In[2]:=
```
<< MB/MB.m
```

```
MB 1.1

by Michal Czakon

more info in hep-ph/0511200

last modified 06 Mar 08
```

In[3]:=
```
(*   The integrand of the MB integral for the one-
  loop propagator diagram with m1=m and m2=0 *)

MB[a1_, a2_] := (-1)^(a1 + a2) / QQ^(a1 + a2 + ep - 2) / Gamma[a1] / Gamma[a2]
   Gamma[2 - ep - a2] Gamma[a1 + a2 + ep - 2 + z] Gamma[2 - ep - a1 - z]
   Gamma[-z] / Gamma[4 - 2 ep - a1 - a2 - z] mm^z / QQ^z;

(*  Notation: mm=m^2, QQ=-q^2;
I Pi^(d/2) is pulled out  *)

(*  The diagram with a1=1 and a2=1  *)
```

In[4]:=
```
P1 = MB[1, 1]
```

Out[4]=
$$\frac{\text{mm}^{z}\,\text{QQ}^{-\text{ep}-z}\,\text{Gamma}[1-\text{ep}]\,\text{Gamma}[1-\text{ep}-z]\,\text{Gamma}[-z]\,\text{Gamma}[\text{ep}+z]}{\text{Gamma}[2-2\,\text{ep}-z]}$$

In[5]:=
```
P1Rules = MBoptimizedRules[P1, ep → 0, {}, {ep}]
```

MBrules::norules : no rules could be found to regulate this integral

Out[5]=
$$\left\{\left\{\text{ep}\to\frac{3}{4}\right\},\ \left\{z\to-\frac{1}{2}\right\}\right\}$$

In[6]:=
```
P1cont = MBcontinue[P1, ep → 0, P1Rules]
```

```
Level 1

Taking +residue in z = -ep

Level 2

Integral {1}

2 integral(s) found
```

Out[6]=
$$\left\{\left\{\text{MBint}\left[\frac{\text{mm}^{-\text{ep}}\,\text{Gamma}[1-\text{ep}]\,\text{Gamma}[\text{ep}]}{\text{Gamma}[2-\text{ep}]},\ \{\{\text{ep}\to 0\},\ \{\}\}\right]\right\},\right.$$
$$\left.\text{MBint}\left[\frac{\text{mm}^{z}\,\text{QQ}^{-\text{ep}-z}\,\text{Gamma}[1-\text{ep}]\,\text{Gamma}[1-\text{ep}-z]\,\text{Gamma}[-z]\,\text{Gamma}[\text{ep}+z]}{\text{Gamma}[2-2\,\text{ep}-z]},\ \left\{\{\text{ep}\to 0\},\ \left\{z\to-\frac{1}{2}\right\}\right\}\right]\right\}$$

In[7]:=
```
P1select = MBpreselect[P1cont, {ep, 0, 0}]
```

Out[7]=
$$\left\{\text{MBint}\left[\frac{\text{mm}^{-\text{ep}}\,\text{Gamma}[1-\text{ep}]\,\text{Gamma}[\text{ep}]}{\text{Gamma}[2-\text{ep}]},\ \{\{\text{ep}\to 0\},\ \{\}\}\right],\right.$$
$$\left.\text{MBint}\left[\frac{\text{mm}^{z}\,\text{QQ}^{-\text{ep}-z}\,\text{Gamma}[1-\text{ep}]\,\text{Gamma}[1-\text{ep}-z]\,\text{Gamma}[-z]\,\text{Gamma}[\text{ep}+z]}{\text{Gamma}[2-2\,\text{ep}-z]},\ \left\{\{\text{ep}\to 0\},\ \left\{z\to-\frac{1}{2}\right\}\right\}\right]\right\}$$

In[8]:=
```
P1exp = MBexpand[P1select, E^(EulerGamma ep), {ep, 0, 0}]
```

Out[8]=
$$\left\{\text{MBint}\left[1+\frac{1}{\text{ep}}-\text{Log}[\text{mm}],\ \{\{\text{ep}\to 0\},\ \{\}\}\right],\right.$$
$$\left.\text{MBint}\left[\frac{\text{mm}^{z}\,\text{QQ}^{-z}\,\text{Gamma}[1-z]\,\text{Gamma}[-z]\,\text{Gamma}[z]}{\text{Gamma}[2-z]},\ \left\{\{\text{ep}\to 0\},\ \left\{z\to-\frac{1}{2}\right\}\right\}\right]\right\}$$

In[9]:=
```
P1exp[[1]]
```

Out[9]=
$$\text{MBint}\left[1+\frac{1}{\text{ep}}-\text{Log}[\text{mm}],\ \{\{\text{ep}\to 0\},\ \{\}\}\right]$$

In[10]:=
```
P1exp[[2]]
```

Out[10]=
$$\text{MBint}\left[\frac{\text{mm}^{z}\,\text{QQ}^{-z}\,\text{Gamma}[1-z]\,\text{Gamma}[-z]\,\text{Gamma}[z]}{\text{Gamma}[2-z]},\ \left\{\{\text{ep}\to 0\},\ \left\{z\to -\frac{1}{2}\right\}\right\}\right]$$

In[11]:=
```
int1 = P1exp[[2]] [[1]] //.
  {Gamma[2 - z] → (1 - z) Gamma[1 - z], Gamma[1 - z] → -z Gamma[-z], Gamma[z] → Gamma[1 + z] / z}
```

Out[11]=
$$\frac{\text{mm}^{z}\,\text{QQ}^{-z}\,\text{Gamma}[-z]\,\text{Gamma}[1+z]}{(1-z)\,z}$$

```
(*  The MB integral can be evaluated by closing the
  integration contour to the right in the complex z-plane.      *)

(*  First two residues  *)
```

In[12]:=
```
res12 = -Residue[int1, {z, 0}] - Residue[int1, {z, 1}]
```

Out[12]=
$$1+\text{Log}[\text{mm}]-\text{Log}[\text{QQ}]-\frac{\text{mm}-\text{mm}\,\text{Log}[\text{mm}]+\text{mm}\,\text{Log}[\text{QQ}]}{\text{QQ}}$$

```
(*  Now we take residues at z=2,3,...  *)
```

In[13]:=
```
int1 /. {Gamma[-z] Gamma[1 + z] → -π Csc[π z]}
```

Out[13]=
$$-\frac{\text{mm}^{z}\,\pi\,\text{QQ}^{-z}\,\text{Csc}[\pi z]}{(1-z)\,z}$$

```
(*  Now we take (minus) the residue at z=n,
To do this we shift the variable and then take
  a residue at z=0 *)
```

In[14]:=
```
% /. z → z + n
```

Out[14]=
$$-\frac{\text{mm}^{n+z}\,\pi\,\text{QQ}^{-n-z}\,\text{Csc}[\pi\,(n+z)]}{(1-n-z)\,(n+z)}$$

In[15]:=
```
% /.  {Csc[π (n + z)] → (-1)^n Csc[π z] }
```

Out[15]=
$$-\frac{(-1)^{n}\,\text{mm}^{n+z}\,\pi\,\text{QQ}^{-n-z}\,\text{Csc}[\pi z]}{(1-n-z)\,(n+z)}$$

In[16]:=
```
-Residue[%, {z, 0}]
```

Out[16]=
$$-\frac{(-1)^{n}\,\text{mm}^{n}\,\text{QQ}^{-n}}{(-1+n)\,n}$$

```
(*  Sum up contributions of the residues at z=2,3,... *)
```

In[17]:=
```
Sum[%, {n, 2, Infinity}]
```

Out[17]=
$$-\frac{-\text{mm}+\text{mm}\,\text{Log}\left[1+\frac{\text{mm}}{\text{QQ}}\right]+\text{QQ}\,\text{Log}\left[1+\frac{\text{mm}}{\text{QQ}}\right]}{\text{QQ}}$$

```
(*  We add the above contributions from the residues
  at z=0 and z=1 as well as B1exp[[1]]               *)
```

In[18]:= `Simplify[% + res12 + P1exp[[1]][[1]]]`

Out[18]= $$\frac{1}{\text{ep QQ}}\left(\text{QQ} + 2\,\text{ep QQ} + \text{ep mm Log}[\text{mm}] - \text{ep mm Log}[\text{QQ}] - \text{ep QQ Log}[\text{QQ}] - \text{ep}\,(\text{mm}+\text{QQ})\,\text{Log}\left[\frac{\text{mm}+\text{QQ}}{\text{QQ}}\right]\right)$$

In[19]:= $\% \;/.\; \text{Log}\left[\frac{\text{mm}+\text{QQ}}{\text{QQ}}\right] \to \text{Log}[\text{mm}+\text{QQ}] - \text{Log}[\text{QQ}]$

Out[19]= $$\frac{1}{\text{ep QQ}}\left(\text{QQ} + 2\,\text{ep QQ} + \text{ep mm Log}[\text{mm}] - \text{ep mm Log}[\text{QQ}] - \text{ep QQ Log}[\text{QQ}] - \text{ep}\,(\text{mm}+\text{QQ})\,(-\text{Log}[\text{QQ}] + \text{Log}[\text{mm}+\text{QQ}])\right)$$

In[20]:= `res = % /. Log[mm + QQ] → Log[(mm + QQ) / mm] + Log[mm]`

Out[20]= $$\frac{1}{\text{ep QQ}}\left(\text{QQ} + 2\,\text{ep QQ} + \text{ep mm Log}[\text{mm}] - \text{ep mm Log}[\text{QQ}] - \text{ep QQ Log}[\text{QQ}] - \text{ep}\,(\text{mm}+\text{QQ})\left(\text{Log}[\text{mm}] - \text{Log}[\text{QQ}] + \text{Log}\left[\frac{\text{mm}+\text{QQ}}{\text{mm}}\right]\right)\right)$$

`(*  result          *)`

In[21]:= $\text{resSimpl} = 2 + \frac{1}{\text{ep}} - \text{Log}[\text{mm}] - \left(1 + \frac{\text{mm}}{\text{QQ}}\right)\text{Log}\left[1 + \frac{\text{QQ}}{\text{mm}}\right];$

`(*   check   *)`

In[22]:= `Simplify[res - resSimpl]`

Out[22]= 0