mu\nu}) - 3\partial_4\sigma\partial_\mu\phi = -\hat{\kappa}T_{\mu 4}, \tag{5}$$

which plays the role of a constraint,

2) 44-component

$$\frac{1}{2}(\partial^\mu\partial^\nu h_{\mu\nu} - \partial_\mu\partial^\mu\tilde{h}) - \frac{3}{2}\partial_4\sigma\partial_4\tilde{h} + 6k^2\phi = -\frac{\hat{\kappa}}{2}T_{44}, \tag{6}$$

where $\tilde{h} = \gamma^{\mu\nu}h_{\mu\nu}$. Equation on the $\mu\nu$-component is the same as that one in [6].

For example, if $T_{MN} = 0$, the physical degrees of freedom of the model can be extracted by the substitution [4]

$$h_{\mu\nu} = b_{\mu\nu} + \gamma_{\mu\nu}(\sigma - c)\phi + \frac{1}{2k^2}\left(\sigma - c + \frac{1}{2} + \frac{c}{2}e^{-2\sigma}\right)\partial_\mu\partial_\nu\phi. \tag{7}$$

with $c = \frac{kR}{e^{2kR}-1}$. It turns out that the field $b_{\mu\nu}(x^\mu, y)$ describes the massless graviton [1, 7] and massive Kaluza-Klein spin-2 fields, whereas $\phi(x)$ describes a scalar field called the radion.

However, the situation is rather different, when there is matter on the branes. The cases in which matter is located on the branes, i.e. the energy-momentum tensor is of the form $T_{\mu\nu} = t_{\mu\nu}(x)\delta(y)$ or $T_{\mu\nu} = t_{\mu\nu}(x)\delta(y - R)$ (and $T_{\mu 4} = 0$, $T_{44} = 0$) were discussed in detail in [6]. It is well known that if we live on the negative tension brane (at $y = R$), the contribution of the radion is e^{2kR} times stronger than the contribution of the massless graviton [6]. It means that in the case of the massless scalar gravity is realized on brane 2, and it is necessary to assume some mechanism for generating the radion mass, for example, the Goldberger-Wise mechanism [8] to make gravity in the zero mode approximation tensor. But we will show below, that this is not the only solution for this problem.

3 A simple example

In the original formulation of the Randall-Sundrum model the mechanism of localization of fields is not taken into account. The branes are treated as infinitely thin objects, and it is assumed that energy-momentum tensor has $\delta(y)$-like profile in the extra coordinate (matter is located on the brane only). Nevertheless in some papers (see, for example, [9, 10, 11, 12, 13]) considering models with extra dimensions, energy-momentum tensors with dependence on the extra coordinates were used for constructing non-trivial background metric. One can also recall models with universal extra dimensions, in which physical fields can propagate in the extra dimension [14], and fat brane scenarios [15, 16]. Thus the existence of energy-momentum tensors of this type seems to be quite reasonable. Here we consider some simple examples, which are sufficient to show attractive features of the non-delta-like localization of matter.

Let us choose as an example the following form of the energy-momentum tensor

$$T_{\mu\nu} = t_{\mu\nu}(x)\frac{e^{-2\sigma}c}{R}, \tag{8}$$

$$T_{\mu 4} \equiv 0, \tag{9}$$

$$T_{44} = t_{\mu\nu}(x)\gamma^{\mu\nu}e^{-2\sigma}\frac{c}{R}\left(\sigma + e^{2kR}c - \epsilon c\right), \tag{10}$$

where ϵ is some arbitrary constant, which will be defined later. Here $t_{\mu\nu}(x)$ is the energy-momentum tensor of matter on the brane, it depends on the four-dimensional coordinates x only. One can see that the matter is localized near the negative tension brane at $y = R$. We would like to note that the energy-momentum tensor of the form (8), (9) and (10) satisfies the covariant energy conservation law, which has the form

$$\nabla^N T_{MN} = 0, \tag{11}$$

and the function of localization in (8) is normalized to unity. Constructions similar to that used in this paper, but in the case of six dimensions, were utilized in [17, 18]. The explanation of such constructions depends on the method of localization, and this issue will not be discussed in this paper.

The substitution, which allows one to decouple the equations of motion with T_{MN} of the form (8), (9), (10) looks like

$$b_{\mu\nu} = u_{\mu\nu} + \frac{1}{2k^2}\left(\frac{ce^{2kR}}{\epsilon} - e^{-2\sigma}\left[\frac{ce^{2kR}}{2\epsilon} + \frac{1}{8\epsilon}\right] - \sigma e^{-2\sigma}\frac{1}{2\epsilon}\right)\partial_\mu\partial_\nu\phi, \tag{12}$$

where $b_{\mu\nu}$ was defined in (7) and $u_{\mu\nu}(x^\mu, y)$ describes the massless graviton and massive Kaluza-Klein spin-2 fields. Substituting (7) and (12) into equations (5), (6) and $\mu\nu$-equation, we get the corresponding equation for the massless modes of the theory (a completely analogous procedure was made in [6]):

$$\Box\phi = \frac{\hat{\kappa}\epsilon c}{3R}t, \tag{13}$$

$$\partial^\nu\left(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha\right) = 0, \tag{14}$$

$$\Box\left(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha\right) = -2\frac{\hat{\kappa}c^2e^{2kR}}{R}t_{\mu\nu}. \tag{15}$$

Now we are ready to find equations of motion for the massless part of $h_{\mu\nu}$. Contributions of massive modes are not interesting for us, because the masses even of the lowest modes are of the order of TeV. Since the matter is localized near the brane, we will calculate the equation for $h_{\mu\nu}|_{y=R}$. Passing to the de Donder gauge condition for the field $h_{\mu\nu}$ *on brane 2* (analogously to what was made in [6]), we get in the zero mode approximation

$$\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right)|_{y=R} = e^{-2kR}\left(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha\right) - 2e^{-2kR}(-kR-c)\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)\phi. \tag{16}$$

An important point is that these equations are written in the coordinates $\{x^\mu\}$, which are *Galilean on brane 1* (not on brane 2) and are inappropriate for studying physical effects on brane 2 (we recall that coordinates are called Galilean, if $g_{\mu\nu} = diag(-1,1,1,1)$ [19]). It is necessary to pass to Galilean coordinates on brane 2 to get a correct result. This problem was discussed in detail in papers [4, 6]. In Galilean coordinates on brane 2 (16) looks like

$$\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right)|_{y=R} = e^{-2kR}\left(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha\right) - 2(-kR-c)\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)\phi, \tag{17}$$

and equations (13), (15) take the form

$$\Box\phi = \frac{\hat{\kappa}\epsilon c}{3R}t, \tag{18}$$

$$\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) = -2\frac{\hat{\kappa}c^2e^{4kR}}{R}t_{\mu\nu}. \tag{19}$$

Thus, we can get

$$\Box\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right)|_{y=R} = -\frac{2\hat{\kappa}c^2e^{2kR}}{R}\left(t_{\mu\nu} - \frac{\epsilon}{3}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)t\right), \tag{20}$$

where $t_{\mu\nu}$ is the energy-momentum tensor of matter in coordinates, which are Galilean at $y = R$. One can see, that this equation coincides with equation for the fluctuations of metric in the linearized Brans-Dicke theory, which looks like

$$\Box\left(\delta g_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\delta g\right) = -16\pi G\left(t_{\mu\nu} - \frac{1}{2\omega+3}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)t\right), \tag{21}$$

where ω is the BD-parameter and G - gravitational constant. Thus, comparing equations (20) and (21), we get

$$G = \hat{G}\frac{2c^2e^{2kR}}{R}, \tag{22}$$

$$\omega = \frac{3(1-\epsilon)}{2\epsilon}. \tag{23}$$

It is easy to see, that in the the Randall-Sundrum model with the δ-function-like localization of matter on the brane there is a factor e^{2kR} instead of ϵ in (20) (compare with the analogous formula in [6]).

¿From the recent experimental data (see, for example, [20]) we know, that $\omega > 3500$. It means that

$$\epsilon < 4,3 \cdot 10^{-4} \tag{24}$$

With this value of ϵ there are already no problems with the radion field.

4 Localization of matter

In (8) the factor $e^{-2\sigma}$, describing distribution of matter in the extra dimension, was utilized. However one can say that matter is spread in the whole bulk rather than localized on the brane (in analogy to the massless graviton whose wave function is $\sim e^{2\sigma}$), and it is not evident, why we make all calculations for $h_{\mu\nu}$ at $y = 0$. It seems that it is more reasonable to consider matter, which is confined to brane 2 much stronger, since the description of gravity on brane 2 by $h_{\mu\nu}|_{y=0}$ is more justified in this case. Let us consider a "toy model" with energy-momentum tensor of the form

$$T_{\mu\nu} = t_{\mu\nu}(x)\frac{k(1+N)}{e^{2kR+2NkR}-1}\, e^{-2\sigma(1+N)}, \tag{25}$$

where the function of localization is normalized to unity as well. Other components of the five-dimensional energy-momentum tensor are chosen to be

$$T_{\mu 4} = 0, \tag{26}$$

$$T_{44} = t(x)\,(k+kN)\, e^{-4\sigma}\left(\frac{c}{2NkR} - \frac{c\epsilon e^{2NkR}}{e^{2kR+2NkR}-1} - \frac{k}{2Nk\,(e^{2kR+2NkR}-1)}\, e^{-2N\sigma}\right). \tag{27}$$

Such a complicated form of (27) is caused by the requirements to satisfy the energy conservation law (11) and to get a weak coupling constant for the radion.

The substitution, which allows one to decouple and solve equations (5), (6) and $\mu\nu$-equation with (25), (26) and (27), has the following form:

$$b_{\mu\nu} = u_{\mu\nu} + \left(\frac{e^{2kR}\left(1-e^{-2NkR}\right)}{4k^2N\epsilon\,(e^{2kR}-1)} - \frac{\left(e^{2kR}-e^{-2NkR}\right)}{8k^2N\epsilon\,(e^{2kR}-1)}\, e^{-2\sigma} \right. \tag{28}$$
$$\left. + \frac{e^{-2NkR}}{4Nk\,(Nk+2k)\,\epsilon}\, e^{-2\sigma-2N\sigma}\right)\partial_\mu\partial_\nu\phi,$$

Analogously to what was made in Section 3, we can get

$$\Box\phi = \frac{\hat{\kappa}\epsilon\,(k+kN)\,e^{2NkR}}{3\,(e^{2kR+2NkR}-1)}t, \tag{29}$$

$$\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) = -\frac{\hat{\kappa}k\,(k+Nk)\left(1-e^{-2NkR}\right)}{Nk\,(e^{2kR}-1)\,(1-e^{-2kR-2NkR})}t_{\mu\nu}. \tag{30}$$

Using (28) and passing to Galilean coordinates at $y = R$, in the zero mode approximation we get

$$\Box\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right)|_{y=R} = \tag{31}$$
$$= -\frac{\hat{\kappa}c\,(1+N)\left(1-e^{-2NkR}\right)}{NR\,(1-e^{-2kR-2NkR})}\left(t_{\mu\nu} - \frac{2NkR\epsilon}{3\,(1-e^{-2NkR})}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)t\right).$$

One can see that in the limit $N \to 0$ equation (31) passes into (20). The four-dimensional gravitational constant has the form

$$G = \hat{G}\frac{c\,(1+N)\left(1-e^{-2NkR}\right)}{NR\,(1-e^{-2kR-2NkR})} \tag{32}$$

(compare with (21)). For a large N we get

$$G \approx \hat{G}\frac{c}{R}. \tag{33}$$

A choice of relatively small ϵ makes the contribution of the radion to be not in contradiction with the present-day experimental data.

5 Conclusion and final remarks

There may arise a question about generation of the new hierarchy instead of the one solved in the Randall-Sundrum model. Actually, the factor ϵ in (10) is much smaller than the factor e^{2kR} in (10). But for the energy-momentum tensor on the brane 2 (at $y = R$) we get

$$T_{\mu\nu}|_{y=R} = t_{\mu\nu}\frac{e^{2kR}c}{R} \approx k t_{\mu\nu},$$

$$T_{44}|_{y=R} = t\frac{e^{4kR}c^2}{R}\left(1 - 4.3 \cdot 10^{-4}\right) \approx t\, k^2 R \left(1 - 4.3 \cdot 10^{-4}\right),$$

and the correction $\sim \epsilon$ is not so small in comparison with the main value ~ 1 (analogous calculations can be made in the case (25) and (27) too). It seems to be quite reasonable. One should take into account that the energy momentum tensors of the form (8), (9), (10) and (25), (26), (27) do not correspond to any real action, and must be interpreted as "toy models". There also arises the question about the origin of such correction $\sim \epsilon$. Explicitly the term $\sim e^{-4\sigma}$ in (10) and (27) corresponds to the homogeneous solution of equation (11) (i.e. to the case of $T_{\mu\nu} = 0$). Nevertheless such term can be taken into consideration, since its existence does not evidently contradict our assumptions. Surely it would be better to derive similar form of the energy-momentum tensor in a more natural way. For example, simple examples with scalar and gauge fields in the bulk in the Randall-Sundrum model were discussed in papers [21, 22]. Wave functions of these fields have their maximum values on brane 2, and these solutions can be treated as a basis for our choice of the energy-momentum tensor. But even in the simplest case of the scalar field only, there arises a nonzero $T_{\mu 4}$-component of the energy-momentum tensor, which does not satisfy the condition (9). In addition, the wave functions are different not only for different fields, but even for different modes. Thus, formulas (8), (9), (10) and (25), (26), (27) must be used as "toy models" for classical objects, but not for the fields (for example, one can choose $t_{00} \sim m\delta(\vec{x})$, $t_{i0} = 0$, $t_{ij} = 0$ for a static point-like source).

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