Renormalization prescriptions in effective theory of pion-nucleon scattering

A. Vereshagin
University of Bergen and St.-Petersburg State University
Alexander.Vereshagin@ift.uib.no

Abstract

Using the results presented in the talk [1] we illustrate the approach developed in [2]–[5] by the calculation of πN elastic scattering amplitude and explain how to derive corresponding bootstrap conditions. In the end, we briefly mention some results of numerical comparison with experimental data.

1 Introduction

The essence of our work is an attempt to develop a self-consistent Dyson perturbation technique for the infinite component effective field theories of strong interactions[1]. The reason to work with Dyson's scheme is that this is the only known way that allows one to combine Lorentz invariance, cluster decomposition principle, and unitarity with general postulates of quantum mechanics [7]. Thus, the problems we have to do with include the infinite number of graphs to be summed at each loop (including the tree) level and the problem of the required number of renormalization conditions, since in such a theory one needs to fix an infinite number of parameters to be able to calculate amplitudes. In [2, 3] it has been shown that already the requirement of summability of tree graphs — the tree level amplitude — leads to strong restrictions on the coupling constants of a theory. However, in those articles many of theoretical statements, like the *meromorphy* and *polynomial boundedness* of the tree amplitude, were taken as postulates and only some general arguments in their favor were given. With these assumptions it turned out possible to obtain *bootstrap equations* for masses and coupling constants of $\pi\pi$ and πK resonances in nice agreement with the experimental data. The main tool used to derive those equations is the *Cauchy expansion*, based on the celebrated Cauchy integral formula, which allows one to represent the tree level amplitude as well defined series in a given domain of the space of kinematical variables [1, 3].

In subsequent publications we fill some gaps left in the previous analysis as well as discuss new concepts. Thus, in [4, 8] we suggested the notion of *minimal parametrization* and in [5] the relevant reduction theorem is proven. This explains why it is sufficient to consider only the minimal ("quasi-on-shell") vertices at each loop order of the perturbation theory — the fact implicitly used in [2, 3] to parametrize amplitudes. Besides, in [4] we discuss what we call the *localizability principle* — the philosophy which, in particular, serves as the background for meromorphy and polynomial boundedness of the tree level amplitude. However, the last point is still not explained clear enough. A brief discussion can be found in [9] where we introduce so-called *summability* and *asymptotic uniformity* principles, and we plan to treat it in more detail in a separate publication. Here we do not focus on this and just assume it is known that the tree level $\pi N \to \pi N$ amplitude is a polynomially bounded meromorphic function of any pair of independent kinematical variables.

This talk is mainly devoted to the derivation of the bootstrap equations for πN elastic scattering and the results of comparison with the experimental data.

2 Structure of the amplitude and minimal vertices

The amplitude $M^{b\beta}_{a\alpha}$ of the reaction

$$\pi_a(k) + N_\alpha(p,\lambda) \to \pi_b(k') + N_\beta(p',\lambda') \tag{1}$$

can be presented in the following form:

$$M^{b\beta}_{a\alpha} = i(2\pi)^4\delta(k+p-k'-p')\left\{\delta_{ba}\delta_{\beta\alpha}M^+ + i\varepsilon_{bac}(\sigma_c)_{\beta\alpha}M^-\right\} \quad . \tag{2}$$

[1] In the sense first suggested by Weinberg [6]; see [4, Sec. 1] and [5, Sec. 2] for comments concerning the quantization scheme.

Here

$$M^{\pm} = \overline{u}(p', \lambda') \left\{ A^{\pm} + \left(\frac{\not{k} + \not{k}'}{2} \right) B^{\pm} \right\} u(p, \lambda) \ ,$$

$a, b = 1, 2, 3$ and $\alpha, \beta = 1, 2$ stand for the isospin indices, λ, λ' — for polarizations of the initial and final nucleon, respectively, $\overline{u}(p', \lambda')$ and $u(p, \lambda)$ — for Dirac spinors and $\sigma_c,\ c = 1, 2, 3$ — for Pauli matrices. The invariant amplitudes $A^{\pm}$ and $B^{\pm}$ are the functions of an arbitrary pair of Mandelstam variables $s \equiv (p + k)^2$, $t \equiv (k - k')^2$, and $u \equiv (p - k')^2$.

Since we rely upon the Dyson's scheme, we must construct the amplitude order by order, starting from the tree level. To construct the tree level amplitude, one needs to collect contributions of the graphs shown in Fig. 1.

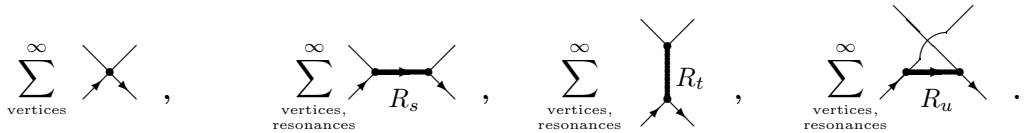

Figure 1: Tree level graphs: R_s, R_t and R_u stand for all admissible resonances in s-, t-, and u-channels, respectively; summation over all possible kinds of vertices is implied, though the summation order is still unspecified.

We deal with the effective theory. This means that the interaction Hamiltonian[2], (in the interaction picture) contains *all* the terms consistent with (algebraic) symmetry properties of strong interactions; there are no limitations on the number and the order of field derivatives. Besides, to avoid model dependence we reserve the possibility to work with infinite number of resonance fields. Altogether this means that the number of items contributing to the tree level amplitude is actually infinite. This creates a problem: there is no guarantee that the sum of these terms taken "as it is" converges, at least, in some domain of the space of kinematical variables. Actually, as it usually happens with functional series, it *diverges* in general, and the tree level amplitude simply *does not exist* until one attracts a guiding principle to fix the order of summation.

The way out of this difficulty was pointed out in [2]–[5]. It consists of switching to the *minimal* parametrization and using the method of Cauchy forms in various regions of the complex space of kinematical variables. The important advantage of this approach is that it results in uniformly converging series of singular terms defining the tree level amplitude as the polynomially bounded meromorphic function — no kind of singularities but simple poles can appear on this way. To construct the Cauchy form for the tree level amplitude, one only needs to fix the residues at corresponding poles and properly choose the bounding polynomial degree — it happens quite sufficient for fixing the amplitude up to few unknown functions. These latter functions, in turn, can usually be found from the *bootstrap equations*.

By the very definition, the residues at poles of the tree level amplitude are nothing but the relevant spin sums (numerators of minimal propagators) dotted by the minimal triple coupling constants (those at minimal triple vertices). This is so just because only the minimal vertices survive on the mass shell and, therefore, do not cancel propagator's numerator. In [1] it is shown that contribution of vertices with four (and more) external lines does not require fixing by independent renormalization prescriptions. That is why in our case the pole graph parameters — minimal triple vertices and masses, together with asymptotic regime and self-consistency requirements completely determine the tree level amplitude and, as a by-product, the contribution of the contact four-leg vertices.

The Hamiltonian density monomials corresponding to the minimal triple vertices contributing to the amplitude of elastic pion-nucleon scattering are listed below[3]. Note, that there is only a *finite* number of minimal triple vertices for each resonance field with given quantum numbers. In this article we consider a concrete process and employ experimental data, therefore it is natural to input some phenomenology already at this stage. Namely, we imply that there are no resonances with isospin $I \geq 2$. Later, when coming to the amplitude calculation, we make another assumption — that the number of resonances with

[2]In [5] it is explained why it is preferably to use the effective interaction Hamiltonian (which, of course, can be written in a Lorentz covariant form), rather than Lagrangian when constructing a theory with unlimited number of field derivatives.

[3]See [5, 9] for rigorous definition of the minimal vertex and minimal parametrization.

the same mass is finite. These restrictions are kept automatically when one substitute phenomenological data. On the other hand, they do not affect the mathematics and can easily be relaxed.

Henceforth we use the Rarita-Schwinger formalism for the higher spin fields. In the formulae below J denotes spin and I — the isotopic spin (we drop the isospin indices and write isovectors in boldface to form scalar and vector products), P is the parity, and $\mathcal{N}$ is the "normality" of the corresponding resonance, the latter equals $(-1)^{(J-1/2)}P$ for baryons and $(-1)^J P$ for mesons. Besides, π is the pion field and N is the nucleon field. The baryon resonance fields are denoted by $\widehat{R}$ and R ($I=1/2$), $\widehat{\Delta}$ and Δ ($I=3/2$)[4], the mesons are S ($I=0$), and V ($I=1$). In the minimal parametrization the vertices (Hamiltonian monomials) we need are the following:

$J = l + \frac{1}{2}$ $(l = 0, 1, 2 \ldots)$

$$
\begin{aligned}
&I = \tfrac{1}{2},\ \mathcal{N} = -1: \qquad g_{\widehat{R}} \overline{N} \boldsymbol{\sigma} \widehat{R}_{\mu_1 \ldots \mu_l} \partial^{\mu_1} \ldots \partial^{\mu_l} \boldsymbol{\pi} + H.c. \\
&I = \tfrac{1}{2},\ \mathcal{N} = +1: \qquad i g_R \overline{N} \boldsymbol{\sigma} \gamma_5 R_{\mu_1 \ldots \mu_l} \partial^{\mu_1} \ldots \partial^{\mu_l} \boldsymbol{\pi} + H.c. \\
&I = \tfrac{3}{2},\ \mathcal{N} = -1: \qquad g_{\widehat{\Delta}} \overline{N} P_{\frac{3}{2}} \widehat{\Delta}_{\mu_1 \ldots \mu_l} \partial^{\mu_1} \ldots \partial^{\mu_l} \boldsymbol{\pi} + H.c. \\
&I = \tfrac{3}{2},\ \mathcal{N} = +1: \qquad i g_\Delta \overline{N} \gamma_5 P_{\frac{3}{2}} \Delta_{\mu_1 \ldots \mu_l} \partial^{\mu_1} \ldots \partial^{\mu_l} \boldsymbol{\pi} + H.c.
\end{aligned}
$$

$J = 0, 2, \ldots,\ I = 0,\ P = +1$:

$$
\begin{aligned}
&\tfrac{1}{2} g_{S\pi\pi} S_{\mu_1 \ldots \mu_J} (\boldsymbol{\pi} \cdot \partial^{\mu_1} \ldots \partial^{\mu_J} \boldsymbol{\pi}) \,, \\
&\left[g^{(1)}_{NNS} \overline{N} \partial_{\mu_1} \ldots \partial_{\mu_J} N + i g^{(2)}_{NNS} J \partial_{\mu_1} \ldots \partial_{\mu_{J-1}} \overline{N} \gamma_{\mu_J} N \right] S^{\mu_1 \ldots \mu_J} \,,
\end{aligned}
$$

$J = 1, 3, \ldots,\ I = 1,\ P = -1$:

$$
\begin{aligned}
&\tfrac{1}{2} g_{V\pi\pi} \boldsymbol{V}_{\mu_1 \ldots \mu_J} (\boldsymbol{\pi} \times \partial^{\mu_1} \ldots \partial^{\mu_J} \boldsymbol{\pi}) \,, \\
&\left[i g^{(1)}_{NNV} \overline{N} \boldsymbol{\sigma} \partial_{\mu_1} \ldots \partial_{\mu_J} N + g^{(2)}_{NNV} J \overline{N} \gamma_{\mu_J} \boldsymbol{\sigma} \partial_{\mu_1} \ldots \partial_{\mu_{J-1}} N \right] \boldsymbol{V}^{\mu_1 \ldots \mu_J} \,. \qquad (3)
\end{aligned}
$$

Here $g^{\cdots}_{\cdots} \equiv g^{\cdots}_{\cdots}(J, I, P)$ stand for real coupling constants, and σ_c $(c = 1, 2, 3)$ — for Pauli matrices.

$$
P_{\frac{3}{2}} \equiv \left(P_{\frac{3}{2}} \right)_{a\alpha b\beta} = \frac{2}{3} \left\{ \delta_{\alpha\beta} \delta_{ab} - \frac{i}{2} \varepsilon_{abc} \left(\sigma_c \right)_{\alpha\beta} \right\},\ (a, b = 1, 2, 3,\ \alpha, \beta = 1, 2)
$$

is the projecting operator on the states with isospin $I = 3/2$, a, α, b, β being the isotopic indices.

There are no more minimal triple vertices contributing to the tree level amplitude. In [9] it is explained that the g's above appears to be not just minimal, but also the *resultant* tree level couplings. As pointed out in [5], the resultant parameters are the natural candidates to impose the renormalization prescriptions on, under the condition that the renormalization point is taken on shell and the *renormalized perturbation theory* is used. In this scheme the action is written in terms of *physical* parameters plus counterterms, the latter ones being tuned in such a way that the values of physical parameters remain unchanged after the renormalization. So, we imply that the Feynman rules are written in the form of physical part plus counterterms at every loop order and it is the *real parts of physical masses* that appear in bare propagators. Simply speaking, we impose the following set of RP's:

$$
\mathbf{Re}\ V(p_1, p_2, p_3) = G_{phys} \text{ at } p_i^2 = M^2_{i_{phys}},
$$

and

$$
\mathbf{Re}\ \Sigma(p) = 0 \text{ at } p^2 = M^2_{phys},
$$

for every self-energy Σ and every three-leg vertex V. Now we are at tree level, thus there are no counterterms relevant, therefore the couplings g are also physical — (in principle) measurable, or *renormalized* ones.

[4]Note that the experimental $\Delta(1.23)$ resonance ($I, J = 3/2,\ P = +1$) has $\mathcal{N} = -1$ and thus belongs to the type $\widehat{\Delta}$ in our notations.

There is no phenomenological evidence that the mass spectrum and spin values of resonances are bounded from above. Therefore we need to reserve the possibility to work with an infinite set of resonances of arbitrary high spin value. In other words, there is still infinite number of minimal couplings coming even from three-leg vertices. One of the main points of our work is that these couplings, or, the same, renormalization conditions fixing them, are *not independent*: there are *self-consistency conditions* that restrict their values. Thus, to make the perturbation theory self-consistent, one should make sure that all the restrictions are met at any loop order. The *bootstrap* constraints discussed below form a part of such restrictions.

3 Cauchy forms

In this Section we show how to construct the Cauchy forms (series): the well-defined expressions for the tree level amplitudes $A^{\pm}$ and $B^{\pm}$ in three mutually intersecting bands (B_s, B_t and B_u) of the Mandelstam plane (Fig. 2). In addition to conventional Mandelstam variables s, t, u it is convenient to define $\nu_s \equiv u - t$,

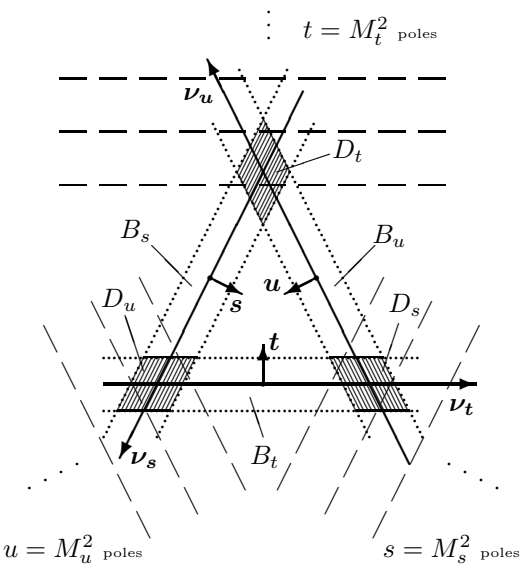

Figure 2: Mandelstam plane: three different Cauchy series uniformly converge in three different bands B_s, B_t and B_u (bounded by dotted lines). The domains D_s, D_t, D_u of the band intersections are hatched. The dashed lines show the pole positions in the relevant variables.

$\nu_t \equiv s - u$ and $\nu_u \equiv t - s$. Every pair (x, ν_x) presents the orthogonal coordinate system in the band (layer)

$$B_x\{x \in \mathbf{R},\ x \sim 0;\ \nu_x \in \mathbf{C}\},\ (x = s, t, u),$$

where the corresponding series converges. The Cauchy form arise as a result of the application of the Cauchy integral formula to the tree level amplitude in one of those layers.

Let us, for definiteness, consider the band B_t. Referring to the talk [1] and keeping in mind that, according to the summability principle [9], the only singularities of the tree level amplitude are the poles appearing in the tree-level exchange graphs, we schematically rewrite [1, Eq. 2] as

$$X(\nu_t, t)|_{B_t} = \sum_{k=0}^{N} \frac{1}{k!}\alpha_k(t)\nu_t^k + \sum_{n}^{\infty}\left\{\frac{p_n(t)}{s - M_{u_n}^2} + \frac{q_n(t)}{u - M_{s_n}^2} - \sum_{m=0}^{N}\beta_{n,m}(t)\nu_t^k\right\}, \tag{4}$$

where X stands for any one of the invariant amplitudes $A^{\pm}$ or $B^{\pm}$ and the summation in order of increasing mass is implied. We call the above expression as *Cauchy form*. In [1] it is shown that, as long

as the bounding polynomial degree N is given, the terms in brackets: poles and the coefficients β of the so-called *correcting polynomials* are easily expressed via the minimal triple couplings (3) and masses M_{s_n} and M_{u_n} of the s- and u-channel resonances. The α's are still unknown at this stage. Analogous forms can be easily written for the tree level amplitude in the cross-symmetric bands B_s and B_u.

According to the *asymptotic uniformity* principle [9], one should admit the tree level amplitude asymptotic behavior to be of the same order as that dictated be the corresponding Regge intercept. One can check that in our case all the invariant amplitudes has decreasing asymptotics in the bands B_s and B_u (behave like negative powers of ν_s and ν_u, respectively), while in B_t: $A^+ \sim \nu_t^1$, $A^- \sim \nu_t^{0.5}$, $B^+ \sim \nu_t^0$, and B^- again behave like negative power of ν_t. Attracting the odd-even properties, we conclude that only the form for A^+ in the band B_t requires correcting polynomials and external part[5] $\alpha(t)$. Cauchy series for all the other amplitudes are just the sums of poles appearing in the corresponding channels (layers).

We have no space here to specify the exact amplitude expansion in each given band, and later on just briefly sketch the way one obtains the bootstrap equations. However, before discussing the bootstrap, it is useful to summarize what we have in hands up to now.

The Cauchy forms above are written for the tree level amplitudes. They are the series of special type, each one being valid (convergent) in certain layer on the Mandelstam plane. The only unknown component is $\alpha(t)$ appearing in A^+ expression in B_t, all the other items are known functions of kinematical variables, triple couplings and masses. In each separately taken form the pole parts originates from those exchange graphs, that contain the poles crossing the band in which the form is valid. Neither cross channel poles nor contact graph (four-leg vertices) contributions appear explicitly[6]. We can formulate it stronger: in almost all of the amplitude expressions in the given channel (layer) the cross channel exchange graphs and four-point vertices do not show up at all — the amplitude is written as just the sum of poles in the given layer. For example, there are no $t = M^2$ poles in (4). It shows that in many cases the asymptotic regime requires the cancellation of contributions coming from cross channel poles with that following from the contact graphs. What is necessary to stress is that it is not always the case, and A^+ expansion in B_t gives an excellent illustration: the α and the correcting polynomials in the Cauchy form for A^+ can be regarded as a joint effect of contact and cross channel exchange graphs.

4 Bootstrap equations and experimental data

As it is seen from the Fig. 2, each pair of the bands B_s, B_t and B_u has non-empty intersection domain: D_s, D_t and D_u. To obtain the bootstrap constraints we just equate each two Cauchy series for a given amplitude in the domain where both forms are valid[7]. After this is done, the function $\alpha(t)$ mentioned in the previous Section can be expressed in terms of the parameters of cross channel Cauchy form (minimal triple couplings and masses). It is at this stage that the tree level amplitude turns out to be completely specified.

Expanding the resulting (bootstrap) equations in powers of kinematical variables (for example, around the $t = u = 0$ point for the equalities valid in D_s), one obtains an infinite set of numerical equations for the resultant couplings and masses with no kinematics involved anymore.

The bootstrap equations have rather complicated form, but what is necessary to mention again, is that according to the consideration in the end of Sec. 2, all the coupling constants and masses that enter are the physical (renormalized) ones. This means that one can check the resulting relations using the experimental values of the known resonances parameters.

Before turning to numerical analysis it looks appropriate to discuss here the meaning of the terms "mass" and "width" often used to specify the parameters describing a resonance.

[5]According to the asymptotics, A^- in B_t could also require the correcting polynomials and external part of zeroth order in ν_t. However, the spectrum is symmetric with respect to $\nu_t \to -\nu_t$ transformation and A^- is odd in ν_t. Thus, the zeroth order polynomials in the corresponding Cauchy form are forbidden (cancel in pairs). Similarly, A^+ is even in ν_t, hence in B_t its correcting polynomials degree equals 0 — not 1 as one could expect from the value of relevant intercept.

[6]Except the trivial cases, each graph with resonance exchange contains a polynomial contribution, as well as the pole. Our minimal (on-shell) parametrization allows one to get rid of the polynomial part, but, of course, the contact vertices still contribute.

[7]Namely D_s, D_t, or D_u. However, in the case under consideration the equations one obtains in D_u are equivalent to those from D_s due to the symmetry of the process. Note also, that these equations are nothing but the requirements of crossing symmetry.

In our approach we are only allowed to calculate the amplitude in question precisely at a given loop order; no kind of partial resummation is allowed[8]. In particular, we are not allowed to perform Dyson's resummation for the propagator. This means that the theoretical expression for the amplitude of, say, elastic pion-nucleon scattering should be constructed from the propagators corresponding to simple poles at real values of the momentum squared. This circumstance creates a problem when one needs to compare the (finite loop order) theoretical expression with experimental data. To circumvent this difficulty one needs to exclude from a consideration those data which correspond to small vicinities of the resonance poles. Technically, this is just a problem of appropriate organization of the fitting procedure; it is easily solvable. In contrast, from the purely theoretical viewpoint this is a problem of definition of the parameters describing a resonance. We would like to stress that the notion of resonance only acquires meaning in the framework of a particular perturbation theory. If we could construct the complete non-perturbative expressions for S-matrix elements we would never need to use the term "resonance".

Since the partial resummation is not allowed, one cannot construct the finite order amplitude with complex poles. Hence, the customary Breit-Wigner description of resonances (mass and width as real and imaginary parts of the pole position, respectively) looses its meaning in terms of the resultant parameters[9]. Instead, we use the term "physical mass" to denote the parameter M_i appearing in the denominator of the free particle propagator (recall that we rely upon the scheme of renormalized perturbation theory). This implies using the renormalization prescription given in Sec.2

When fitting data with the finite loop order amplitude one obtains the values of relevant physical couplings which, in turn, can be used to compute (formally!) the decay amplitude and, hence, the resonance width. Thus, in principle, it is possible to avoid using the ill-defined term "width" and work in terms of coupling constants instead. However, when comparing our bootstrap equations with experiment, we are forced to use the numbers quoted, say, in [10]. Those numbers are given in terms of mass and width corresponding to the parameters describing the T-matrix poles at an unphysical sheet of the complex energy plane. In many other sources the data on resonances are quoted in terms of Breit-Wigner or K-matrix parameters. Clearly, in the case of well-separated narrow resonance the different methods of parametrization of the amplitude (including that based on field theory models operating directly with coupling constants) result in approximately the same values of mass and coupling constant. This is, however, not true with respect to broad resonances like famous light scalars[10]. In this latter case the connection between the mass parameters appearing in propagators, coupling constants and the parameters describing poles in complex energy plane (real and imaginary part) becomes very complicated. In other words, the values of mass and coupling constant extracted from the same data set with the help of different theoretical amplitudes may differ considerably. Surely, this is explained by well known difficulties of analytic continuation of the approximate expressions obtained in the framework of certain perturbation scheme. This is the reason why we find it necessary to clarify the terminology.

As an example of comparison with experimental data we shall mention a calculation of the ratio for two ρ-meson coupling constants. The quantities $G^T_{NN\rho}$ and $G^V_{NN\rho}$ were defined and fitted in [12] as couplings in the effective Hamiltonian

$$H^{NN\rho}_{\rm eff} = -\overline{N}\left[G^V_{NN\rho}\gamma_\mu \boldsymbol{\rho}^\mu - G^T_{NN\rho}\frac{\sigma_{\mu\nu}}{4m}\left(\partial^\mu \boldsymbol{\rho}^\nu - \partial^\nu \boldsymbol{\rho}^\mu\right)\right]\frac{1}{2}\boldsymbol{\sigma} N \ , \tag{5}$$

where σ_a are Pauli matrices and m is the proton mass. The existing experimental data [12] give:

$$\frac{G^T_{NN\rho}}{G^V_{NN\rho}} \approx 6.1 \ , \quad \frac{G_{\pi\pi\rho}G^V_{NN\rho}}{4\pi} \approx 2.4 \ , \quad G_{\pi\pi\rho} \approx 6.0 \ . \tag{6}$$

These values can be deduced from the bootstrap equations for πN elastic scattering with 15% accuracy[11]. It should be also mentioned here that the same value for the first ratio was obtained during the similar analysis of KN elastic scattering [9].

[8] The latter can lead, e.g., to the violation of perturbative unitarity.

[9] Generally speaking, this is just a consequence of the fact that the notion of resonance is ill-defined in the framework of perturbative quantum field theory [7].

[10] An excellent discussion of this point can be found in the series of papers [11].

[11] The problem of the convergence rapidity of various types of the bootstrap equations is discussed in [13].

Besides, we checked the tree level bootstrap equations for $\pi\pi$ and πK elastic scattering amplitudes (see [3] and references therein). Similar calculations were performed for the cases of πN [8] and, recently, KN elastic scattering [9]. There were no contradiction found so far, and in most cases examined the experimental data seem to support our approach nicely.

Apart from the question of formal compatibility with experiment, there is a question of efficiency. One can ask how many loops should be taken into account and how many parameters fixed to obtain the amplitude that could fit well the data at least in some kinematical region. To check this point we performed a calculation of low energy coefficients[12] for the πN amplitude. This coefficients measured and fitted in [12] are reproduced in our approach with very good accuracy already at tree level[13], and to gain reasonable precision it is enough to specify the parameters of just few lightest resonances. The results of this analysis were summarized in [8]; the details will be published elsewhere.

I am grateful to K. Semenov-Tian-Shansky and V. Vereshagin for helpful collaboration as well as to V. Cheianov, H. Nielsen, S. Paston, J. Schechter, A. Vasiliev and M. Vyazovsky for stimulating discussions. This work is supported by INTAS (project 587, 2000), Ministry of Education of Russia (Programme "Universities of Russia") and L. Meltzers Høyskolefond (Studentprosjektstipend, 2004).

References

[1] V. Vereshagin et.al., *"Renormalization prescriptions and bootstrap in effective theories"*, (this conference).
[2] V. Vereshagin, Phys. Rev. D **55** (1997) 5349.
[3] A. Vereshagin and V. Vereshagin, Phys. Rev. D **59** (1999) 016002.
[4] A. Vereshagin, V. Vereshagin, and K. Semenov-Tian-Shansky, Zap. Nauchn. Sem. POMI **291** (2002) 78; *the English version published in* J. Math. Sci. **125**, iss. 2 (2005) 114.
[5] A. Vereshagin, V. Vereshagin, Phys. Rev. D **68** (2004) 025002.
[6] S. Weinberg, Physica A **96** (1979) 327.
[7] S. Weinberg, The Quantum Theory of Fields, vols. 1-3, Cambridge, 1996.
[8] A. Vereshagin and V. Vereshagin, πN Newsletter **15** (1999) 288-291; A. Vereshagin, πN Newsletter **16** (2002) 426-428.
[9] V. Vereshagin et.al., *"Renormalization programme for effective theories"*; A. Vereshagin, *"Explicit equations for renormalization prescriptions in the case of pion-nucleon scattering"*, K. Semenov-Tian-Shansky, *"Renormalization in effective theories: prescriptions for kaon-nucleon resonance parameters"*; *talks given at International Workshop HSQCD'2004, to be published in Proceedings.*
[10] Particle Data Group, *Review of Particle Physics*, Phys. Letters B **592** (2004).
[11] M. Harada, F. Sannino, J. Schechter, Phys. Rev. D **54** (1996) 1991; D. Black, A. Fariborz, F. Sannino, and J. Schechter, Phys. Rev. D **58** (1998) 054012; *ibid*, D **59** (1999) 074026;
[12] M. M. Nagels et al. Nucl. Phys. **B109** (1976) 1; O. Dumbrajs et al, Nucl. Phys. **B216** (1983) 277.
[13] K. Semenov-Tian-Shanski, *"Bootstrap equations for string-like amplitude"*, (this conference).

[12] Taylor expansion coefficients around the crossing symmetry point.
[13] Of course, it is partly because this region is relatively far from the branch points. In case if the latter appears close to the investigated region one should necessary include loops.