Renormalization prescriptions and bootstrap in effective theories

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Abstract

We discuss the peculiar features of the renormalization procedure in the case of infinite-component effective theory. It is shown that in the case of physically interesting theories (namely, those leading to the amplitudes with asymptotic behavior governed by known Regge intercepts) the full system of required renormalization prescriptions only contains those needed to fix the minimal counterterm vertices with one, two, three and (in some cases) four lines. There is no necessity in imposing the prescriptions for n-leg counterterm vertices with $n > 4$. Moreover, the prescriptions for $n \leq 4$ cannot be taken arbitrary: an infinite system of bootstrap constraints must be taken into account. The general method allowing one to write down the explicit form of this system is explained and illustrated by the example of elastic scattering process.

1 Introduction

In this talk we give a brief description of the mathematical scheme that allows one to put in order an infinite set of renormalization prescriptions (RP's) needed to fix the physical content of the *localizable effective scattering theory*[1] of strong interactions. For simplicity we only consider the non-strange sector. This means that all the true asymptotic states are constructed from pions and nucleons and the free Hamiltonian is solely defined by the operators of these fields (we rely upon the intrinsically quantum construction of the field theory described in [8]). An infinite set of auxiliary resonance fields only appears in the *extended perturbation scheme* which we use to assign rigorous perturbative meaning to the formal Dyson's series of the initial effective theory. It is this scheme which we discuss below. Here we have no space for detailed discussion — it will be published elsewhere [9].

2 The Cauchy formula in hyper-layers

In this Section we remind the main features of the mathematical tool that turns out especially useful for constructing the explicit form of the system of bootstrap conditions.

Consider the function $f(z, \mathbf{x})$ analytic in complex variable z and smoothly depending on a set of parameters $\mathbf{x} \equiv \{x_i\}$. Let us further suppose that when $\mathbf{x} \in D$ (here D is a small domain in the space of parameters) this function has only a finite number of singular points in every finite domain of the complex-z plane. On Fig. 1 it is shown the geography of singular points s_k ($k = 0, \pm 1, \pm 2, \ldots$) typical for the finite loop order scattering amplitudes in quantum field theory. Note that the cuts are drawn in unconventional way — this is done for the sake of convenience. If the point s_k corresponds to the pole type singularity there is no need in cut, but its presence makes no influence on the results discussed below.

Let us recall the definition of the polynomial boundedness property adjusted [1] for the case of many variables. Consider the system of closed embedded contours $C(i) \equiv C_{-m_i, n_i}$ (see Fig. 1) such that $m_i \geq m_j$ and $n_i \geq n_j$ when $i \geq j$ (both left and right singular points are enumerated in order of increasing modulo). We say that the function $f(z, \mathbf{x})$ is N-bounded in the hyper-layer $B_x\{z \in \mathbf{C},\, \mathbf{x} \in D\}$ if there is an infinite system of contours $C(i)$ ($i = 1, 2, \ldots$) and an integer N such that, when $i \to \infty$,

$$\max_{\mathbf{x} \in D;\ z \in C(i)} \left| \frac{f(z, \mathbf{x})}{z^{N+1}} \right| \longrightarrow 0 \ . \tag{1}$$

[1] The preliminary discussion of the problem and definitions of the terms can be found in [1]–[3], see also our talks [4]–[6] given at HSQCD'2004 and two another talks [7] at this conference.

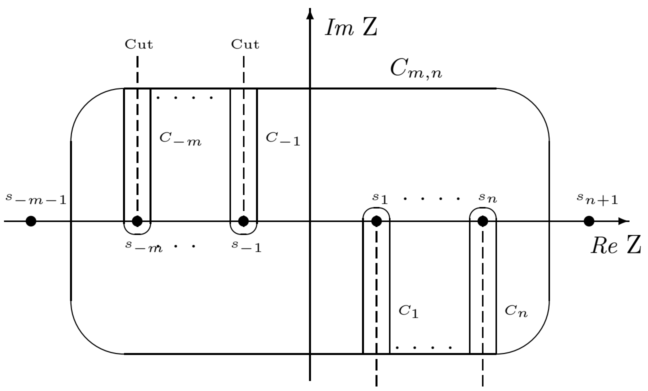

Figure 1: *System of contours on the complex-z plane.*

The *minimal* N (possibly, negative) providing the correctness of the uniform (in x) estimate (1) we call the degree of bounding polynomial in B_x.

Let us now make use of the famous Cauchy's integral formula for the function $f(z,\mathbf{x})/z^{N+1}$. Consider the closed contour (see Fig. 1) consisting of $C(i)$ (except small segments crossing the cuts) and the corresponding parts of the contours C_{-p}, C_q ($p = 1, 2, ..., m_i;\ q = 1, 2, ..., n_i;$) surrounding cuts[2]. Taking the limit $i \to \infty$ and keeping in mind (1) one obtains

$$f(z,\mathbf{x}) = \sum_{k=0}^{N} \frac{1}{k!} f^{(k)}(0,\mathbf{x}) z^k + \frac{z^{N+1}}{2\pi i} \sum_{k=-\infty}^{\infty} \int\limits_{C_k} \frac{f(\xi,\mathbf{x})}{\xi^{N+1}(\xi - z)}\, dz\ , \tag{2}$$

where the last sum is done in order of increasing modulo of the singularities s_k which the contours C_k are drawn around. The relation (2) provides a mathematically correct form of the result. In the case when the number of singular points is infinite, every contour integral on the right side should be considered as a single term of the series. The mentioned above order of summation provides a guarantee of the uniform (in z and $\mathbf{x}$) convergence of the series.

The formula (2) plays the key role in the renormalization programme discussed below.

3 Tentative consideration

As noted in [2], the problem of ordering the full set of renormalization prescriptions needed to fix the physical content of the effective scattering theory should be considered in terms of the resultant parameters [3]. The simple example below explains the main idea of our renormalization procedure and shows the source of the bootstrap conditions.

Let us consider the case of elastic scattering process

$$a(p_1) + b(k_1) \to a(p_2) + b(k_2)\ . \tag{3}$$

For simplicity, we consider both a and b particles to be spinless. This considerably simplifies the purely technical details without changing the logical line of the analysis.

Along with the conventional kinematical variables $s = (k_1 + p_1)^2$, $t = (k_1 - k_2)^2$, $u = (k_1 - p_2)^2$ we introduce three equivalent pairs of independent ones:

$$(x, \nu_x), \quad x = s, t, u; \quad \text{where } \nu_s = u - t,\ \nu_t = s - u,\ \nu_u = t - s. \tag{4}$$

The pair (x, ν_x) provides a natural coordinate system in 3-dimensional (one complex and one real coordinate) layer $B_x\{\nu_x \in \mathbf{C};\ x \in (a, b) \in \mathbf{R}\}$.

[2]We assume that f is regular at the origin. However $f(z,\mathbf{x})/z^{N+1}$ may have a singularity there and to apply the Cauchy theorem one should add a circle around the origin to the contour of integration. It is this part of the contour that gives the first sum in the right side of Eq. (2).

Let us now suppose that in $B_t\{\nu_t \in \mathbf{C};\ t \in \mathbf{R},\ t \sim 0\}$ the full (non-perturbative) amplitude of the process (3) is described by the 0-bounded function $f(\nu_t, t)$ ($N_t = 0$). According to the uniformity principle[3], we have to construct the perturbation series

$$f(\nu_t, t) = \sum_{l=0}^{\infty} f_l(\nu_t, t)$$

in such a way that the full sum $f_l(...)$ of the l-th loop order graphs must also present the 0-bounded function in B_t. Hence, from the relation (2) it follows that in this layer

$$f_l(\nu_t,\ t) = f_l(0, t) + \frac{\nu_t}{2\pi i} \sum_{k=-\infty}^{\infty} \int_{C_k(t)} \frac{f_l(\xi, t)}{\xi(\xi - \nu_t)}\, d\xi \quad . \tag{5}$$

Here the notation $C_k(t)$ is used to stress that we deal with singularities (and, hence, with cuts) in the complex-ν_t plane; the variable t should be considered as a parameter.

We suppose that all the numerical parameters, needed to fix the finite (renormalized) amplitudes of the previous loop orders, are known and one only needs to carry out the renormalization of the l-th order graphs. To do this let us assume (this assumption is justified below) for the moment that the infinite sum of integrals in (5) solely depends on the parameters already fixed on the previous steps of renormalization procedure. Thus it only remains to fix the function $f_l(0, t)$ — then (5) will give the complete renormalized expression for the l-th order contribution in B_t. It is sufficient to fix the values of the coefficients in power series expansion of $f_l(0, t)$. This can be done with the help of self-consistency requirement.

To make use of this requirement we need to consider the cross-conjugated process

$$a(p_1) + \overline{a}(-p_2) \to \overline{b}(-k_1) + b(k_2)\ . \tag{6}$$

Let us suppose that in $B_u\{\nu_u \in \mathbf{C};\ u \in \mathbf{R},\ u \sim 0\}$ it is described by the (-1)-bounded ($N_u = -1$) amplitude

$$\phi(\nu_u,\ u) = \sum_{l=0}^{\infty} \phi_l(\nu_u,\ u)\ .$$

The uniformity principle tells us that every function $\phi_l(\nu_u,\ u)$, in turn, must be (-1)-bounded in B_u. Hence in this layer

$$\phi_l(\nu_u,\ u) = \frac{1}{2\pi i} \sum_{k=-\infty}^{\infty} \int_{C_k(u)} \frac{\phi_l(\xi, u)}{(\xi - \nu_u)}\, d\xi \quad . \tag{7}$$

Again, it is assumed (and proved below) that the sum of integrals in the right side only depends on the parameters already fixed on the previous steps of the renormalization procedure.

Remembering now that both expressions (5) and (7) follow from the same infinite sum of l-loop graphs we conclude that in the intersection domain $D_s \equiv B_t \cap B_u$ they must coincide with one another:

$$f_l(0, t) + \frac{\nu_t}{2\pi i} \sum_{k=-\infty}^{\infty} \int_{C_k(t)} \frac{f_l(\xi, t)}{\xi(\xi - \nu_t)}\, d\xi = \frac{1}{2\pi i} \sum_{k=-\infty}^{\infty} \int_{C_k(u)} \frac{\phi_l(\xi, u)}{(\xi - \nu_u)}\, d\xi \quad , \tag{8}$$

which means that in D_s

$$f_l(0, t) = -\frac{\nu_t}{2\pi i} \sum_{k=-\infty}^{\infty} \int_{C_k(t)} \frac{f_l(\xi, t)}{\xi(\xi - \nu_t)}\, d\xi + \frac{1}{2\pi i} \sum_{k=-\infty}^{\infty} \int_{C_k(u)} \frac{\phi_l(\xi, u)}{(\xi - \nu_u)}\, d\xi \ \equiv \Psi^{(0,-1)}(t, u)\ . \tag{9}$$

The relation (9) only makes sense in D_s. It is not difficult to construct two more relations of this kind, one of them being valid in $D_t \equiv B_u \cap B_s$, and other one — in $D_u \equiv B_s \cap B_t$. These relations

[3]The precise formulation of this principle is given in [6]. It looks as follows. *The degrees of the bounding polynomials which specify the asymptotic properties of individual items of the loop series expansion must be taken equal to the degree of the polynomial specifying the asymptotics of the full (non-perturbative) amplitude of the process under consideration.*

play a key role in our approach because they provide us with a source of an infinite system of bootstrap conditions. To explain this point, let us consider (9) in more detail and make two statements.

First one: *despite of the fact that (9) only makes sense in D_s, it allows one to express all the coefficients $c_k(l)$ of the power series expansion*

$$f_l(0,t) = \sum_{k=0}^{\infty} c_k(l) t^k$$

in terms of the parameters which, by suggestion, have been fixed on the previous steps of renormalization procedure. The set of those coefficients completely defines this function everywhere in B_t. When translated to the language of Feynman rules, this means that in our model example there is no necessity in attracting special renormalization prescriptions fixing the finite part of the four-leg counterterms. Instead, the relation (9) should be treated as that generating the relevant RP's iteratively — step by step. In what follows we call this — generating — part of self-consistency equations as *bootstrap conditions of the first kind.*

Second: *the relation (9) strongly restricts the allowed values of the parameters which are assumed to be fixed on the previous stages.* To show this it is sufficient to note that $f_l(0,t)$ only depends on the variable t while the function $\Psi^{(0,-1)}(t,u)$ formally depends on both variables t and u. Thus we are forced to require the dependence on u to be fictitious. It is this requirement which provides us with an additional infinite set of restrictions for the resultant parameters. We call these restrictions as *the bootstrap conditions of the second kind.*

The proof of both statements is simple. The domain $D_s\{t \sim 0,\ u \sim 0\}$ contains the point $(t = 0,\ u = 0)$ which can be taken as the origin of the local coordinate system. Using the definitions (4) and expanding both sides of (9) in power series in t and u we obtain the full set of Taylor coefficients

$$c_k(l) = \frac{1}{n!}\ f_l^{(k)}(0,0) = \frac{1}{n!}\ \frac{\partial^k}{\partial t^k}\ \Psi_l^{(0,-1)}(t=0, u=0), \qquad (k = 0, 1, \dots)\ . \tag{10}$$

This is quite sufficient for fixing $f_l(0,t)$ everywhere in B_t. Further, from (9) it follows an infinite system of the bootstrap conditions of the second kind:

$$\frac{\partial^k}{\partial t^k}\frac{\partial^{m+1}}{\partial u^{m+1}}\Psi_l^{(0,-1)}(t=0, u=0)\ = 0, \qquad (k, m = 0, 1, \dots)\ ; \tag{11}$$

they restrict the allowed values of the parameters fixed on the previous steps of renormalization procedure.

Thus we see that the system of bootstrap equations[4] is naturally divided into two subsystems. The bootstrap conditions of the first kind just allow one to express the resultant parameters of higher levels in terms of the lower level parameters which, by condition, already have been expressed in terms of the *fundamental observables*[5] on the previous steps. In other words, they provide a possibility to express the higher level parameters in terms of observable quantities. This subsystem does not restrict the admissible values of the latter quantities.

In contrast, the subsystem consisting of the bootstrap conditions of the second kind does impose extremely strong limitations on the allowed values of the physical (observable!) parameters of effective scattering theory. In fact, it provides us with the system of physical predictions which — at least, in principle — can be verified experimentally.

To make our analysis complete it remains to discuss two key points. First, we need to explain the above-made suggestions concerning the degrees of bounding polynomials. Second, it is necessary to show that all the contour integrals in (5) and (7) only depend on the parameters already fixed on the previous stages of the renormalization procedure. The first point can be easily explained: we choose the bounding polynomial degrees in accordance with known data on Regge intercepts. The proof of the second one is based on the relation (2) and on the results of [3].

First of all we need to review the main stages of the renormalization procedure. As usually, we discuss

[4] This is not the full system: it only mirrors the self-consistency requirements in certain domains of the complex space of relevant kinematical variables!

[5] The parameters appearing in the right sides of RP's.

this point in terms of 1PI (one-particle irreducible) graphs[6] (see, e.g., [10], [11]). Again, we consider first the elastic two body scattering (3). As follows from the analysis in [3], the reduction procedure does not change the structure of singularities of a given graph; it only allows one to re-express this graph in terms of minimal parameters of various levels. This relates also to the full sum of graphs of a given loop order. The reduction procedure just converts it into another sum of graphs, each one being written in terms of resultant parameters. We always imply that the reduction is done and work with these latter parameters.

Let us begin from the consideration of one loop contributions. In this case one deals with the graphs solely constructed from the minimal propagators and resultant 1-, 2-, 3- and 4-leg vertices with the level indices $l = 0, 1$. First, there are graphs with one *explicit loop*: 1) Those containing one minimal tadpole (1-leg) insertion, 2) Those with one minimal self-energy type (2-leg) insertion, 3) Graphs with one-loop minimal triple vertex and, at last, 4) One-particle irreducible one-loop graphs. All these graphs only depend on the parameters of the lowest level $l = 0$. Second, there are graphs with one *implicit loop*: those containing the completely reduced 1-, 2-, 3- and 4-leg vertices and the same graphs with insertions of 1-loop counterterms for 1-, 2-, 3- and 4-leg minimal effective vertices. All the parameters of the highest level $l = 1$ are concentrated in the graphs of this latter kind. As usually, to fix the one-loop order counterterms for 4-point amplitude, one shall first perform the one-loop renormalization of the resultant vertices with one, two and three legs (tadpole, self-energy and triple vertex). To fix the remaining 4-leg counterterm one needs to formulate the corresponding renormalization prescription.

Clearly, this conclusion can be easily generalized for the case of inelastic processes involving an arbitrary number of particles. This is precisely the situation known from the conventional renormalization theory. The only difference is that in the case of effective scattering theory with unstable particles the wave function renormalization process has certain peculiar features[7]. Besides, in this latter case the full number of counterterms, needed to construct the finite expressions for all one-loop S-matrix elements, is infinite.

The generalization of the above reasoning for the case of arbitrary loop order is straightforward. Namely, the procedure of renormalization of the L-loop contribution to the amplitude of the process involving N ($N = 4, 5, \ldots$) particles consists of L stages. Every l-th stage ($l = 1, 2, \ldots, (L-1)$) presents a certain (depending on N and l) number of steps each of which corresponds to the renormalization of l-loop S-matrix elements with fixed number n of external lines ($n = 1, 2, \ldots, n_{max}(N, l)$). The last — L-th — stage consists of $(N-1)$ preliminary steps (renormalization of the L-loop n-leg graphs with $1 \leq n < N$) and one final step — fixing the L-th order N-leg counterterm vertices.

Now we can turn to a consideration of contour integrals in (5) and (7). It is implied that all the previous steps already have been done and we only need to fix the l-th level coefficients in the series expansion of 4-leg vertex.

It is clear[8] that the only contributions to the contour integrals in question follow from those graphs which have at least one internal line (otherwise the graph does not contain a singularity). It is easy to show that the graphs with internal lines may only depend on the parameters of the resultant vertices of lower levels and on the l-th level resultant parameters describing the vertices with 1, 2, or 3 legs. By condition, all those parameters have been already fixed on the previous steps of renormalization procedure. All this means that, indeed, the contour integrals in (5) and (7) should be considered as known functions. This completes our proof.

To summarize: in our example, as far as we consider all the resultant parameters of the previous levels $l' = 1, 2, \ldots, (l-1)$ being fixed, one does not need to attract any new RP's in addition to those fixing the l-th level resultant vertices with $n = 1, 2, 3$ legs; the 4-leg counterterms of the l-th loop level automatically become fixed by the bootstrap conditions of the first kind.

The above analysis offers a hint about the structure of the full system of requirements sufficient for performing the renormalization procedure in effective scattering theory.

[6] In [3] we considered tadpoles (1-leg graphs) attached to a given vertex on the same footing as self-closed lines. Here, however, we consider tadpoles as independent elements of Feynman rules for constructing graphs in terms of resultant parameters. This allows us to avoid problems connected with the definition of one-particle irreducibility.

[7] This point is briefly discussed in [3]; the detailed analysis will be published elsewhere.

[8] This point is justified by the *summability* principle formulated in [6].

4 Renormalization programme

The model example considered in the previous section allows us to make the following statement. In those cases when it is possible to point out two intersecting hyper-layers B_x and B_y such that in one of them (say, in B_x) the amplitude of a given process $2 \to 2$ is presented by the (-1)-bounded function of the corresponding complex variable (and, possibly, of several parameters) there is no need in formulating the RP's for 4-leg amplitudes: the summability principle provides us with a tool for generating those prescriptions order by order. This conclusion remains in force also if in another layer B_y the degree of the relevant bounding polynomial is greater than one $N_y \geq 1$. As to the 1-, 2- and 3-leg vertices (which all are just constants in terms of the resultant parameters), one does need to formulate the relevant RP's, but this cannot be done arbitrarily — the bootstrap requirements must be taken into account.

It is well known that the amplitudes of inelastic processes involving $n > 4$ particles decrease with energy, at least in the physical area of the other relevant variables. Thus it looks natural to suggest that in corresponding hyper-layers these amplitudes can be described with the help of at most (-1)-bounded functions of one complex energy-like variable (and several parameters). Also, it is always possible to choose the variables in such a way that the domains of mutual intersections of every two hyper-layers are non-empty[9]. Therefore, from the above analysis it follows that the system of RP's needed to fix the physically interesting effective scattering theory only contains those prescriptions which fix the 1-, 2-, 3- and, possibly, 4-leg amplitudes[10].

We would like to recall that we are interested in constructing the renormalization procedure which could provide a possibility to manage the effective hadron scattering theory. For simplicity, we only consider here the non-strange sector[11]. This means that the only stable particles in our case are pions and nucleons. As known, the high-energy behavior of the amplitudes of elastic pion-pion, pion-nucleon and nucleon-nucleon scattering processes is governed by the Regge asymptotic law. It is not difficult to check that every one of these processes is described by the amplitude (more precisely, by several scalar formfactors) characterized by the *negative* degree of bounding polynomial, at least in one of the three cross-conjugated channels. That is why the analysis in Sec. 3 is relevant, and we conclude that 4-leg graphs with external lines corresponding to pions and nucleons do not require formulating RP's.

Now we need to consider the 4-leg graphs with at least one external line corresponding to unstable particle. Here the situation looks more complicated owing to the absence of direct experimental information about the processes with unstable hadrons. In other words, the problem of choice of the degrees of relevant bounding polynomials is to a large measure nothing but a matter of postulate. The only way to check the correctness of the choice is to construct the corresponding bootstrap relations and compare them with existing data on resonance parameters. This work is in progress now. Here, however, we consider the relatively simple situation when all the 4-leg amplitudes of the processes involving unstable particles decrease with energy, at least, at sufficiently small values of the momentum transfer[12]. As we already mentioned, the same relates also to the amplitudes of the processes with $n > 4$ external particles. All this means that, according to the results discussed above, the only RP's needed to fix the physical content of the effective theory under consideration are those restricting the allowed values of the coupling constants at 1-, 2- and 3-leg minimal counterterm vertices.

We stress that this conclusion is based on the analysis of the bootstrap equations only valid in the intersection domain of two layers $B_x\{x \in \mathbf{R},\ x \sim 0;\ \nu_x \in \mathbf{C}\}$ and $B_y\{y \in \mathbf{R},\ y \sim 0;\ \nu_y \in \mathbf{C}\}$, each of which contains small vicinity of the origin of the relevant coordinate system. The reason for this choice of layers is explained by the existence of experimental information on Regge intercepts. This choice is, however, justified from the field-theoretic point of view [1]–[3]. Here are the arguments. A formal way to construct the $(L+1)$-th order amplitude of the process $X \longrightarrow Y$ is to close the external lines of the relevant L-th order graphs corresponding to the process $X + p_1 \longrightarrow Y + p_2$ with two additional particles carrying the momenta p_1 (let it be incoming) and p_2 (outgoing). This means that those latter graphs

[9] This follows from the fact that the number of pair energies is much larger then that of independent kinematic variables.

[10] We would have no need in RP's even for 4-leg amplitudes if the experimental information on hyper-layers where they decrease with energy were more complete.

[11] The situation in the strange sector is discussed in [4].

[12] Surely, this is just a model suggestion which, however, seems us quite reasonable. One of the arguments in its favor follows from the fact that unstable particles cannot appear in true asymptotic states.

should be calculated at $p_1 = p_2 \equiv q$, dotted by the relevant propagator and integrated over q. To ensure the correctness of this procedure (see [12]), one needs to require the polynomial boundedness (in p_1) of the L-th order amplitude of the process $X + p_1 \longrightarrow Y + p_2$ at $x \equiv (p_1 - p_2)^2 = 0$ and, by continuity, in a small vicinity of this value. Clearly, this argumentation applies to arbitrary graphs with $N \geq 4$ external lines.

Thus we arrive at the following conclusion. *To perform the complete renormalization of the effective scattering theory with the above-specified asymptotic conditions it is quite sufficient to attract the system of RP's that fix the finite parts of counterterm coupling constants at minimal* $1-$, $2-$ *and* $3-$*leg vertices. Moreover, this system cannot be taken arbitrary — the bootstrap constraints must be taken into account.*

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