Model independent radiative corrections to deep inelastic scattering for future polarization experiments at colliders

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Abstract

The one-loop electroweak radiative correction of the lowest-order to lepton current for deep inelastic scattering (DIS) of longitudinally polarized leptons by polarized protons are obtained in a model independent way. The numerical analysis is performed for kinematics of future polarized experiments at colliders.

1 Introduction

Deep inelastic lepton-hadron scattering, starting with the discovery of the Bjorken scaling in the end of nineteen-sixties has played a crucial role in the development of our present understanding of the nature of particle interaction. The appearance of the first data on the polarization DIS opened a new field of experimental and theoretical investigations and, as a result it led to come the disturbing data of EMC in 1988 [1].

It is natural, that data processing of the modern experiments on DIS of the polarized lepton on the polarized nuclear target requires correct account of the radiative corrections (RC). Up to now the series of works exist where RC are taken into account in the frame of QED-theory for polarization experiments (see f.e. [2, 3, 4, 5]). Those results are used for experiments on a fixed target. However the polarization DIS experiment at collider will be possible in the future [6].

For the calculation of RC to such kind of experiments we can not be restricted only by γ-exchange graphs, because in this case the squared transfer momentum Q^2 is so high that weak effects begin to play an essential role in the total cross section and spin asymmetries. At the same time the ratio m^2/Q^2 (where m is the mass of a scattering lepton) becomes so small that calculation of electroweak RC could be restricted by non-vanishing terms for $m \to 0$.

The similar work was already done by us [7], but there we used only the naive parton model. In this report we present the explicit expressions for one-loop lowest-order RC to polarized lepton-nucleon scattering which are obtained in the terms of the structure functions. Moreover, the target has arbitrary polarization and the lepton has longitudinal one. The contributions appearing from additional virtual particles in the on-mass renormalization scheme and t'Hooft-Feynman gauge are presented. The results of calculation for unitary gauge which were obtained by D.Bardin's group can be found in ref. [8]. We note also that the separation of variables in accordance with [2] allows to write down all formulae in a compact form that make results more clear as compared to ref. [8].

The present report is organized as follows. In the section 2 the Born contribution and all necessary kinematic invariants are presented. The section 3 is devoted to the electroweak one-loop correction. Numerical analysis for a collider experiment is presented in the section 4. The conclusion is given in section 5.

2 Born contribution

Let us consider deep inelastic polarized lepton-nucleon scattering

$$\ell(k_1,\xi) + N(p,\eta) \to \ell(k_2) + X \tag{1}$$

in the framework of the electroweak standard theory.

The lepton has always a longitudinal polarization, so an expression for the polarization vector of this particle can be represented in the form [2]:

$$\xi = \frac{1}{\lambda_s^{1/2}}\,(\,\tfrac{S}{m}k_1 - 2mp\,)\,. \tag{2}$$

The quantities $k_1(k_2)$, p define the momenta of an initial (final) lepton and target respectively ($k_1^2 = k_2^2 = m^2$, $p^2 = M^2$).

The double-differential cross-section ($d^2\sigma^B/dxdy = \sigma^B$) of lepton-nucleon scattering on the Born level in the framework of electroweak theory is read as

$$\sigma^B = \frac{4\pi\alpha^2 y}{Q^4}\sum_{i=1}^{8}\theta_i^B \mathcal{F}_i. \tag{3}$$

The quantities θ_i^B depend only on the target polarization vector and kinematical invariants:

$$\begin{array}{ll}
\theta_1^B = Q^2, & \theta_5^B = \eta q Q^2 S_p/2M^3, \\
\theta_2^B = (SX - M^2Q^2)/2M^2, & \theta_6^B = (X\eta k_1 + S\eta k_2)/M, \\
\theta_3^B = Q^2 S_p/2M^2, & \theta_7^B = \eta q(SX - M^2Q^2)/2M^3, \\
\theta_4^B = -Q^2\eta(k_1+k_2)/M, & \theta_8^B = -\eta q Q^2/M,
\end{array} \tag{4}$$

which are defined in a standard way:

$$\begin{gathered}
S = 2k_1p,\ X = 2k_2p = (1-y)S,\ Q^2 = -q^2 = -(k_1-k_2)^2 = xyS, \\
S_x = S - X,\ Q_m^2 = Q^2 + 2m^2,\ S_p = S + X,\ \lambda_s = S^2 - 4m^2M^2,\ W^2 = S_x - Q^2 + M^2.
\end{gathered} \tag{5}$$

The generalized structure functions $\mathcal{F}_i$ are some linear combinations of the standard electroweak ones [9], namely:

$$\begin{aligned}
\mathcal{F}_1 &= R_V^\gamma F_1^\gamma + \chi R_V^{\gamma Z} F_1^{\gamma Z} + \chi^2 R_V^Z F_1^Z, \\
\mathcal{F}_2 &= \epsilon(R_V^\gamma F_2^\gamma + \chi R_V^{\gamma Z} F_2^{\gamma Z} + \chi^2 R_V^Z F_2^Z), \\
\mathcal{F}_3 &= \epsilon(\chi R_A^{\gamma Z} F_3^{\gamma Z} + \chi^2 R_A^Z F_3^Z), \\
\mathcal{F}_4 &= \epsilon P_N(R_A^\gamma(g_1^\gamma + g_2^\gamma) + \chi R_A^{\gamma Z}(g_1^{\gamma Z} + g_2^{\gamma Z}) + \chi^2 R_A^Z(g_1^Z + g_2^Z)), \\
\mathcal{F}_5 &= \epsilon^2 P_N(R_A^\gamma g_2^\gamma + \chi R_A^{\gamma Z} g_2^{\gamma Z} + \chi^2 R_A^Z g_2^Z), \\
\mathcal{F}_6 &= \epsilon P_N(\chi R_V^{\gamma Z} g_3^{\gamma Z} + \chi^2 R_V^Z g_3^Z), \\
\mathcal{F}_7 &= \epsilon^2 P_N(\chi R_V^{\gamma Z} g_4^{\gamma Z} + \chi^2 R_V^Z g_4^Z), \\
\mathcal{F}_8 &= \epsilon P_N(\chi R_V^{\gamma Z} g_5^{\gamma Z} + \chi^2 R_V^Z g_5^Z),
\end{aligned} \tag{6}$$

where $\chi = Q^2/(Q^2 + M_Z^2)$ and $\epsilon = M^2/pq$. The quadratic combinations of the electroweak coupling constants are defined as

$$\begin{array}{ll}
R_V^\gamma = 1, & R_A^\gamma = -P_L, \\
R_V^{\gamma Z} = 2(v_l - P_L a_l), & R_A^{\gamma Z} = 2(a_l - P_L v_l), \\
R_V^Z = (v_l^2 + a_l^2) - 2P_L v_l a_l, & R_V^Z = 2v_l a_l - P_L(v_l^2 + a_l^2).
\end{array} \tag{7}$$

The quantities P_L and P_N define polarization degrees of lepton and nucleon respectively.

3 One-Loop Correction

A full set of graphs contributing to the one-loop lowest order correction is presented in fig. 1. The total one-loop lowest order correction which includes the contribution from additional virtual particles (σ_V) and from the radiation of a real photon (σ_R) can be presented as a sum of four infrared stable terms:

$$\sigma_V + \sigma_R = \frac{\alpha}{\pi}\delta_{VR}\sigma_0 + \sigma_V^r + \sigma_R^F + \hat{\sigma}_R \tag{8}$$

The factor

$$\delta_{VR} = (\ln\frac{Q^2}{m^2} - 1)\left[\ln\frac{(W^2 - (M+m_\pi)^2)^2}{(X+Q^2)(S-Q^2)} + \frac{3}{2}\right] - \frac{1}{2} - \frac{1}{2}\ln^2\frac{X+Q^2}{S-Q^2} + \mathrm{Li}_2\frac{SX - Q^2M^2}{(X+Q^2)(S-Q^2)} - \frac{\pi^2}{6}, \tag{9}$$

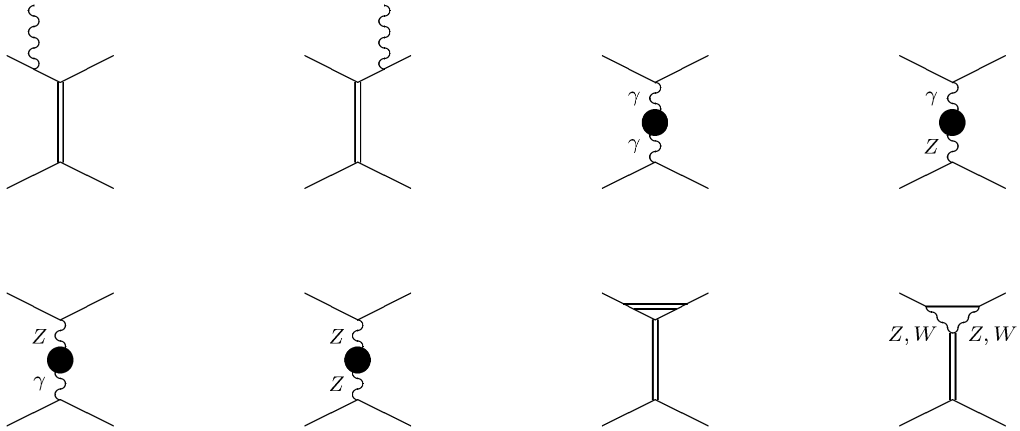

Figure 1: *A complete set of electroweak graphs, contributed to lepton-nucleon scattering in a model-independent way. The double line corresponds to the contribution of γ- or Z-exchange. All possible graphs, which give the contribution to vacuum polarization,are designated by the symbol $\bullet$ [10].*

appears in front of the Born cross section after cancelation of infrared divergence by summarize of infrared part separated from σ_R and so-called 'QED-part' of loop contribution which corresponds to the lepton vertex graphs including an additional virtual photon.

A contribution from electroweak loops with the exception of 'QED-part' can be written in the term of the Born cross section with the follow replacement

$$\sigma_V^r = \sigma_0 \left(R^i_{V,A} \to \delta R^i_{V,A} \right), \tag{10}$$

where

$$\begin{aligned}
\delta R^{\gamma}_{\pm} = \quad & -2\Pi^{\gamma} R^{\gamma}_{\pm} - 2\Pi^{\gamma Z}\chi R^{Z\gamma}_{\pm} + \tfrac{\alpha}{4\pi}[2R^{Z}_{\pm}\Lambda_2(-Q^2, M_Z) + (1-P_l)\tfrac{3}{4s_w^2}\Lambda_3(-Q^2, M_W)], \\
\delta R^{\gamma Z}_{\pm} = \quad & -4(\Pi^{\gamma} + \Pi^{Z})R^{\gamma Z}_{\pm} - 4\Pi^{\gamma Z}(R^{\gamma\gamma}_{\pm} + \chi R^{ZZ}_{\pm}) + \tfrac{\alpha}{2\pi}[2\left(v_l R^{Z}_{\pm} + a_l R^{Z}_{\mp}\right)\Lambda_2(-Q^2, M_Z) \\
& +(1-P_L)\{\tfrac{1}{8s_w^2}\Lambda_2(-Q^2, M_W) + \tfrac{3}{4s_w^2}(v_l + a_l - \tfrac{c_w}{s_w})\Lambda_3(-Q^2, M_W)\}], \\
\delta R^{ZZ}_{\pm} = \quad & -2\Pi^{Z} R^{ZZ}_{\pm} - 2\Pi^{\gamma Z} R^{\gamma Z}_{\pm} + \tfrac{\alpha}{4\pi}[(2\left(v_l^2 + a_l^2\right) R^{ZZ}_{\pm} + 4v_l a_l R^{ZZ}_{\mp})\Lambda_2(-Q^2, M_Z) \\
& +(1-P_L)(v_l + a_l)\{\tfrac{1}{4s_w^2}\Lambda_2(-Q^2, M_W) - 3\tfrac{c_w}{2s_w^3}\Lambda_3(-Q^2, M_W)\}].
\end{aligned} \tag{11}$$

Here $M_{Z,W}$ are the masses of Z and W bosons and

$$\Pi^{\gamma} = -\frac{\hat{\Sigma}^{\gamma}(-Q^2)}{Q^2}, \quad \Pi^{Z} = -\frac{\hat{\Sigma}^{Z}(-Q^2)}{Q^2 + M_Z^2}, \quad \Pi^{\gamma Z} = -\frac{\hat{\Sigma}^{\gamma Z}(-Q^2)}{Q^2}. \tag{12}$$

Quantities $\hat{\Sigma}^{\gamma,\gamma Z,Z}$ are defined by formulae (A.2,3.17,B.2-5) of [10] and $\Lambda_{2,3}$ are given by (B.4,B.6) of [11].

Using approach suggested in ref. [2] infrared free contribution from radiation of a real photon can be written by

$$\begin{aligned}
\sigma_R^F = -\alpha^3 y \int\limits_{\tau_{min}}^{\tau_{max}} d\tau \sum_{i=1}^{8} \Bigg\{ & \theta_{i1}(\tau) \int\limits_0^{R_{max}} \frac{dR}{R} \left[\frac{\mathcal{F}_i(R,\tau)}{(Q^2 + R\tau)^2} - \frac{\mathcal{F}_i(0,0)}{Q^4} \right] \\
& + \sum_{j=2}^{k_i} \theta_{ij}(\tau) \int\limits_0^{R_{max}} dR \frac{R^{j-2}}{(Q^2 + R\tau)^2} \mathcal{F}_i(R,\tau) \Bigg\},
\end{aligned} \tag{13}$$

where

$$\tau_{max,min} = \frac{S_x \pm \sqrt{S_x^2 + 4M^2Q^2}}{2M^2}, \qquad R_{max} = \frac{W^2 - (M + m_\pi)^2}{1 + \tau}. \tag{14}$$

Here m_π is the pion mass.

The structure functions $\mathcal{F}_i(R, \tau)$ are calculated by formulae (6) with the next replacement of arguments

$$Q^2 \to Q^2 + R\tau, \; x \to \frac{Q^2 + R\tau}{S_x - R}. \tag{15}$$

All results presented above are originated from the first term $\sim k_1$ of the lepton polarization vector (2). A part of the cross section which appears for the second term in (2) is very small ($\sim m^2$) both on the Born level and for the contribution with additional virtual particles. However, the radiation of a real photon gives the non-vanishing contribution coming from it:

$$\hat{\sigma}_{in} = \frac{\alpha y}{\pi} P_L \int\limits_0^{R^s_{max}} \frac{RdR}{(S - Q^2)(S_x - yQ^2 - R)} \tilde{\sigma}_0^{pl}, \tag{16}$$

where the upper limit $R^s_{max} = (1 - x)(S - Q^2)$ and σ_0^{pl} is lepton polarization part of the Born cross section with the replacement of kinematical variables: $S \to S - RS/(S - Q^2)$,$X \to X$ and $Q^2 \to Q^2 - RQ^2/(S - Q^2)$.

4 Numerical Analysis

Both at the Born level and at the level of RC the double differential cross section σ as a function of the polarization characteristics can be represented as a sum of four terms:

$$\sigma_{0,1} = \sigma_{0,1}^{uu} - P_L\sigma_{0,1}^{pu} + P_N\sigma_{\|0,1}^{up} - P_N P_L \sigma_{\|0,1}^{pp} \tag{17}$$

for the longitudinally polarized target, and

$$\sigma_{0,1} = \sigma_{0,1}^{uu} - P_L\sigma_{0,1}^{pu} + P_N\sigma_{\perp 0,1}^{up} - P_N P_L \sigma_{\perp 0,1}^{pp} \tag{18}$$

for the transversely polarized one.

So we can construct six independent quantities

$$\begin{aligned} \delta^{uu} &= \sigma_1^{uu}/\sigma_0^{uu}, \quad \delta^{pu} = \sigma_1^{pu}/\sigma_0^{pu}, \\ \delta_{\|}^{up} &= \sigma_{\|1}^{up}/\sigma_{\|0}^{up}, \quad \delta_{\|}^{pp} = \sigma_{\|1}^{pp}/\sigma_{\|0}^{pp}, \\ \delta_{\perp}^{up} &= \sigma_{\perp 1}^{up}/\sigma_{\perp 0}^{up}, \quad \delta_{\perp}^{pp} = \sigma_{\perp 1}^{pp}/\sigma_{\perp 0}^{pp}, \end{aligned} \tag{19}$$

which describe the ratio of the radiative cross-sections to the Born ones. It is clear that RC to all observable quantities can be constructed from these six ratios.

Dependence of the quantities (19) on the scaling variable y are presented in fig. 2 for different x. These results were obtained using the electroweak structure functions $F_i^{\gamma\,\gamma Z\,Z}$ and $g_i^{\gamma\,\gamma Z\,Z}$ defined by formulae (3.5.5-3.5.7) of ref.[9] and the GRV parameterization for parton distributions [12].

5 Conclusions

1. The one-loop electroweak RC to the lepton current for DIS of longitudinally polarized leptons by polarized target are obtained in a model independent way.

2. The numerical analysis for future experiments at colliders shows:

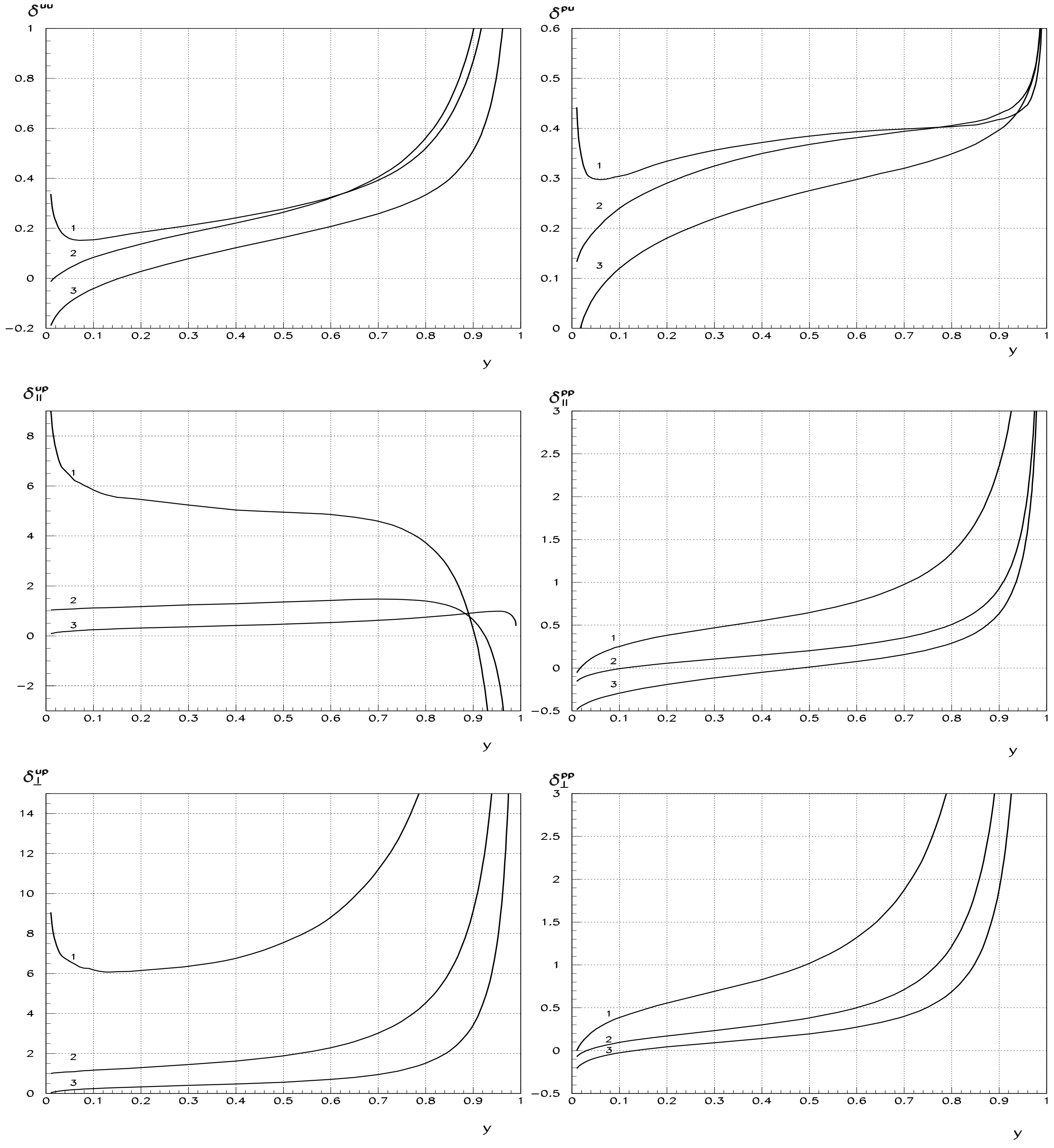

Figure 2: *y-dependence of the quantities δ (19) for deep inelastic electron-proton scattering. Initial energies of electron $E_e = 27.5 GeV$ and proton $E_p = 810 GeV$. 1) $x = 0.001$; 2) $x = 0.01$; 3) $x = 0.1$*

(a) the relative RC to a polarized part of the cross section has the same behavior as for unpolarized one: it goes up
— when y tends to kinematical boundaries ($y \to 0, y \to 1$)
— when x goes down.

(b) at the same time the corrections to polarized parts can in the several times exceed the unpolarized corrections.

Appendix

In this Appendix explicit expressions for θ functions are represented. Due to factorization of infrared terms all θ_{i1} are proportional to the Born contributions:

$$\theta_{i1} = 4F_{IR}\theta_i^B. \tag{20}$$

The others θ functions can be written as follows:

$$\begin{aligned}
\theta_{12} &= 4\tau F_{IR}, \\
\theta_{13} &= -2(2F + F_d\tau^2), \\
\theta_{22} &= (F_{1+}S_xS_p - F_dS_p^2\tau + 2m^2F_{2-}S_p - 2(2M^2\tau - S_x)F_{IR})/2M^2, \\
\theta_{23} &= (4FM^2 + 2F_dM^2\tau^2 - F_dS_x\tau - F_{1+}S_p)/2M^2, \\
\theta_{32} &= (F_{1+}S_xQ^2 + 2m^2F_{2-}Q^2 - m^2F_{2+}S_p\tau + 3F_{IR}S_p\tau)/M^2, \\
\theta_{33} &= (2m^2F_{2-}\tau - F_dS_p\tau^2 - 2F_{1+}Q^2)/M^2, \\
\theta_{34} &= -\tau F_{1+}/M^2, \\
\theta_{42} &= 2(\eta\mathcal{K}\tau(m^2F_{2+} - 3F_{IR}) - \eta qF_{1+}Q^2 - 2m^2F_{2-}^{\eta}Q^2)/M, \\
\theta_{43} &= 2(\eta\mathcal{K}F_d\tau^2 + 2F_{1+}^{\eta}Q^2 - 2m^2F_{2-}^{\eta}\tau)/M, \\
\theta_{44} &= 2\tau F_{1+}^{\eta}/M, \\
\theta_{62} &= (\eta\mathcal{K}(F_{1+}S_x - 2F_dS_p\tau + 2m^2F_{2-})/M, \\
&\quad +\eta q(F_{1+}S_p + 2F_{IR}) + 2F_{1+}^{\eta}S_x + 2m^2F_{2-}^{\eta}S_p)/M, \\
\theta_{63} &= -(\eta\mathcal{K}F_{1+} + \eta qF_d\tau + F_d^{\eta}S_x\tau + F_{1+}^{\eta}S_p)/M,
\end{aligned} \tag{21}$$

and for $i = 5, 7, 8$:

$$\begin{array}{lll}
\theta_{51} = \eta q\theta_{31}/M, & \theta_{71} = \eta q\theta_{21}/M, & \theta_{81} = \eta q\theta_{11}/M, \\
\theta_{52} = (\eta q\theta_{32} - \theta_{31}^{\eta})/M, & \theta_{72} = (\eta q\theta_{22} - \theta_{21}^{\eta})/M, & \theta_{82} = (\eta q\theta_{12} - \theta_{11}^{\eta})/M, \\
\theta_{53} = (\eta q\theta_{33} - \theta_{32}^{\eta})/M, & \theta_{73} = (\eta q\theta_{23} - \theta_{22}^{\eta})/M, & \theta_{83} = (\eta q\theta_{13} - \theta_{12}^{\eta})/M, \\
\theta_{54} = (\eta q\theta_{34} - \theta_{33}^{\eta})/M, & \theta_{74} = -\theta_{23}^{\eta}/M, & \theta_{84} = -\theta_{13}^{\eta}/M, \\
\theta_{55} = -\theta_{34}^{\eta}/M. & &
\end{array} \tag{22}$$

The definitions of quantities F and θ_{ij}^{η} can be found in Appendix B of ref.[13].

References

[1] J.Ashman et al., *Phys. Lett.* **B206** 364 (1988).

[2] I.Akushevich, N.Shumeiko, *J. Phys.* **G20** 513 (1994).

[3] T.Kukhto, M.Shumeiko *Nucl. Phys.* **B219** 412 (1983).

[4] A.Soroko, N.Shumeiko *Yad.Fiz.* **49** 1348 (1989).

[5] N.Shumeiko, S.Timoshin *Journal of Physics* **G17** 1145 (1991).

[6] J.Feltesse, A.Schäfer, in Proc. of the Workshop 1995/96 *Future Physics at HERA*, Eds. G.Ingelman, A.De Roeck, R. Klanner, DESY, 1996, p.760 .

[7] I.Akushevich, A.Ilyichev, N.Shumeiko *Yad. Fiz.* **58** 2029 (1995).

[8] D.Bardin, J.Blumlein, P.Christova, L. Kalinovskaya *DESY-96-189, Dec 1996. 36pp.*

[9] M.Anselmino, A.Efremov, E.Leader *Phys. Rep.* **261** 1 (1995).

[10] W. Hollik *Fortschr. Phys.* **38**165 (1990).

[11] M. Böhm , W.Hollik, H.Spiesberger*Fortschr. Phys.* **34** 687 (1986).

[12] M.Glück, E.Reya, A.Vogt 1992 *Z. Phys.* **C53** 127 (1992).

[13] I.Akushevich , A.Ilyichev, A.Soroko, N.Shumeiko, A.Tolkachev 1997 *Comput.Phys.Commun.* **104** 201 (1997).