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ln[1]:= SetDirectory["c:/diskE/job2008/Zurich"];

(* http://www-ttp.particle.uni-karlsruhe.de/~asmirnov *)

ln[2]:= << MB/MB.m;
<< MB/MBresolve.m

MB 1.1

by Michal Czakon

more info in hep-ph/0511200

last modified 06 Mar 08

MBresolve 1.0

by Alexander Smirnov

last modified 22 Oct 08

(* The integrand of the MB integral for the one-
loop propagator diagram with m1=m and m2=0 *)

ln[4]:= MB[a1_, a2_] := (-1)^(a1 + a2) / QQ^(a1 + a2 + ep - 2) / Gamma[a1] / Gamma[a2]
Gamma[2 - ep - a2] Gamma[a1 + a2 + ep - 2 + z] Gamma[2 - ep - a1 - z]
Gamma[-z] / Gamma[4 - 2 ep - a1 - a2 - z] mm^z / QQ^z;

(* Notation: mm=m^2, QQ=-q^2;
I Pi^(d/2) is pulled out *)

(* The diagram with a1=1 and a2=1 *)

ln[5]:= P1 = MB[1, 1]

Out[5]= 
$$\frac{mm^z QQ^{-ep-z} \Gamma[1-ep] \Gamma[1-ep-z] \Gamma[-z] \Gamma[ep+z]}{\Gamma[2-2ep-z]}$$


ln[6]:= MBresolve[P1, ep]

CREATING RESIDUES LIST.....0.2812 seconds
EVALUATING RESIDUES.....0.1875 seconds

Out[6]= 
$$\left\{ \text{MBint} \left[ \frac{QQ^{-ep} \Gamma[1-ep]^2 \Gamma[ep]}{\Gamma[2-2ep]}, \{ \{ep \rightarrow 0\}, \{\} \} \right], \text{MBint} \left[ \frac{mm^z QQ^{-ep-z} \Gamma[1-ep] \Gamma[1-ep-z] \Gamma[-z] \Gamma[ep+z]}{\Gamma[2-2ep-z]}, \{ \{ep \rightarrow 0\}, \{z \rightarrow 0.511912\} \} \right] \right\}$$


ln[7]:= Box1[a1_, a2_, a3_, a4_] :=
(S^{2-a1-a2-a3-a4-ep-z} T^z Gamma[a1 + a2 + a3 + a4 - 2 + ep + z] Gamma[a2 + z] Gamma[a4 + z]
Gamma[2 - a1 - a2 - a4 - ep - z] Gamma[2 - a2 - a3 - a4 - ep - z] Gamma[-z]) /
(Gamma[a1] Gamma[a2] Gamma[a3] Gamma[a4] Gamma[4 - a1 - a2 - a3 - a4 - 2 ep]);

ln[8]:= Box1[1, 1, 1, 1]

Out[8]= 
$$\frac{S^{-2-ep-z} T^z \Gamma[-1-ep-z]^2 \Gamma[-z] \Gamma[1+z]^2 \Gamma[2+ep+z]}{\Gamma[-2ep]}$$


ln[9]:= B1 = % /. {S -> 1, T -> x}

Out[9]= 
$$\frac{x^z \Gamma[-1-ep-z]^2 \Gamma[-z] \Gamma[1+z]^2 \Gamma[2+ep+z]}{\Gamma[-2ep]}$$


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In[10]:= **MBresolve**[B1, ep]

CREATING RESIDUES LIST.....0.4688 seconds  
EVALUATING RESIDUES.....0.3125 seconds

Out[10]= 
$$\left\{ \text{MBint} \left[ -\frac{\text{EulerGamma} \Gamma[-\text{ep}]^2 \Gamma[1+\text{ep}]}{x \Gamma[-2\text{ep}]} + \frac{\Gamma[-\text{ep}]^2 \Gamma[1+\text{ep}] \text{Log}[x]}{x \Gamma[-2\text{ep}]} - \frac{2 \Gamma[-\text{ep}]^2 \Gamma[1+\text{ep}] \text{PolyGamma}[0, -\text{ep}]}{x \Gamma[-2\text{ep}]} + \frac{\Gamma[-\text{ep}]^2 \Gamma[1+\text{ep}] \text{PolyGamma}[0, 1+\text{ep}]}{x \Gamma[-2\text{ep}]} \right], \{\{\text{ep} \rightarrow 0\}, \{\}\} \right\},$$

$$\left\{ \text{MBint} \left[ \frac{x^2 \Gamma[-1-\text{ep}-z]^2 \Gamma[-z] \Gamma[1+z]^2 \Gamma[2+\text{ep}+z]}{\Gamma[-2\text{ep}]} \right], \{\{\text{ep} \rightarrow 0\}, \{z \rightarrow -1.81305\}\} \right\} \right\}$$

In[11]:= **NPMB**[a1\_, a2\_, a3\_, a4\_, a5\_, a6\_] := ((-1)^(a1+a2+a3+a4+a5+a6) /  
(Gamma[a1] Gamma[a2] Gamma[a3] Gamma[a4] Gamma[a5] Gamma[a6])  
Gamma[2-ep-a3-a5] Gamma[2-ep-a4-a6] / Gamma[4-2ep-a3-a4-a5-a6] /  
Gamma[6-3ep-a1-a2-a3-a4-a5-a6]  
Gamma[a1+a2+a3+a4+a5+a6+2ep-4+z1+z2] Gamma[-z1] Gamma[-z2]  
Gamma[2-ep-a1-a2-z1] Gamma[a4+z2] Gamma[a1+z1+z2]  
Gamma[4-2ep-a1-a3-a4-a5-a6-z2] Gamma[4-2ep-a1-a2-a4-a5-a6-z1-z2]  
Gamma[a5+z2] Gamma[4-2ep-a1-a2-a3-a4-a5-z1-z2] /  
Gamma[4-2ep-a1-a2-a4-a6-z1] / Gamma[4-2ep-a1-a2-a3-a5-z1]);

In[12]:= **V2** = **NPMB**[1, 1, 1, 1, 1, 1]

Out[12]= 
$$\left( \frac{\Gamma[-\text{ep}]^2 \Gamma[-\text{ep}-z1] \Gamma[-z1] \Gamma[-1-2\text{ep}-z2] \Gamma[-1-2\text{ep}-z1-z2]^2}{\Gamma[-z2] \Gamma[1+z2]^2 \Gamma[1+z1+z2] \Gamma[2+2\text{ep}+z1+z2]} \right) /$$

$$\left( \Gamma[-3\text{ep}] \Gamma[-2\text{ep}] \Gamma[-2\text{ep}-z1]^2 \right)$$

In[13]:= **MBresolve[V2, ep]**

CREATING RESIDUES LIST.....0.625 seconds

EVALUATING RESIDUES.....0.25 seconds

Out[13]=  $\left\{ \text{MBint} \left[ \frac{\Gamma[-2 \text{ep}]^4 \Gamma[-\text{ep}]^2 \Gamma[1+2 \text{ep}]^2}{\Gamma[-4 \text{ep}]^2}, \{\{\text{ep} \rightarrow 0\}, \{\}\}\right], \right.$   
 $\text{MBint} \left[ \frac{(\Gamma[-2 \text{ep}] \Gamma[-\text{ep}]^2 \Gamma[1+2 \text{ep}] \Gamma[-\text{ep}-z1] \Gamma[-z1]^3 \Gamma[1+z1] \Gamma[-2 \text{ep}+z1])}{(\Gamma[-3 \text{ep}] \Gamma[-2 \text{ep}-z1]^2)}, \{\{\text{ep} \rightarrow 0\}, \{z1 \rightarrow -0.859981\}\}\right],$   
 $\text{MBint} \left[ -\frac{1}{\Gamma[-3 \text{ep}]} \text{EulerGamma} \Gamma[-\text{ep}]^2 \Gamma[-1-2 \text{ep}-z2] \Gamma[-z2] \right.$   
 $\Gamma[1+\text{ep}+z2] \Gamma[1+2 \text{ep}+z2] - \frac{1}{\Gamma[-3 \text{ep}]} \Gamma[-\text{ep}]^2 \Gamma[-1-2 \text{ep}-z2]$   
 $\Gamma[-z2] \Gamma[1+\text{ep}+z2] \Gamma[1+2 \text{ep}+z2] \text{PolyGamma}[0, -2 \text{ep}] - \frac{1}{\Gamma[-3 \text{ep}]}$   
 $2 \Gamma[-\text{ep}]^2 \Gamma[-1-2 \text{ep}-z2] \Gamma[-z2] \Gamma[1+\text{ep}+z2] \Gamma[1+2 \text{ep}+z2]$   
 $\text{PolyGamma}[0, 1+z2] + \frac{1}{\Gamma[-3 \text{ep}]} \Gamma[-\text{ep}]^2 \Gamma[-1-2 \text{ep}-z2]$   
 $\Gamma[-z2] \Gamma[1+\text{ep}+z2] \Gamma[1+2 \text{ep}+z2] \text{PolyGamma}[0, 1+\text{ep}+z2] +$   
 $\frac{1}{\Gamma[-3 \text{ep}]} \Gamma[-\text{ep}]^2 \Gamma[-1-2 \text{ep}-z2] \Gamma[-z2] \Gamma[1+\text{ep}+z2]$   
 $\left. \Gamma[1+2 \text{ep}+z2] \text{PolyGamma}[0, 1+2 \text{ep}+z2], \{\{\text{ep} \rightarrow 0\}, \{z2 \rightarrow -0.859981\}\}\right],$   
 $\text{MBint} \left[ \frac{(\Gamma[-\text{ep}]^2 \Gamma[-\text{ep}-z1] \Gamma[-z1] \Gamma[-1-2 \text{ep}-z2] \Gamma[-1-2 \text{ep}-z1-z2]^2)}{\Gamma[-z2] \Gamma[1+z2]^2 \Gamma[1+z1+z2] \Gamma[2+2 \text{ep}+z1+z2]} \right. /$   
 $\left. \frac{(\Gamma[-3 \text{ep}] \Gamma[-2 \text{ep}] \Gamma[-2 \text{ep}-z1]^2)}{(\Gamma[-3 \text{ep}] \Gamma[-2 \text{ep}] \Gamma[-2 \text{ep}-z1]^2)}, \{\{\text{ep} \rightarrow 0\}, \{z1 \rightarrow -0.72274, z2 \rightarrow -0.274294\}\}\right\}$

In[14]:= **Simplify[%]**

Out[14]=  $\left\{ \text{MBint} \left[ \frac{\Gamma[-2 \text{ep}]^4 \Gamma[-\text{ep}]^2 \Gamma[1+2 \text{ep}]^2}{\Gamma[-4 \text{ep}]^2}, \{\{\text{ep} \rightarrow 0\}, \{\}\}\right], \right.$   
 $\text{MBint} \left[ \frac{(\Gamma[-2 \text{ep}] \Gamma[-\text{ep}]^2 \Gamma[1+2 \text{ep}] \Gamma[-\text{ep}-z1] \Gamma[-z1]^3 \Gamma[1+z1] \Gamma[-2 \text{ep}+z1])}{(\Gamma[-3 \text{ep}] \Gamma[-2 \text{ep}-z1]^2)}, \{\{\text{ep} \rightarrow 0\}, \{z1 \rightarrow -0.859981\}\}\right],$   
 $\text{MBint} \left[ \frac{1}{\Gamma[-3 \text{ep}]} \Gamma[-\text{ep}]^2 \Gamma[-1-2 \text{ep}-z2] \Gamma[-z2] \Gamma[1+\text{ep}+z2] \right.$   
 $\Gamma[1+2 \text{ep}+z2] (-\text{EulerGamma} - \text{PolyGamma}[0, -2 \text{ep}] - 2 \text{PolyGamma}[0, 1+z2] +$   
 $\text{PolyGamma}[0, 1+\text{ep}+z2] + \text{PolyGamma}[0, 1+2 \text{ep}+z2]), \{\{\text{ep} \rightarrow 0\}, \{z2 \rightarrow -0.859981\}\}\right],$   
 $\text{MBint} \left[ \frac{(\Gamma[-\text{ep}]^2 \Gamma[-\text{ep}-z1] \Gamma[-z1] \Gamma[-1-2 \text{ep}-z2] \Gamma[-1-2 \text{ep}-z1-z2]^2)}{\Gamma[-z2] \Gamma[1+z2]^2 \Gamma[1+z1+z2] \Gamma[2+2 \text{ep}+z1+z2]} \right. /$   
 $\left. \frac{(\Gamma[-3 \text{ep}] \Gamma[-2 \text{ep}] \Gamma[-2 \text{ep}-z1]^2)}{(\Gamma[-3 \text{ep}] \Gamma[-2 \text{ep}] \Gamma[-2 \text{ep}-z1]^2)}, \{\{\text{ep} \rightarrow 0\}, \{z1 \rightarrow -0.72274, z2 \rightarrow -0.274294\}\}\right\}$

(**\***      **Example 4a**      **\***)

In[15]:=  $F = \text{Gamma}\left[\frac{3}{2} + \text{ep} + z\right] \text{Gamma}\left[-1 - 2 \text{ep} - z\right]$   
 $\text{Gamma}\left[4 \text{ep} + z\right] \text{Gamma}\left[-z\right] \text{Gamma}\left[\frac{1}{2} - \text{ep} - z\right] / \text{Gamma}\left[1 - 2 \text{ep} - z\right]$

Out[15]=  $\frac{1}{\text{Gamma}\left[1 - 2 \text{ep} - z\right]}$

$\text{Gamma}\left[-1 - 2 \text{ep} - z\right] \text{Gamma}\left[\frac{1}{2} - \text{ep} - z\right] \text{Gamma}\left[-z\right] \text{Gamma}\left[\frac{3}{2} + \text{ep} + z\right] \text{Gamma}\left[4 \text{ep} + z\right]$

In[16]:= **MBresolve[F, ep]**

CREATING RESIDUES LIST.....0.3281 seconds  
 EVALUATING RESIDUES.....0.3125 seconds

Out[16]=  $\left\{ \text{MBint}\left[\frac{\text{Gamma}\left[\frac{3}{2} - 3 \text{ep}\right] \text{Gamma}\left[4 \text{ep}\right] \text{Gamma}\left[-1 + 2 \text{ep}\right] \text{Gamma}\left[\frac{1}{2} + 3 \text{ep}\right]}{\text{Gamma}\left[1 + 2 \text{ep}\right]}, \{\{\text{ep} \rightarrow 0\}, \{\}\}\right], \right.$   
 $\text{MBint}\left[-\frac{\text{Gamma}\left[\frac{1}{2} - 3 \text{ep}\right] \text{Gamma}\left[2 \text{ep}\right] \text{Gamma}\left[\frac{3}{2} + 3 \text{ep}\right] \text{Gamma}\left[1 + 4 \text{ep}\right]}{\text{Gamma}\left[2 + 2 \text{ep}\right]}, \{\{\text{ep} \rightarrow 0\}, \{\}\}\right],$   
 $\left. \text{MBint}\left[\left(\text{Gamma}\left[-1 - 2 \text{ep} - z\right] \text{Gamma}\left[\frac{1}{2} - \text{ep} - z\right] \text{Gamma}\left[-z\right] \text{Gamma}\left[\frac{3}{2} + \text{ep} + z\right] \text{Gamma}\left[4 \text{ep} + z\right]\right) / \right.$   
 $\left. \text{Gamma}\left[1 - 2 \text{ep} - z\right], \{\{\text{ep} \rightarrow 0\}, \{z \rightarrow -1.48302\}\}\right] \right\}$

In[17]:= **MBmerge[%]**

Out[17]=  $\left\{ \text{MBint}\left[\frac{\text{Gamma}\left[\frac{3}{2} - 3 \text{ep}\right] \text{Gamma}\left[4 \text{ep}\right] \text{Gamma}\left[-1 + 2 \text{ep}\right] \text{Gamma}\left[\frac{1}{2} + 3 \text{ep}\right]}{\text{Gamma}\left[1 + 2 \text{ep}\right]} - \right.$   
 $\left. \frac{\text{Gamma}\left[\frac{1}{2} - 3 \text{ep}\right] \text{Gamma}\left[2 \text{ep}\right] \text{Gamma}\left[\frac{3}{2} + 3 \text{ep}\right] \text{Gamma}\left[1 + 4 \text{ep}\right]}{\text{Gamma}\left[2 + 2 \text{ep}\right]}, \{\{\text{ep} \rightarrow 0\}, \{\}\}\right],$   
 $\left. \text{MBint}\left[\left(\text{Gamma}\left[-1 - 2 \text{ep} - z\right] \text{Gamma}\left[\frac{1}{2} - \text{ep} - z\right] \text{Gamma}\left[-z\right] \text{Gamma}\left[\frac{3}{2} + \text{ep} + z\right] \text{Gamma}\left[4 \text{ep} + z\right]\right) / \right.$   
 $\left. \text{Gamma}\left[1 - 2 \text{ep} - z\right], \{\{\text{ep} \rightarrow 0\}, \{z \rightarrow -1.48302\}\}\right] \right\}$

(\* **Example 4b** \*)

In[18]:=  $F = \text{Gamma}\left[-1/2 + \text{ep} + z\right] \text{Gamma}\left[1 + \text{ep} + z\right] \text{Gamma}\left[3/2 - \text{ep} - z\right] \text{Gamma}\left[-z\right]$

Out[18]=  $\text{Gamma}\left[\frac{3}{2} - \text{ep} - z\right] \text{Gamma}\left[-z\right] \text{Gamma}\left[-\frac{1}{2} + \text{ep} + z\right] \text{Gamma}\left[1 + \text{ep} + z\right]$

In[19]:= **MBresolve[F, ep]**

CREATING RESIDUES LIST.....0.2188 seconds  
 EVALUATING RESIDUES.....0.1562 seconds

Out[19]=  $\left\{ \text{MBint}\left[\frac{1}{2} \sqrt{\pi} \text{Gamma}\left[-\frac{1}{2} + \text{ep}\right], \{\{\text{ep} \rightarrow 0\}, \{\}\}\right], \text{MBint}\left[\right.$   
 $\left. \text{Gamma}\left[\frac{3}{2} - \text{ep} - z\right] \text{Gamma}\left[-z\right] \text{Gamma}\left[-\frac{1}{2} + \text{ep} + z\right] \text{Gamma}\left[1 + \text{ep} + z\right], \{\{\text{ep} \rightarrow 0\}, \{z \rightarrow -0.294497\}\}\right] \right\}$