Use of Foldy-Wouthuysen Representation in Relativistic Quantum Theory

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Abstract

The method of the transformation to the Foldy-Wouthuysen representation for relativistic spin-1/2 particles in external fields is suggested allowing for determining the Hamilton operator in this representation to any accuracy. The interaction of particles, having an anomalous magnetic moment, with nonstationary electromagnetic and electroweak fields is investigated.

1 Introduction

The Foldy-Wouthuysen (FW) representation [1] occupies a specific place in quantum theory. Its distinguishing feature is an even (quasidiagonal in two spinors) form of relativistic wave equations and of the operators. This makes the procedure of the extraction of the even operator parts unnecessary and allows for using a single spinor (for spin-1/2 particles). This condition makes the FW representation in the relativistic quantum theory similar to the non-relativistic quantum theory. The main advantages of that in FW representation were described in the first work of Foldy and Wouthuysen [1].

For relativistic particles in external fields, the operators in this representation have the same form as in the non-relativistic quantum theory.

It is very important that the relations between the operators in this representation are similar to those between the respective classical values, and this simplifies considerably the transformation to the quasiclassical description. The FW representation is useful in the description of the particle polarization. If only one spinor is used, then the three-dimensional polarization operator in this representation has a simple form and is reduced to the matrix $\mathbf{\Pi} = \beta\mathbf{\Sigma}$ [2, 3].

In the Dirac representation this operator is described by a clumsy expression and depends on the particle momentum and external field parameters. Note that the FW representation can be used for particles with spin $s > 1/2$ [4].

In the FW representation the equations of motion of a particle and spin are derived very simply. To do this, one should calculate the commutator of the Hamiltonian with the same operators as in the non-relativistic theory. The transformation to the quasiclassical description is also simple.

However, the transformation to the FW representation for relativistic particles in external fields is difficult (see [5, 6]). Therefore, this representation does not occupy an important place in quantum field theory. The Dirac and Melosh [7] representations are commonly used.

The methods of the transformation to the FW representation are well known for the cases when the non-relativistic approximation is used, and the relativistic corrections are taken into account [1, 8, 9]. There are several methods of the transformation to the FW representation for relativistic particles in external fields [5, 6, 10]. However, these methods either lead to clumsy calculations or the domain of their applicability is limited to the weak field approximation.

In several cases the FW transformation may be performed exactly [11, 12, 13, 14].

The exact FW transformation for the wide class of external fields has been obtained in Ref. [15]. In this transformation the involutive symmetries of relativistic wave equations have been used. However, the transformed Hamiltonians contain "non-traditional" space reflection operators. The reduction of the Hamiltonians to the "traditional" form is a separate problem which has not been investigated in [15]. This reduction is necessary for solution of some problems (for example, of particle and spin motion).

The suggested method of the transformation to the FW representation for relativistic particles in external fields differs from these methods. This method is similar to the classical method of Foldy and Wouthuysen for non-relativistic particles [1]. Using the suggested method, the Hamilton operator can be obtained to any accuracy, as a series in terms of the external field potentials and their derivatives, by successive approximations.

The relativistic system of units $\hbar = c = 1$ is used.

2 Exact Transformation to the Foldy-Wouthuysen Representation

Let us consider some cases of the exact transformation to the FW representation.

The transformation to the new representation described by the wave function Ψ' is performed using the unitary operator U:

$$\Psi' = U\Psi = e^{iS}\Psi,$$

where Ψ is the wave function (bispinor) in the Dirac representation. As

$$\Psi = U^{-1}\Psi', \quad i\frac{\partial}{\partial t}\Psi = \mathcal{H}\Psi, \quad i\frac{\partial}{\partial t}\Psi' = \mathcal{H}'\Psi',$$

the following transformation can be performed:[1)]

$$i\frac{\partial}{\partial t}\Psi = \mathcal{H}U^{-1}\Psi', \quad i\frac{\partial}{\partial t}\Psi = i\frac{\partial}{\partial t}\left(U^{-1}\Psi'\right) = i\frac{\partial U^{-1}}{\partial t}\Psi' + iU^{-1}\frac{\partial \Psi'}{\partial t} =$$
$$\left(i\frac{\partial U^{-1}}{\partial t} + U^{-1}\mathcal{H}'\right)\Psi', \quad U\mathcal{H}U^{-1}\Psi' = \left(iU\frac{\partial U^{-1}}{\partial t} + \mathcal{H}'\right)\Psi'.$$

Hence, the Hamilton operator in the new representation has the form [1, 16]:

$$\mathcal{H}' = U\mathcal{H}U^{-1} - iU\frac{\partial U^{-1}}{\partial t}, \tag{1}$$

or

$$\mathcal{H}' = U\left(\mathcal{H} - i\frac{\partial}{\partial t}\right)U^{-1} + i\frac{\partial}{\partial t}. \tag{2}$$

The suggested method of the transformation to the Foldy-Wouthuysen representation consists in the following. Firstly, the transformation, similar to the FW transformation for a free particle, is performed for a particle in an external field. Secondly, the transformation, similar to the FW one for a non-relativistic particle, is performed.

To do this, the Hamiltonian is split into the operators commuting and non-commuting with the operator β:

$$\mathcal{H} = \beta m + \mathcal{E} + \mathcal{O}, \quad \beta\mathcal{E} = \mathcal{E}\beta, \quad \beta\mathcal{O} = -\mathcal{O}\beta. \tag{3}$$

Let us assume that the operator S is defined by the expression:

$$S = -i\frac{\beta\mathcal{O}}{C}\theta, \tag{4}$$

where C, θ are functions of $\mathcal{O}^2$.

The operators β and $\mathcal{O}^2$ commute. Therefore, the operator $\mathcal{O}^2$ is quasidiagonal. Let an operator C satisfy the following conditions:

$$C^2 = \mathcal{O}^2, \qquad [\beta, C^2] = 0. \tag{5}$$

It follows from the conditions (5) that the operator C is quasidiagonal too. It can be defined by the following formal equality:

$$C = \sqrt{\mathcal{O}^2}. \tag{6}$$

In this case relations (5),(6) define the square root of matrix operators. To unambiguously define this operator, let us complete these relations by the condition that the square root of the unit matrix $\mathcal{I}$ is equal to the unit matrix.[2)] For example, for free particle

$$\mathcal{O} = \boldsymbol{\alpha}\cdot\mathbf{p}, \quad \mathcal{O}^2 = \mathcal{I}\mathbf{p}^2, \quad C = \mathcal{I}\sqrt{\mathbf{p}^2}, \quad \mathbf{p} = -i\nabla.$$

Further, the symbol of the unit matrix $\mathcal{I}$ will be omitted.

[1)] There is an inaccuracy in this transformation in Ref. [8].
[2)] The definition of the square root coincides with that in Refs. [1, 8, 15].

Since

$$\mathcal{O}^2 = -\beta\mathcal{O}\beta\mathcal{O}, \quad C = \sqrt{-\beta\mathcal{O}\beta\mathcal{O}}, \quad f(\mathcal{O}^2) = f(C^2),$$

the operators $\beta\mathcal{O}, C, \mathcal{O}^2$ and θ commute with each other. The angle θ is the angle of rotation of the basic vector set in the spinor space.

The unitary transformation operator U can be written in the form:

$$U = \cos\theta + \frac{\beta\mathcal{O}}{C}\sin\theta.$$

The transformation to the FW representation is exact, if an external field is stationary and the operators $\mathcal{E}$ and $\beta\mathcal{O}$ commute:

$$[\mathcal{E}, \beta\mathcal{O}] = \beta[\mathcal{E}, \mathcal{O}] = 0. \tag{7}$$

Condition (7) is the sufficient but not necsssary condition of the exact transformation.

In this case the Hamilton operator in the new representation has the form:

$$\mathcal{H}' = \left(\cos\theta + \frac{\beta\mathcal{O}}{C}\sin\theta\right)\mathcal{H}\left(\cos\theta - \frac{\beta\mathcal{O}}{C}\sin\theta\right) = (\beta m + \mathcal{O})\left(\cos\theta - \frac{\beta\mathcal{O}}{C}\sin\theta\right)^2 + \mathcal{E} =$$
$$\beta\left(m\cos 2\theta + C\sin 2\theta\right) + \mathcal{O}\left(\cos 2\theta - \frac{m}{C}\sin 2\theta\right) + \mathcal{E}.$$

The Hamiltonian $\mathcal{H}'$ is even, if the odd term (proportional to $\mathcal{O}$) vanishes. This occurs when

$$\tan 2\theta = \frac{C}{m}. \tag{8}$$

This condition is satisfied for two angles differing by $\pi/2$. They are defined by the conditions:

$$\tan\theta_1 = \frac{C}{\epsilon + m}, \quad \tan\theta_2 = -\frac{C}{\epsilon - m}, \quad \epsilon = \sqrt{m^2 + C^2} = \sqrt{m^2 + \mathcal{O}^2}. \tag{9}$$

Thus, there are two unitary transformations, characterized by the angles θ_1 and θ_2 and leading to the operator $\mathcal{H}$ in a quasidiagonal form. The rotation by the angle θ_1 leads to the FW representation.

In both representations, one of the spinors (lower for θ_1 and upper for θ_2) becomes zero as for a free particle.

Note that the transformation considered is also similar to the Melosh transformation [7].

If condition (7) is satisfied, then the Hamilton operator in the FW representation is defined exactly:

$$\mathcal{H}' = \beta\epsilon + \mathcal{E}. \tag{10}$$

Unlike the works [5, 6, 10], this equation contains all the exact expressions earlier derived for the Hamilton operator as particular cases.

The transformation operator U can be written in a non-exponential form. In this case the operator is described by the following expression where $U^+ \equiv U,\ U^- \equiv U^{-1}$:

$$U^{\pm} = \frac{\epsilon + m \pm \mathcal{O}}{\sqrt{2\epsilon(\epsilon + m)}}. \tag{11}$$

3 Exact Transformation for Particles in Electroweak Fields

Let us consider the interaction of a relativistic spin-1/2 particle, having an anomalous magnetic moment (AMM), with stationary electromagnetic and electroweak fields.[3)] For electromagnetic interaction, the Hamilton operator in the Dirac representation is defined by the Dirac-Pauli equation [17]:

$$\mathcal{H}_{DP} = \boldsymbol{\alpha\pi} + \beta m + e\Phi + \mu'(-\boldsymbol{\Pi}\mathbf{H} + i\gamma\mathbf{E}), \quad \boldsymbol{\pi} = \mathbf{p} - e\mathbf{A}, \quad \boldsymbol{\Pi} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix}, \tag{12}$$

[3)] Formulae (12),(13) do not change if external fields are non-stationary.

where μ' is AMM, $\Phi, \mathbf{A}$ and $\mathbf{E}, \mathbf{H}$ are the potentials and the strength vectors of an external field, $\mathbf{p} = -i\nabla$ is the momentum operator, $\boldsymbol{\alpha} = \beta\boldsymbol{\gamma}$, $\beta \equiv \gamma^0$; $\boldsymbol{\sigma}$ is the Pauli matrix, 0 stands for the respective 2×2 matrix. This equation was derived in terms of the one particle approximation and is, naturally, insufficient to describe the effects of a strong field. However, it is often used when an external field is not extremely strong (see Ref. [10]).

We should add the Hamiltonian of the weak interaction to the Hamiltonian (12). The weak interaction does not conserve the spatial parity. For the interaction mediated by neutral currents, the standard model gives the following expression for the parity nonconserving Hamiltonian of the weak interaction in the approximation of a small transferred momentum [18]:[4)]

$$\mathcal{H}_{PNC} = -\frac{G}{\sqrt{2}}\left(C_1\gamma^5 + C_2\boldsymbol{\alpha\sigma}'\right) n(\mathbf{r}), \quad \gamma^5 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \tag{13}$$

where G is the Fermi constant; $\boldsymbol{\sigma}'$ is the Pauli matrix for matter particles; $n(\mathbf{r})$ is density of matter particles that in the general case is the superposition of δ-functions. In the interactions with nuclei, $n(\mathbf{r})$ characterizes the density of nucleons of a certain kind, and $\boldsymbol{\sigma}'$ should be replaced by the nucleus spin. The coefficients C_1, C_2 are different for different pairs of interacting particles. The Hamiltonians, corresponding to the interactions with different matter particles, should be summed-up. The signs in formula (13) depend on the definition of the coefficients C_1 C_2 and matrix γ^5. The summed-up Hamiltonian equals:

$$\mathcal{H} = \mathcal{H}_{DP} + \mathcal{H}_{PNC}. \tag{14}$$

In this case in formulae (3),(9)–(11)

$$\begin{gathered}\mathcal{E} = e\Phi - \mu'\boldsymbol{\Pi}\mathbf{H}, \qquad \mathcal{O} = \boldsymbol{\alpha\pi} + i\mu'\boldsymbol{\gamma}\mathbf{E} - \frac{G}{\sqrt{2}}\left(C_1\gamma^5 + C_2\boldsymbol{\alpha\sigma}'\right) n(\mathbf{r}) = \\ \beta\left[\boldsymbol{\gamma\pi} + i\mu'\boldsymbol{\alpha}\mathbf{E} - \frac{G}{\sqrt{2}}\left(C_1\beta\gamma^5 + C_2\boldsymbol{\gamma\sigma}'\right) n(\mathbf{r})\right].\end{gathered} \tag{15}$$

The exact expression for the Hamiltonian in the FW representation is given by formula (10) where $\mathcal{E}$ is defined by formula (15), and

$$\begin{gathered}\epsilon = \left\{m^2 - \left[\boldsymbol{\gamma\pi} + i\mu'\boldsymbol{\alpha}\mathbf{E} - \frac{G}{\sqrt{2}}\left(C_1\beta\gamma^5 + C_2\boldsymbol{\gamma\sigma}'\right) n(\mathbf{r})\right]\right\}^{1/2} = \\ \{m^2 + \boldsymbol{\pi}^2 - e\boldsymbol{\Sigma}\mathbf{H} + \beta\mu'\left(\boldsymbol{\Sigma}[\boldsymbol{\pi}\mathbf{E}] - \boldsymbol{\Sigma}[\mathbf{E}\boldsymbol{\pi}] - \nabla\cdot\mathbf{E}\right) + \mu'^2\mathbf{E}^2 + \\ \frac{G}{\sqrt{2}}\left(C_1\{\boldsymbol{\Sigma\pi}, n(\mathbf{r})\}_+ - C_2\{\boldsymbol{\sigma}'\boldsymbol{\pi}, n(\mathbf{r})\}_+ + C_2[\boldsymbol{\Sigma\sigma}']\cdot\nabla n(\mathbf{r}) - 2\beta\mu' C_2[\boldsymbol{\Sigma\sigma}']\mathbf{E}n(\mathbf{r})\right) + \\ \left.\frac{G^2}{2}n^2(\mathbf{r})\left[C_1^2 + 3C_2^2 - 2C_2(C_1 + C_2)\boldsymbol{\Sigma\sigma}'\right]\right\}^{1/2}.\end{gathered} \tag{16}$$

Hence, relation for the operator $U^{\pm}$ is defined by formulae (11), (15),(16). Note that relation (16) is formally exact, however, inclusion of small terms proportional to C_1^2, C_1C_2, C_2^2 has no sense in the approximation of a small transferred momentum.

Formulae (10),(15),(16) define the exact form of the Hamilton operator in the FW representation for the following particular cases:

a) in the presence of only the weak interaction ($\phi = 0, \mathbf{A} = 0, \mathbf{E} = 0, \mathbf{H} = 0$);

b) for the Dirac particles ($\mu' = 0$) in magnetic and weak fields ($\phi = 0, \mathbf{E} = 0$);

c) for uncharged particles with AMM in electric and weak fields ($e = 0, \mathbf{A} = 0, \mathbf{H} = 0$);

d) for the particles with AMM, moving in the plane, ortogonal to a static homogeneous magnetic field ($\boldsymbol{\Phi} = 0, \mathbf{E} = 0, P_z = 0, C_1 = C_2 = 0$);

e) for the uncharged particles with AMM, moving in the plane, ortogonal to a static homogeneous magnetic field. A static magnetic field (possibly non-homogeneous) is also ortogonal to an electric field ($e = 0, \mathbf{E} \perp \mathbf{H}, P_z = 0, C_1 = C_2 = 0$).

[4)]The matter particles are considered to be at rest.

If in case (d) $\mathbf{H}=H\mathbf{e}_z$, and in case (e) $\mathbf{H}=H\mathbf{e}_z$, and the electric field strength does not depend on z,[5)] the operator $p_z=-i(\partial/\partial z)$ commutes with the Hamilton operator. Hence, it has an eigenvalue $P_z=$const, and consideration of the particular case $P_z=0$ is quite reasonable. Condition (7) is satisfied in all these cases.

Formulae (10),(15),(16) agree with all exact expressions for the operator $\mathcal{H}'$, obtained in [11] for Dirac particles in a static magnetic field, and in [12] for uncharged particles with AMM in electrostatic field, and in [13, 14] for particles with AMM, moving in the plane, ortogonal to a static homogeneous magnetic field. The weak interaction has not been considered in [11, 12, 13, 14].

4 General Case

We assume that the external field is not extremely strong, and the transformed Hamiltonian could be expressed as a series of terms in powers of field potentials and their derivatives.

As in the general case $\mathcal{E}$ depends on coordinates and contains the Dirac matrices, formula (10) is not exact. We should determine the commutator of the operators U and $\mathcal{E}$:

$$U\mathcal{E}U^{-1}=\mathcal{E}+[U,\mathcal{E}]U^{-1}.$$

In this case we should calculate the commutators containing inverse operators and the square root of operators. This commutators can be calculated using the following exact formulae valid for arbitrary operators A and B [19]:

$$[A^{-1},B]=A^{-1}[B,A]A^{-1}, \tag{17}$$

$$[A,B]=\frac{1}{4}\{A^{-1},[A^2,B]\}_+-\frac{1}{4}\left[[A,[A,B]],A^{-1}\right], \tag{18}$$

where $A^{-1}\equiv 1/A$, $\{\ldots,\ldots\}_+$ stands for the anticommutator. When A is the square root of operators and the commutator of operators is small by the order of magnitude in comparison with their product, formulae (17),(18) allow for obtaining the value $[A,B]$ to any accuracy by the method of successive approximations (see [19]). This condition is satisfied, as a rule, because in present case it is equivalent to the condition of smallness either of the Compton wave length of a particle, or of the value $\hbar c/E$ for ultra-relativistic particles in comparison with the characteristic non-uniformity scale of an external field.

After the unitary transformation with the operator (11), the Hamiltonian $\mathcal{H}'$ still contains odd terms proportional to the derivatives of the potentials. However, these terms will be small in comparison with ϵ. At the second stage, this condition allows for applying the usual scheme of the non-relativistic FW transformation [1, 8]. Let us write the operator $\mathcal{H}'$ as:

$$\mathcal{H}'=\beta m+\mathcal{E}'+\mathcal{C}', \quad \beta\mathcal{E}'=\mathcal{E}'\beta, \quad \beta\mathcal{O}'=-\mathcal{O}'\beta. \tag{19}$$

In this case the following transformation operator can be used:

$$S'=-\frac{i}{4}\beta\left\{\mathcal{C}',\frac{1}{\epsilon}\right\}_+=-\frac{i}{4}\left[\frac{\beta}{\epsilon},\mathcal{O}'\right]. \tag{20}$$

Further calculations are similar to those given in [8]. The single difference is the necessity to allow for the non-commutation of the operator ϵ with $\mathcal{E}',\mathcal{C}'$. If only the largest corrections are included, then the Hamiltonian in the FW representation, as a result of the unitary transformation, gets the form:

$$\mathcal{H}''=\beta\epsilon+\mathcal{E}'+\frac{1}{4}\beta\left\{\mathcal{O}'^2,\frac{1}{\epsilon}\right\}_+.$$

To obtain the desired exactness, the calculation procedure with the transformation operator (20) may be repeated multiply.

Let us calculate the Hamiltonian in the FW representation for a relativistic particle with AMM, interacting with a non-stationary electroweak field. The Hamiltonian in the Dirac representation is

[5)] In the opposite case rot$\mathbf{E}\neq 0$, and the magnetic field is not constant ($\partial\mathbf{H}/\partial t\neq 0$).

defined by formulae (12)–(14). Using the method described above method of the transformation to the FW representation, we obtain the following relation:

$$\begin{aligned}\mathcal{H}' = \beta\epsilon + e\Phi + \frac{e}{8}\left\{\frac{1}{\epsilon(\epsilon+m)}, (\boldsymbol{\Sigma}[\boldsymbol{\pi}\mathbf{E}] - \boldsymbol{\Sigma}[\mathbf{E}\boldsymbol{\pi}] - \nabla\cdot\mathbf{E})\right\}_+ + \\ \frac{e}{32}\left\{\frac{2\epsilon^2+2\epsilon m+m^2}{\epsilon^4(\epsilon+m)^2}, \boldsymbol{\pi}\cdot\nabla(\boldsymbol{\pi}\mathbf{E}+\mathbf{E}\boldsymbol{\pi})\right\}_+ - \mu'\boldsymbol{\Pi}\mathbf{H} + \\ \beta\frac{\mu'}{4}\left\{\frac{1}{\epsilon(\epsilon+m)}, \Big[(\mathbf{H}\boldsymbol{\pi})(\boldsymbol{\Sigma}\boldsymbol{\pi}) + (\boldsymbol{\Sigma}\boldsymbol{\pi})(\boldsymbol{\pi}\mathbf{H}) + 2\pi(\boldsymbol{\pi}\mathbf{j}+\mathbf{j}\boldsymbol{\pi})\Big]\right\}_+,\end{aligned} \tag{21}$$

where $\mathbf{j}$ is the external current density, and ϵ is described by formula (16). Relation (21) has been obtained neglecting the derivatives the field potentials of the third order and higher (of the second order and higher of the field strengths) and of the terms squared with respect to field strengths. Note that this relation has been already found as result of the transformation with the operator (11).

Unlike works [5, 6, 10], formula (21) includes, as particular cases, the exact expressions for the Hamiltonian in the FW representation, obtained in [11, 12, 13, 14]. This formula agrees also with the results obtained in Refs. [5, 6, 10, 20, 21]. The detailed analysis shows that the method of the transformation to the FW representation, used in Ref. [10], does not allow for terms proportional to the double commutators of the operators ϵ with $e\Phi$ and $\mu'\boldsymbol{\Pi}\mathbf{H}$ $\big(e[\epsilon,[\epsilon,\Phi]]$ and $\mu'[\epsilon,[\epsilon,\boldsymbol{\Pi}\mathbf{H}]]\big)$. The terms proportional to $\mathbf{E}$ and $\mathbf{H}$ in the Hamiltonian obtained in Ref. [10] coincide with the respective tarms both in relation (21) and in Refs. [5, 6]. Note that only Dirac particles have been considered in [5], the derivatives of strengths have not been included in [6], and relativistic corrections to the non-relativistic Hamiltonian have been calculated in [20].

5 Summary

This work suggests the method allowing for the transformation to the FW representation to any precision and applicable in the cases of the arbitrary energy of particles. This method is simple enough, and bases on the earlier investigations [1, 8]. As an example, the Hamilton operator in the FW representation for relativistic particles with AMM in a non-stationary electroweak field has been found to the precision including the second derivatives of potentials.

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