

The first Barnes lemma

$$\begin{aligned} & \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \Gamma(\lambda_1 + z) \Gamma(\lambda_2 + z) \Gamma(\lambda_3 - z) \Gamma(\lambda_4 - z) \\ &= \frac{\Gamma(\lambda_1 + \lambda_3) \Gamma(\lambda_1 + \lambda_4) \Gamma(\lambda_2 + \lambda_3) \Gamma(\lambda_2 + \lambda_4)}{\Gamma(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)} \end{aligned}$$

Multiple corollaries, e.g.,

$$\begin{aligned} & \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \Gamma(\lambda_1 + z) \Gamma^*(\lambda_2 + z) \Gamma(-\lambda_2 - z) \Gamma(\lambda_3 - z) \\ &= \Gamma(\lambda_1 - \lambda_2) \Gamma(\lambda_2 + \lambda_3) [\psi(\lambda_1 - \lambda_2) - \psi(\lambda_1 + \lambda_3)] \end{aligned}$$

$$\begin{aligned}
& \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} \frac{dz}{z} \Gamma(\lambda_1 + z) \Gamma(\lambda_2 + z) \Gamma(\lambda_3 - z) \Gamma(\lambda_4 - z) \\
& \quad = \frac{\Gamma(2 - \lambda_1 - \lambda_3) \Gamma(1 - \lambda_2 - \lambda_3) \Gamma(\lambda_1 + \lambda_3 - 1) \Gamma(\lambda_2 + \lambda_3)}{\Gamma(1 - \lambda_1) \Gamma(1 - \lambda_2)} \\
& \quad \times [\Gamma(1 - \lambda_1) \Gamma(1 - \lambda_2) - \Gamma(2 - \lambda_1 - \lambda_2 - \lambda_3) \Gamma(\lambda_3)]
\end{aligned}$$

The second Barnes lemma

$$\begin{aligned}
& \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \frac{\Gamma(\lambda_1 + z) \Gamma(\lambda_2 + z) \Gamma(\lambda_3 + z) \Gamma(\lambda_4 - z) \Gamma(\lambda_5 - z)}{\Gamma(\lambda_6 + z)} \\
& \quad = \frac{\Gamma(\lambda_1 + \lambda_4) \Gamma(\lambda_2 + \lambda_4) \Gamma(\lambda_3 + \lambda_4) \Gamma(\lambda_1 + \lambda_5)}{\Gamma(\lambda_1 + \lambda_2 + \lambda_4 + \lambda_5) \Gamma(\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5)} \\
& \quad \times \frac{\Gamma(\lambda_2 + \lambda_5) \Gamma(\lambda_3 + \lambda_5)}{\Gamma(\lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)}, \quad \lambda_6 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5
\end{aligned}$$