Decays of Positronium

Andrzej Czarnecki[a] and Savely G. Karshenboim[b]

[a] *Physics Department, Brookhaven National Laboratory*
Upton, New York 11973, USA

[b] *D. I. Mendeleev Institute for Metrology, St. Petersburg, Russia*

Abstract

We briefly review the theoretical and experimental results concerning decays of positronium. Possible solutions of the "orthopositronium lifetime puzzle" are discussed. Positronium annihilation into neutrinos is examined and disagreement is found with previously published results.

1 Introduction

Positronium (Ps), an electron-positron bound state, is the lightest known atom. Its gross spectrum is similar to that of the hydrogen, except for a different reduced mass[1] $m_R = m_e/2$. Ps is of significant theoretical and experimental interest. In contrast to the hydrogen, it does not contain a proton and its theoretical description is not limited by hadronic uncertainties, at least at the present level of experimental accuracy. Spectrum and lifetimes of Ps states can be predicted within Quantum Electrodynamics (QED) with very high accuracy. Recently, significant progress in QED theory of positronium spectrum has been achieved. Effects $\mathcal{O}\left(\alpha^6 m_e\right)$ [1, 2, 3, 4, 5] and $\mathcal{O}\left(\alpha^7 \ln^2(\alpha) m_e\right)$ [6, 7, 8] were calculated. Some progress has also been made in the study of the positronium annihilation rate, which we will discuss below. Combined with precise experimental measurements, these results provide unique opportunities for testing bound-state theory based on QED. For example, one can study recoil effects in detail because in Ps they are not suppressed by a small mass ratio (as opposed to e.g. muonium).

In addition, precise studies of positronium decays provide bounds on exotic particles, such as axions, paraphotons, or millicharged particles. This short note is devoted to a review of the standard and some exotic positronium decay channels. In particular, we will be interested in the decay into neutrinos, where we find disagreement with previously published results.

2 Photonic decays of positronium

2.1 Parapositronium decays

In the S state of Ps, spins of the electron and positron can combine to give either a total spin 0 singlet state (parapositronium, p-Ps), or a spin 1 triplet (orthopositronium, o-Ps). Because of the possibility of e^+e^- annihilation, both states are unstable. Barring C-violating effects (e.g. caused by the weak interactions), the ground state of parapositronium (singlet) can annihilate into only even number of photons (see Fig. 1).

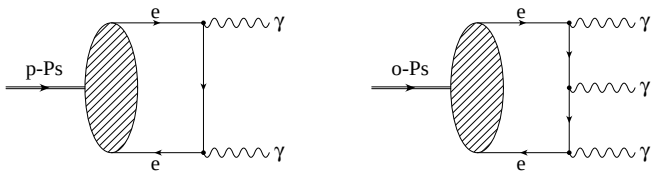

Figure 1: Lowest order decay channels of p-Ps and o-Ps.

The decay rate of the p-Ps ground state, 1^1S_0, can be calculated as a series in α. The two-photon decay rate is

$$\Gamma(\text{p-Ps} \to \gamma\gamma) \quad = \quad \frac{\alpha^5 m_e}{2}\left[1 - \left(5 - \frac{\pi^2}{4}\right)\frac{\alpha}{\pi} + 2\alpha^2 \ln\frac{1}{\alpha} + 1.75(30)\left(\frac{\alpha}{\pi}\right)^2 - \frac{3\alpha^3}{2\pi}\ln^2\frac{1}{\alpha} + \mathcal{O}\left(\alpha^3 \ln\frac{1}{\alpha}\right)\right]$$

[1] We use $\hbar = c = 1$, $\alpha = e^2/4\pi \simeq 1/137$, and denote electron's mass and charge by m_e and $-e$, respectively.

$$= \quad 7989.50(2)\ \mu\mathrm{s}^{-1}\,, \tag{1}$$

where the non-logarithmic terms $\mathcal{O}\left(\alpha^2\right)$ [9, 10] and leading-logarithmic terms $\mathcal{O}\left(\alpha^3 \ln^2 \alpha\right)$ [6] have been obtained only recently. The four–photon branching ratio is of the relative order α^2 [11, 12, 13]:

$$\mathrm{BR}(\text{p-Ps} \to 4\gamma) = \frac{\Gamma(\text{p-Ps} \to 4\gamma)}{\Gamma(\text{p-Ps} \to \gamma\gamma)} = 0.277(1) \left(\frac{\alpha}{\pi}\right)^2 \simeq 1.49 \cdot 10^{-6}\,. \tag{2}$$

The theoretical prediction, (1), agrees well with the experiment [14],

$$\Gamma_{\mathrm{exp}}(\text{p-Ps}) = 7990.9(1.7)\ \mu\mathrm{s}^{-1}\,. \tag{3}$$

2.2 Orthopositronium decays

The ground state of the orthopositronium, 1^3S_1, can decay into an odd number of the photons only (if C is conserved). The three–photon (see Fig. 1) decay rate formula is [15, 6]

$$\begin{aligned}\Gamma(\text{o-Ps} \to \gamma\gamma\gamma) &= \frac{2(\pi^2-9)\alpha^6 m_e}{9\pi}\left[1 - 10.28661(1)\frac{\alpha}{\pi} - \frac{\alpha^2}{3}\ln\frac{1}{\alpha} + B_o\left(\frac{\alpha}{\pi}\right)^2 - \frac{3\alpha^3}{2\pi}\ln^2\frac{1}{\alpha} + \mathcal{O}\left(\alpha^3 \ln\alpha\right)\right] \\ &\simeq \quad \left(7.0382 + 0.39 \cdot 10^{-4}\ B_o\right)\ \mu\mathrm{s}^{-1}\,. \end{aligned} \tag{4}$$

Because of the three-body phase space and a large number of diagrams, a theoretical analysis of o-Ps decays is more difficult than in the case of p-Ps. The non-logarithmic two-loop effects, parameterized by B_o, have not been evaluated yet, except for a subset of the so-called soft corrections. Those partial results depend on the scheme adopted for regularizing ultraviolet divergences and do not give a reliable estimate of the complete B_o. Further theoretical work is needed to find that potentially important correction.

Reference	Method	Γ(o-Ps) $[\mu s^{-1}]$	B_o
London [17]	Gas	7.0450(60)	174(154)
Mainz [18]	Vacuum	7.0310(70)	−185(179)
Ann Arbor [19]	Gas	7.0514(14)	338(36)
Ann Arbor [20]	Vacuum	7.0482(16)	256(41)
Tokyo [21]	Powder	7.0398(29)	41(74)
Theory without α^2		7.0382	0

Table 1: Some experimental results for the o-Ps lifetime. "Method" in the second column refers to the medium in which o-Ps decays. The last column shows the value of the two-loop coefficient B_o, necessary to bring the theoretical prediction (4) into agreement with the given experimental value. The last line gives the theoretical prediction with $B_o = 0$.

Five-photon decay branching ratio is of order α^2 [13, 16],

$$\mathrm{BR}(\text{o-Ps} \to 5\gamma) = \frac{\Gamma(\text{o-Ps} \to 5\gamma)}{\Gamma(\text{o-Ps} \to \gamma\gamma\gamma)} = 0.19(1) \left(\frac{\alpha}{\pi}\right)^2 \simeq 1.0 \times 10^{-6}\,, \tag{5}$$

and does not significantly influence the total width.

Ref.	Mode	Branching ratio	Boson mass
[29]	$X^L + \gamma$	$< 6.4 \cdot 10^{-5} - 7.6 \cdot 10^{-6}$ (at 90% CL)	$m_X < 800$ keV
[30]	$X + \gamma$	$< \left(1-6\right) \cdot 10^{-4}$ (at 95% CL)	$300 < m_X < 900$ keV
[31]	$X^L + \gamma$	$< 1.1 \cdot 10^{-6}$ (at 90% CL)	$m_X < 800$ keV
[32]	$X^S + \gamma$	$< 3.0 \cdot 10^{-4}$ (at 90% CL)	$m_X < 500$ keV
[33]	$X^S + \gamma$	$< 2.0 \cdot 10^{-4}$ (at 90% CL)	$847 < m_X < 1013$ keV

Table 2: Searches for exotic decays of orthopositronium with one detected photon. X denotes a neutral boson: L – long-lived and S – short-lived.

Experimentally, o-Ps lifetime is somewhat easier to measure than that of p-Ps, because o-Ps lives about 1000 times longer. However, the interactions of o-Ps with the cavity walls and/or the buffer gas are more important. Several experiments have been performed, but their results are in part inconsistent with one another. The present experimental situation is summarized in Table 1. The last column indicates the value of the two-loop coefficient B_o necessary to reconcile a given experimental value with the theoretical prediction (4). We see that the most precise Ann Arbor experiments require an anomalously large value of B_o. This has been known as the "o-Ps lifetime puzzle."

It should be mentioned that the perturbative coefficients are moderate in all known QED predictions for observables studied with high accuracy (e.g. anomalous magnetic moments of the electron and muon, Lamb shift in the hydrogen atom, or the ground state hyperfine interval in muonium). The known corrections to the positronium gross, fine [3, 5] and hyperfine structure [1, 2, 3, 4] and to parapositronium decay rate [9, 10] are also moderate. In any case, before definite conclusions about comparison of theory and experiment can be made, it is necessary to clarify the experimental situation. Efforts to improve the experiments are underway [22, 23].

3 Exotic decays of positronium

The o-Ps lifetime found in the precise experiments in Ann Arbor [19, 20] is significantly shorter than expected in QED. It has been speculated that there might be other decay channels which contribute to the faster decay rate. A number of dedicated experimental searches of rare Ps decays were undertaken.

An example of rare decays predicted by QED are the multi-photon channels. For p-Ps the experimental results are

$$\begin{aligned} \mathrm{BR}(\text{p-Ps} \to 4\gamma) &\simeq 1.48(18) \cdot 10^{-6}, \qquad [24,\ 25,\ 26], \\ \mathrm{BR}(\text{p-Ps} \to 4\gamma) &\simeq 1.50(11) \cdot 10^{-6}, \qquad [27], \end{aligned} \tag{6}$$

in agreement with the theoretical expectation (2). For o-Ps one also finds agreement, with (5):

$$\mathrm{BR}(\text{o-Ps} \to 5\gamma) \simeq 2.2^{+2.6}_{-1.8} \cdot 10^{-6}, \qquad [28,\ 26]. \tag{7}$$

There have also been a number of searches for forbidden photonic decays and for modes with some unregistered particles. Some of such studies for o-Ps are summarized in Tables 2 and 3. In Table 2 we list searches for decay modes involving an exotic spinless particle X, accompanied by a single photon. Table 3 refers to decays into invisible particles.

Standard Model predicts some "invisible" decay channels, namely a conversion of Ps into a neutrino–antineutrino pair. We present a calculation of the corresponding decay rate in the next section.

Ref.	Mode	Branching ratio
[34]	o-Ps $\to X$	$< 5.8 \cdot 10^{-4}$ (at 90% CL)
[35]	p-Ps $\to X$	$< 1.7 \cdot 10^{-6}$ (at 90% CL)
[35]	o-Ps $\to X$	$< 2.8 \cdot 10^{-6}$ (at 90% CL)

Table 3: Searches for invisible decays of Ps. X denotes any invisible products.

4 Weak decays of orthopositronium

Of the two spin states of Ps, only o-Ps (spin 1 state) can decay weakly into a pair $\nu\bar{\nu}$, at least if the neutrinos are massless and if there is no emission of photons.

Two amplitudes contribute to the decay o-Ps $\to \nu_e\bar{\nu}_e$: W exchange in t channel and an annihilation via Z. The rate of this decay is

$$\Gamma(\text{o-Ps} \to \nu_e\bar{\nu}_e) = \frac{1}{3M^2}\frac{1}{8\pi}|\psi(0)|^2|\mathcal{M}|^2\,, \tag{8}$$

where the factor $1/3$ is due to the average over the spin orientations, $M \approx 2m_e$ is the mass of the Ps, $1/8\pi$ comes from the phase space, and the square of the wave function at the origin is

$$|\psi(0)|^2 = \frac{1}{\pi a^3} = \frac{\alpha^3 m_e^3}{8\pi}\,, \tag{9}$$

where $a = \frac{2}{\alpha m_e}$ is the Bohr radius of the Ps. The transition matrix element is given by

$$\begin{aligned}\mathcal{M} = \frac{iG_F}{\sqrt{2}}\,[&\bar{u}_\nu\gamma^\mu(1-\gamma_5)u_e \cdot \bar{v}_e\gamma_\mu(1-\gamma_5)v_\nu \\ &-\bar{u}_\nu\gamma^\mu(1-\gamma_5)v_\nu \cdot \bar{v}_e\gamma_\mu\left(2s^2-\frac{1}{2}+\frac{1}{2}\gamma_5\right)u_e\Big]\,,\end{aligned} \tag{10}$$

$$s \equiv \sin\theta_W\,. \tag{11}$$

The minus sign between the two terms is due to the different ordering of the fermion fields. It leads to a destructive interference of the W and Z exchange amplitudes. The decay rate is

$$\Gamma(\text{o-Ps} \to \nu_e\bar{\nu}_e) = \frac{G_F^2\alpha^3 m_e^5}{24\pi^2}\left(1+4s^2\right)^2\,, \tag{12}$$

which corresponds to a totally negligible branching ratio of about 6.2×10^{-18}. We note that after Fierz transformation of the W exchange diagram, the effective Hamiltonian for the o-Ps decay coincides with the well known Hamiltonian for the neutrino-electron scattering [36]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\bar{u}_\nu\gamma^\mu(1-\gamma_5)v_\nu \cdot \bar{v}_e\gamma_\mu(a-b\gamma_5)u_e\,, \tag{13}$$

$$a = 2s^2+\frac{1}{2}\,, \qquad b = \frac{1}{2}\,. \tag{14}$$

The decay o-Ps $\to \nu_e\bar{\nu}_e$ was discussed in Ref. [37, 38]. Our result differs from both publications. In Ref. [37] the relative minus sign between the W and Z exchange amplitudes was left out, so that the interference was constructive. In Ref. [38] there is a mistake in the trace calculation so that the decay seems to arise only due to the axial, rather than vector, part of the effective Hamiltonian.

For other flavors of the final state neutrinos only the annihilation diagram contributes. Since the vector coupling of Z to electrons is proportional to $1-4s^2$, these channels are strongly suppressed compared to the electron–neutrino final state. We find for $l \neq e$

$$\Gamma(\text{o-Ps} \to \nu_l\bar{\nu}_l) = \frac{G_F^2\alpha^3 m_e^5}{24\pi^2}\left(1-4s^2\right)^2\,. \tag{15}$$

The branching ratio for each non-electron neutrino flavor is about 9.5×10^{-21}, much smaller than the experimental limits (cf. Table 3). Certainly it cannot explain the "orthopositronium lifetime puzzle."

5 Weak decays of other leptonic bound states

In this section we briefly summarize the influence of weak interactions on the lifetimes of the $\mu^+\mu^-$ and μ^+e^- bound states.

Dimuonium (Dm) is a bound state of a muon and an anti-muon. The spectrum [39, 40], QED [40, 41] decay modes and possible production [41] of such a system have been studied recently. In contrast to orthopositronium, the decay rate of orthodimuonium (o-Dm) is of order $\alpha^5 m_\mu$ because of conversion into the the electron–positron pair via a virtual single–photon annihilation,

$$\Gamma^{(0)}(\text{o-Dm} \to e^+e^-) = \frac{\alpha^5 m_\mu}{6}\,. \tag{16}$$

In addition to the dominant QED decay channel, o-Dm can decay into neutrinos or electrons via an annihilation amplitude with a Z-boson exchange. The decay width into muon neutrinos is given by a formula analogous to eq. (12), with muon mass substituted for m_e; the width into other neutrino flavors is obtained similarly from eq. (15).

In case of the decay o-Dm$\to e^+e^-$, the Z contribution is a tiny correction to the photon exchange:

$$\Gamma(\text{o-Dm} \to e^+e^-) \;=\; \frac{\alpha^5 m_\mu}{6}\left(1 + \frac{G_F m_\mu^2 (1-4s^2)^2}{4\sqrt{2}\pi\alpha} + \frac{G_F^2 m_\mu^4 (1-4s^2)^2\left[1+(1-4s^2)^2\right]}{32\pi^2\alpha^2}\right). \tag{17}$$

The two correction terms correspond, respectively, to the interference between the photon and Z amplitudes, and to the square of the Z amplitude.

Muonium (Mu) consists of an electron and an antimuon. Its decays have recently been discussed in some detail in Ref. [42]. The electroweak interactions can lead to its decay via a W exchange in the t channel. For the decay width of a bound state of particles with unequal masses we use

$$\Gamma(\text{o-Mu} \to \nu_e\bar\nu_\mu) = \frac{1}{3\cdot 4m_\mu m_e}\frac{1}{8\pi}|\psi(0)|^2|\mathcal{M}|^2 \tag{18}$$

which leads to

$$\Gamma(\text{o-Mu} \to \nu_e\bar\nu_\mu) = \frac{G_F^2\alpha^3 m_e^3 m_\mu^3}{3\pi^2(m_\mu+m_e)}\,. \tag{19}$$

This formula can also be derived from known results in B_c decays [43].

6 Summary

We have reviewed the confrontation of theoretical and experimental results concerning decays of positronium. We believe that the "orthopositronium lifetime puzzle" is not likely to be solved by large two-loop QED corrections, nor by a discovery of an exotic decay channel. We have investigated in some detail the annihilation amplitude into neutrinos and found that its contribution is negligible. In fact it is even smaller than the results previously cited in the literature. Most likely, the situation will be clarified by the future improved experiments, and we are looking forward to their results.

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