The problem of classicalization of quantum Universe and quark confinement

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Abstract

We apply developed earlier proposal for the choice of the preferred basis in the many-worlds interpretation of quantum mechanics and quantum cosmology to the systems possessing some non-Abelian symmetry. It is shown that in this case the subdivision of the system into subsystems implying the choice of the preferred basis should be arranged in such a way that the wave functions of the subsystems belong to the trivial representation of the corresponding non-Abelian group. In the opposite case the procedure of the choice of the preferred basis does not work. Obtained result is used for a "toy" explanation of the phenomenon of quark confinement in quantum chromodynamics.

This paper is dedicated to the memory of Nikita Alexeevich Sveshnikov. I had a happy opportunity to work with him in the end of 70th and at the beginning of 80th on the problem of infrared divergences in the quantum-field-theoretical models possessing non-Abelian symmetry. As known this problem is closely connected with the problem of quark confinement. It was N.A. Sveshnikov who first understood that the consistent application of the method of asymptotic dynamics developed for fighting with infrared divergences in quantum electrodynamics [1] to massless theories with non-Abelian symmetries can imply non-trivial restrictions on the spectrum of the theory including the absence of color states in chromodynamics [2]. We worked together until the middle of 80th, but after that I often used to discuss with Nikita different topics and he was always able to give a good advice.

I believe that his presence in the science can not be reduced to the list of his publications. His indirect influence on people around him was very strong and important and I can find its traces in works rather distant from his activity in such fields as, for example, quantum cosmology.

In the work which I present here some distant topics are mixed– interpretation of quantum mechanics, problem of classicalization of quantum Universe and quark confinement. I shall try to show how the phenomenon of quark confinement could be understood (or illustrated) from the simple quantum mechanical considerations connected with the many-worlds interpretation of quantum mechanics [3].

Let me describe briefly the basic points of many-worlds interpretation of quantum mechanics. In the Copenhagen interpretation there are two types of fundamental processes: evolution of the wave function according to Schrödinger equation and reduction of the wave packet. This second process is connected with the special role of classical observer or even consciousness [4].

In the many-worlds interpretation of quantum mechanics only quantum objects are primary ones and only one fundamental process exists: evolution according to Schrödinger equation without reduction of the wave packet. Instead one can speak of a simultaneous realization of different outcomes of quantum measurement in different branches of the wave function of the Universe or in different "parallel" Everett worlds.

Resolving some old problems of interpretation of quantum theory the many-worlds interpretation invokes new questions. The special place between them occupies the problem of the choice of the preferred basis or in other words the choice of the procedure which determines what namely pieces of the wave function of the Universe can be regarded as those describing the autonomous Everett worlds or branches. In our papers [5] we have revived the old proposal of Zeh [6] and gave somewhat different and more transparent arguments in favor of this proposal. It is necessary to stress also that this basis, which we shall call bi-orthogonal, was first introduced at the dawn of quantum mechanics (and many years before the many-worlds interpretation came into existence) by Schrödinger for the purpose of describing the quantum correlations between the interacting subsystems [7].

Here I would like to discuss briefly the relation between the procedure of the choice of the preferred basis and the symmetry of the system under consideration. It is shown that in this case when the system possesses some non- Abelian symmetry the subdivision of this system into subsystems implying the choice of the preferred basis should be arranged in such a way that the wave functions of the subsystems belong to the trivial representation of the corresponding non-Abelian group. In the opposite case the procedure of the choice of the preferred basis does not work [8].

For illustrative purposes we begin the consideration with the well-known example of the Stern-Gerlach experiment. Let the system consist of the device which is initially in the state $|\Phi\rangle_0$ and is ready to measure the spin z-component of an atom, which is in the state

$$(c_1|\uparrow\rangle + c_2|\downarrow\rangle),$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ are $s_z = \pm\frac{1}{2}$ orthonormal eigenstates of an atom. The initial state of the system as a whole is

$$|\Psi\rangle_{in} = (c_1|\uparrow\rangle + c_2|\downarrow\rangle)|\Phi\rangle_0. \tag{1}$$

We see that before the measurement both the device and atom are in the pure state. Let us introduce now the unitary operator $\hat{U}$ describing the interaction between the atom and the device. The measurement operator $\hat{U}$ satisfies the following rule [21]:

$$\begin{aligned}\hat{U}|\uparrow\rangle|\Phi\rangle_0 &= |\uparrow\rangle|\Phi_\uparrow\rangle \\ \hat{U}|\downarrow\rangle|\Phi\rangle_0 &= |\downarrow\rangle|\Phi_\downarrow\rangle\end{aligned} \tag{2}$$

where $|\Phi_\uparrow\rangle$ and $|\Phi_\downarrow\rangle$ are the orthonormal eigenstates of the device indicating the fact of measuring respectively the values $s_z = \frac{1}{2}$ and $s_z = -\frac{1}{2}$. Under the action of $\hat{U}$ the state (1) transforms into

$$|\Psi\rangle_{out} = \hat{U}|\Psi\rangle_{in} = c_1|\uparrow\rangle|\Phi_\uparrow\rangle + c_2|\downarrow\rangle|\Phi_\downarrow\rangle. \tag{3}$$

¿From the point of view of Copenhagen interpretation the unitary transition from the state (1) to the state (3) is only the first part of the process of quantum measurement. We know that really we can measure only one value of s_z, but in (3) there are two terms corresponding to both possible results of measurement. The second part of the quantum measurement consists in the elimination of one of these terms:

$$|\Psi\rangle_{out} \Rightarrow |\uparrow\rangle|\Phi_\uparrow\rangle$$

or

$$|\Psi\rangle_{out} \Rightarrow |\downarrow\rangle|\Phi_\downarrow\rangle. \tag{4}$$

The process (4) is nothing else but a reduction of the wave packet. However, in the framework of the many-worlds interpretation we reject the process (4) and recognize the simultaneous existence of both terms in the superposition (3): the measurement process is reduced to the arrangement of correlations between the atom and the device and both outcomes of the experiment exist in parallel worlds. Thus, instead of the reduction of the wave packet we have the "branching" of our world. From the mathematical point of view the branching is merely defactorization of the wave function of a system with respect to the subdivision of this system into subsystems (see eq.(3)) and these subsystems undergo the transition from pure quantum states to mixed ones.

Generally speaking, every transition of the wave function of a system which consists of two subsystems from the factorized state

$$|\Psi\rangle_{in} = |\varphi\rangle|\chi\rangle \tag{5}$$

to the defactorized state

$$|\Psi\rangle_{out} = \sum_{i=1}^{n} |\varphi_i\rangle|\chi_i\rangle, \tag{6}$$

where the states $|\varphi\rangle$, $|\varphi_i\rangle$ describe one subsystem and states $|\chi\rangle, |\chi_i\rangle$ - another one, and $n > 1$ can be regarded as a process of branching or defactorization of the wave function.

We see that in the decomposition (6) every state of one subsystem $|\varphi_i\rangle$ has the uniquely determined counterpart $|\chi_i\rangle$, which is usually called the "relative state". However at the same time we can choose another set of basis states $|\tilde{\varphi}_i\rangle$ instead of $|\varphi_i\rangle$. In this case we shall have another set of relative states $|\tilde{\chi}_i\rangle$ and the wave function (6) will be written in the following form:

$$|\Psi\rangle_{out} = \sum_{i=1}^{n} |\tilde{\varphi}_i\rangle|\tilde{\chi}_i\rangle. \tag{7}$$

¿From the mathematical point of view there is not essential difference between formulas (6) and (7), they simply describe the same wave function, but written in different bases. But from the physical point of view the choice of different bases implies different decompositions of the wave function into some sets of branches which correspond to different real worlds, which can be perceived experimentally. Thus, we have to make a definite choice of some preferred decomposition or preferred basis.

One can say that within the usual "common sense" it is impossible to choose in the case of Gerlach-Stern experiment another basis but

$$|\uparrow\rangle|\Phi_\uparrow\rangle \ and \ |\downarrow\rangle|\Phi_\downarrow\rangle, \tag{8}$$

because our device is constructed and arranged to measure namely spin z-component of an atom. It is true, but in order to separate other possible choices of the basis we have to resort to some classical properties of the device. However, this deprives us of the main advantage of the many-worlds interpretation - its purely quantum nature. Moreover, the common sense are applicable only in simple quantum mechanical experiments. In a more complicated case of quantum cosmology common sense does not always work. The point is that it is necessary to work out the procedure of constructing the preferred basis originating from purely quantum notions. We can try to extract this procedure from the consideration of rather simple quantum mechanical problems, generalize it, then apply it to more general and complicated situations amounting to problems of quantum cosmology.

The construction of the preferred basis, which we would like to advocate, consists of two steps: (i) splitting of the system under consideration into certain subsystems and (ii) choosing the proper basis for one subsystem and the basis of relative states for another subsystem. It is necessary to stress that from the point of view of our prescription, we can choose any subdivision of the system into subsystems, because for every such subdivision our prescription gives an unique preferred basis. In contradistinction to the Copenhagen interpretation, where observation and measurement play a special role, in the many-worlds interpretation they are simply interactions between subsystems. Therefore, we shall treat our subsystems on an equal footing without indication which of them is the observer and which is the observable. All notations concerning the corresponding subsystem will be marked by subscripts I and II.

It seems rather reasonable to require that the belonging of different terms of the decomposition of the wave function to different worlds must be accompanied by their mutual orthogonality; moreover, we require also orthonormality of substates corresponding to our subdivision of the system into subsystems. It leads immediately to the following decomposition:

$$|\Psi\rangle = \sum_n c_n|n\rangle_I|n\rangle_{II}, \tag{9}$$

where the both sets of basis vectors $|n\rangle_I$ and their relative states $|n\rangle_{II}$ are orthonormal

$$_I\langle n|m\rangle_I = \delta_{nm}, \quad _{II}\langle n|m\rangle_{II} = \delta_{nm} \tag{10}$$

and c_n are some complex coefficients which determine a priory probabilities p_n of realizing the nth Everett world:

$$\begin{aligned} p_n &= |c_n|^2 \\ \sum_n |c_n|^2 &= \langle\Psi|\Psi\rangle = 1. \end{aligned} \tag{11}$$

This prescription for the choice of the preferred basis yields also a constructive algorithm for finding it. Consider the density matrix of the total system $\hat{\rho} = |\Psi\rangle\langle\Psi|$ and the density matrix of the first subsystem I, obtained by tracing out the degrees of freedom of the subsystem II

$$\hat{\rho}_I = Tr_{II}|\Psi\rangle\langle\Psi|. \tag{12}$$

Substituting the decomposition (9) and using the orthonormality condition (10) one finds that the latter has the form

$$\hat{\rho}_I = \sum_n |c_n|^2|n\rangle_I \ _I\langle n| \tag{13}$$

whence one concludes that the vectors $|n\rangle_I$ of the preferred basis solve the eigenvalue problem for the density matrix $\hat{\rho}_I$

$$\hat{\rho}_I|n\rangle_I = p_n|n\rangle_I,$$

$$p_n = |c_n|^2 \tag{14}$$

with eigenvalues which exactly coincide with the probability weights (11) of Everett worlds. Similarly to $|n\rangle_I$ the basis vectors $|n\rangle_{II}$ are the eigenvectors of the density matrix of the second subsystem

$$\hat{\rho}_{II} = Tr_I|\Psi\rangle\langle\Psi|, \quad \hat{\rho}|n\rangle_{II} = p_n|n\rangle_{II}. \tag{15}$$

The both density matrices $\hat{\rho}_I$ and $\hat{\rho}_{II}$ are Hermitian, positive semi-definite and (in view of the normalizability of $|\Psi\rangle$) bounded operators. Therefore they possess a countable eigenvalue spectrum and their orthonormal eigenvectors are unique up to inessential phase factors. The only exceptions are the invariant subspaces of $\hat{\rho}_I$ and $\hat{\rho}_{II}$ corresponding to possible degenerate eigenvalues, in which the eigenvectors can be determined up to unitary rotations. These subspaces can only be finite-dimensional, because the sum of eigenvalues over each subspace is less than (or equal to) 1 in view of (11).

The basis (9), to which we came due to reasoning of the above type, coincides with the bi-orthogonal basis of Schrödinger [7], introduced for the study of quantum correlations between interacting quantum systems or with the "Schmidt canonical basis" of Zeh [6], used in the context of the many-worlds interpretation. This basis can be obtained by solving two eigenvalue problems (13) and (14) for the density matrices of relevant subsystems. The non-uniqueness of this solution in the invariant subspaces of the degenerate eigenvalues was also mentioned in Ref. [6], but without a concrete proposal for its resolution. Here we shall go somewhat further and resort to an additional principle in order to fix this basis uniquely. From Eqs. (13)–(14) it follows that the preferred basis depends on the quantum state $|\Psi\rangle$ of the total system and consequently evolves in accordance with the Schrödinger evolution of $|\Psi\rangle = |\Psi(t)\rangle$. Then it seems natural to require that the decomposition of these invariant Hilbert subspaces into equally probable Everett worlds should be stable against this dynamical evolution. To demonstrate the efficiency of this principle we return to the above mentioned example of the Gerlach-Stern experiment.

The density matrices of measurement device and atom corresponding to the state of the whole system after measurement (3) take the form

$$\hat{\rho}_{device} = |c_1|^2|\Phi_\uparrow\rangle\langle\Phi_\uparrow| + |c_2|^2|\Phi_\downarrow\rangle\langle\Phi_\downarrow|$$

$$\hat{\rho}_{atom} = |c_1|^2|\uparrow\rangle\langle\uparrow| + |c_2|^2|\downarrow\rangle\langle\downarrow|$$

and according to our algorithm preferred basis is given in the non-degenerate case $|c_1|^2 \neq |c_2|^2$ by the vectors (4).

For $|c_1|^2 = |c_2|^2 = \frac{1}{2}$ this procedure becomes insufficient, because the preferred basis in our two-dimensional Hilbert space is already not unique. For example, instead of (4) one can take the basis of vectors $|\rightarrow\rangle|\Phi_\rightarrow\rangle$ and $|\leftarrow\rangle|\Phi_\leftarrow\rangle$,where

$$\begin{aligned}
|\rightarrow\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle), \\
|\leftarrow\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle), \\
|\Phi_\rightarrow\rangle &= \frac{1}{\sqrt{2}}(|\Phi_\uparrow\rangle + |\Phi_\downarrow\rangle), \\
|\Phi_\leftarrow\rangle &= \frac{1}{\sqrt{2}}(|\Phi_\uparrow\rangle - |\Phi_\downarrow\rangle).
\end{aligned} \tag{16}$$

For the case $c_1 = c_2 = \frac{1}{\sqrt{2}}$ Eq. (15) also solves the eigenvalue problems. The new basis is however unstable with respect to the measurement-like interaction between the device and the atom. Indeed, two branches of $|\Psi\rangle$ in this basis

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|\rightarrow\rangle|\Phi_\rightarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow\rangle|\Phi_\leftarrow\rangle. \tag{17}$$

In contrast to branches (15) of the decomposition (16) those branches (4) of the decomposition (3) are not destroyed during the dynamical evolution by the operator $\hat{U}$ and thus form the unique preferred basis which gives the reasonable many-worlds picture of the Gerlach-Stern experiment.

The procedure of the choice of preferred basis considered above is constructed in such a manner to be independent of the particular splitting of the system under consideration into subsystems. However, in some cases the requirement of definiteness and uniqueness of our choice can effect a procedure of splitting. We can represent one example of such "back reaction" of the prescription for preferred basis to the splitting.

Let us assume that there is some conserved non-Abelian symmetry in our Universe i.e. Hamiltonian, governing the evolution of the Universe contains operators, belonging to some non-trivial representations of a non-Abelian group in such a way, that this Hamiltonian as a whole belongs to singlet representation of this group. Besides, we suppose that the wave function of the Universe belongs also to a singlet representation of the symmetry group. Under a singlet representation of some group one usually means the trivial representation of this group. When we speak about the wave function in the singlet (trivial) representation we mean that this wave function is intact under the action of all group elements $g|\Psi\rangle = |\Psi\rangle$ and is annihilated by every generator of the corresponding Lie algebra. In the case of an operator belonging to a singlet representation (in our problem we consider Hamiltonian) the following equality is correct

$$gHg^{-1} = H,$$

where g is an arbitrary group element. In other terms it means that the Hamiltonian commutes with all the generators of the corresponding Lie algebra. It is necessary to stress that the wave function can be constructed in such a way, that being in singlet (trivial) state it can contain components, belonging to the non-singlet (non-trivial) states. Mathematical background of this phenomenon consists in the fact that the direct products of elements belonging to non-trivial representations of a non-Abelian group can contain in the decomposition into the direct sum the trivial representation. For example, the couple of particles with the one-half spin can have total spin which equals to zero. Analogously, baryon, which consists of three color quarks belonging to non-trivial (fundamental) representation of $SU(3)$ group can be colorless and belonging to the trivial representation of this group. In accordance with written above we can choose the splitting of the system under consideration (Universe) in such a way that both subsystems are in non-singlet states, while the Universe as a whole is in a singlet state. All subsequent consideration is valid for arbitrary gauge theory with non-Abelian symmetry, but tending to clearness of our picture we shall speak about color group $SU(3)$ in quantum chromodynamics and as a subsystem with a non-zero color charge we can take a single quark or another color combination of quarks and antiquarks. In the case then one subsystem is quark and another—the rest of the Universe the wave function takes the following form:

$$|\Psi\rangle = \sum_{i=1}^{3} \frac{1}{\sqrt{3}} |\varphi_i\rangle |\chi_i\rangle. \tag{18}$$

Here, three functions $|\varphi_i\rangle$ belong to the fundamental representation of $SU(3)$ group while functions $|\chi_i\rangle$ belong to one contragradient to fundamental. The density matrices for both subsystems have degenerate spectrum ($p_i = \frac{1}{3}$ for $i = 1, 2, 3$). Thus in order to get a unique preferred basis we must resort to the principle of dynamical stability of this basis. The Hamiltonian governing the evolution of the wave function (18) can be represented in the following form

$$H = H_\varphi \bigotimes I_\chi + I_\varphi \bigotimes H_\chi + J_\varphi \bigotimes J_\chi. \tag{19}$$

Here the first term in (19) denotes free Hamiltonian of first subsystem which must belong to trivial (singlet) representation of $SU(3)$ (and I_χ is the unite operator acting in the subspace, describing the second subsystem), the second term is free Hamiltonian for the second subsystem and the third term in the right-hand side of (19) describes the color interaction between subsystems. The color currents J_φ and J_χ must belong to some nontrivial representations of $SU(3)$ and their direct product must be constructed in such way to be belonging to the singlet representation of symmetry group. It implies that term $J_\varphi \bigotimes J_\chi$ contains more than one item [9]. Let us now act by the evolution operator determined by Hamiltonian (19) on the one branch $|\varphi_k\rangle|\chi_k\rangle$ from the decomposition (18). Due to the structure of the term $J_\varphi \bigotimes J_\chi$, including several items our branch under consideration turns into several branches analogously to the splitting of the branch $|\rightarrow\rangle|\Phi_\rightarrow\rangle$ in the Gerlach-Stern experiment under the action of measurement operator $\hat{U}$. Every branch $|\varphi_k\rangle|\chi_k\rangle$ will undergo this additional splitting and this phenomenon does not depend on the particular way of the decomposition of the wave function (18). Independently of the choice of our preferred basis its elements ("branches") will rotate with permanent velocity in the space of internal color degrees of freedom. Thus we do not have the preferred basis for this decomposition because we do not have the branches which are stable against the quantum evolution. All the above considerations remain true in the cases when the subsystem I is not a single quark but an arbitrary combination of quarks and antiquarks which belongs to some non-singlet representation of $SU(3)$. Moreover, our arguments are applicable to every non-Abelian symmetry group. At the same time the splitting of the system into singlet subsystems resolves the problem of stability of the preferred basis because in this case the non-trivial (color) interaction between subsystems can not take place. Thus, we have seen that the rules for the choice of the preferred basis formulated in Sec. II in some cases can effect the choice of splitting of the system into subsystems. The requirement of consistency imposes some restrictions on the arbitrariness of this splitting. In the case of $SU(3)$ color group of quantum chromodynamics the impossibility of the consideration of a single quark or some color combination of quarks and antiquarks as the subsystem can be treated as some hint for a purely quantum mechanical proof of the confinement phenomenon in QCD or as some "toy" approach to this phenomenon. Let us stress once again: in the presence of exact non-Abelian symmetry for the wave function of the Universe and for Hamiltonian governing its evolution decomposition of the Universe into non-singlet parts do not induce the unique and consistent choice of the preferred basis because all the possible bases will be unstable against interaction Hamiltonian belonging to the trivial representation of the corresponding non-Abelian group but composed from non-trivial currents. Note that treating Stern-Gerlach experiment we could choose the unique preferred basis even in the case of degenerate density matrix of the atom (degeneration of density matrix is equivalent to the fact that the wave function is invariant under the action of rotation group $SO(3)$) because the Hamiltonian of interaction between atom and device was not invariant under $SO(3)$ and had a chosen direction —orientation of magnetic field.

In the conclusion I would like to mention once again Nikita Alexeevich Sveshnikov. Work presented here is literally penetrated by ideas and thoughts which I have caught from him during our joint work and permanent discussions. Sometimes, I feel that my attitudes to different questions of physics (often distant from immediate interests of N.A. Sveshnikov) are molded by my communication with him. The very interest to the phenomenon of confinement and to the role of non-Abelian symmetries in quantum field theory was connected with my work with Nikita. Moreover, long before I first heard about many-worlds interpretation of quantum mechanics I have learned from him the attitude to quantum theory as to the theory more fundamental than classical mechanics. It was in strong contrast with the Copenhagen-style ideas represented in the standard text-books and namely conversation with Nikita helped me to understand many-worlds interpretation and in indirect way to start my work in quantum cosmology. One can add also that Nikita was fond of papers of Yu.M. Shirokov [10] devoted to elaboration of universal Hamilton mechanics and studying these papers (again under Nikita's influence) helped me very much in my work.

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