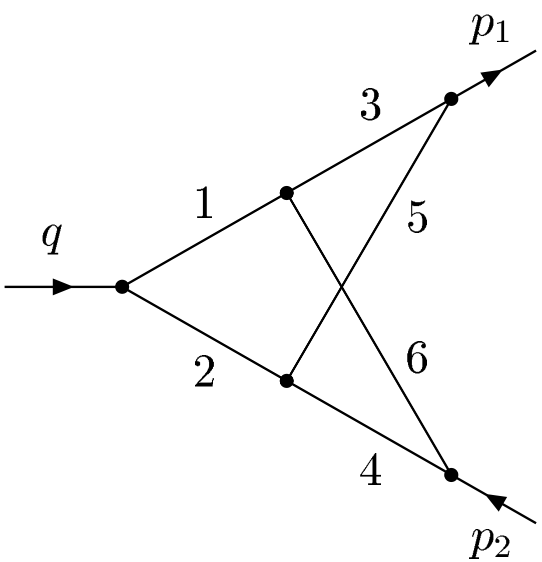
Non-planar two-loop massless vertex diagram with $p_1^2 = p_2^2 = 0$, $Q^2 = -(p_1 - p_2)^2 = 2p_1 \cdot p_2$



$$F_\Gamma(Q^2; a_1, \ldots, a_6, d) = \int\int \frac{\mathrm{d}^d k\, \mathrm{d}^d l}{[(k+l)^2 - 2p_1 \cdot (k+l)]^{a_1}}$$

$$\times \frac{1}{[(k+l)^2 - 2p_2 \cdot (k+l)]^{a_2} (k^2 - 2p_1 \cdot k)^{a_3} (l^2 - 2p_2 \cdot l)^{a_4} (k^2)^{a_5} (l^2)^{a_6}}$$

$$\frac{1}{(k^2 - 2p_1 \cdot k)^{a_3} (k^2)^{a_5}} = \frac{(-1)^{a_3+a_5} \Gamma(a_3 + a_5)}{\Gamma(a_3)\Gamma(a_5)} \times \int_0^1 \frac{\mathrm{d}\xi_1 \, \xi_1^{a_3-1} (1 - \xi_1)^{a_5-1}}{[-(k - \xi_1 p_1)^2 - \mathrm{i}0]^{a_3+a_5}}$$

and, similarly, for the second pair, with the replacements

$$\xi_1 \to \xi_2, \; p_1 \to p_2, \; k \to l, \; a_3 \to a_4, \; a_5 \to a_6$$

Change the integration variable $l \to r = k + l$ and integrate over k by means of our massless one-loop formula

$$\int \frac{\mathrm{d}k}{[-(k-\xi_1 p_1)^2]^{a_3+a_5}[-(r-\xi_2 p_2-k)^2]^{a_4+a_6}} = \mathrm{i}\pi^{d/2} \frac{G(a_3+a_5, a_4+a_6)}{[-(r-\xi_1 p_1-\xi_2 p_2)^2]^{a_3+a_4+a_5+a_6+\epsilon-2}}$$

Apply Feynman parametric formula to the propagators 1 and 2 and the propagator arising from the previous integration, with a resulting integral over r evaluated in terms of gamma functions:

$$\int \frac{\mathrm{d}^d r}{[-(r^2-Q^2 A(\xi_1,\xi_2,\xi_3,\xi_4))]^{a+\epsilon-2}} = \mathrm{i}\pi^{d/2} \frac{\Gamma(a+2\epsilon-4)}{\Gamma(a+\epsilon-2)} \frac{1}{(Q^2)^{a+2\epsilon-4} A(\xi_1,\xi_2,\xi_3,\xi_4)^{a+2\epsilon-4}}$$

where $a = a_1 + \ldots + a_6$ and

$$A(\xi_1,\xi_2,\xi_3,\xi_4) = \xi_3\xi_4 + (1-\xi_3-\xi_4)[\xi_2\xi_3(1-\xi_1) + \xi_1\xi_4(1-\xi_2)]$$

Gonsalves'83:

$$F_\Gamma(Q^2; a_1, \ldots, a_6, d) \ = \ \frac{(-1)^a \left(\mathrm{i}\pi^{d/2}\right)^2 \Gamma(2-\epsilon-a_{35})\Gamma(2-\epsilon-a_{46})}{(Q^2)^{a+2\epsilon-4} \prod \Gamma(a_l)\ \Gamma(4-2\epsilon-a_{3456})}$$

$$\times\Gamma(a+2\epsilon-4)\int_0^1 \mathrm{d}\xi_1 \ldots \int_0^1 \mathrm{d}\xi_4\, \xi_1^{a_3-1}(1-\xi_1)^{a_5-1}\xi_2^{a_4-1}(1-\xi_2)^{a_6-1}$$

$$\times\xi_3^{a_1-1}\xi_4^{a_2-1}(1-\xi_3-\xi_4)_+^{a_{3456}+\epsilon-3}A(\xi_1,\xi_2,\xi_3,\xi_4)^{4-2\epsilon-a}$$

$$\frac{\Gamma(a+2\epsilon-4)}{[\eta\xi(1-\xi)+(1-\eta)(\xi\xi_2(1-\xi_1)+(1-\xi)\xi_1(1-\xi_2))]^{a+2\epsilon-4}}$$

$$= \frac{1}{2\pi \mathrm{i}}\int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \frac{\mathrm{d}z_1\,\Gamma(-z_1)\eta^{z_1}\xi^{z_1}(1-\xi)^{z_1}}{(1-\eta)^{a+2\epsilon-4+z_1}}$$

$$\times \frac{\Gamma(a+2\epsilon-4+z_1)}{[\xi\xi_2(1-\xi_1)+(1-\xi)\xi_1(1-\xi_2)]^{a+2\epsilon-4+z_1}}$$

The last line $\rightarrow$

$$\frac{1}{2\pi \mathrm{i}}\int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \frac{\mathrm{d}z_2\,\Gamma(a+2\epsilon-4+z_1+z_2)\Gamma(-z_2)\xi^{z_2}\xi_2^{z_2}(1-\xi_1)^{z_2}}{(1-\xi)^{a+2\epsilon-4+z_1+z_2}\xi_1^{a+2\epsilon-4+z_1+z_2}(1-\xi_2)^{a+2\epsilon-4+z_1+z_2}}$$

$$F_\Gamma(Q^2; a_1, \ldots, a_6, d) = \frac{(-1)^a \left(\mathrm{i}\pi^{d/2}\right)^2 \Gamma(2-\epsilon-a_{35})}{(Q^2)^{a+2\epsilon-4}\Gamma(6-3\epsilon-a)\prod\Gamma(a_l)}$$

$$\times \frac{\Gamma(2-\epsilon-a_{46})}{\Gamma(4-2\epsilon-a_{3456})} \frac{1}{(2\pi\mathrm{i})^2} \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \mathrm{d}z_1 \mathrm{d}z_2 \Gamma(a+2\epsilon-4+z_1+z_2)$$

$$\times \Gamma(-z_1)\Gamma(-z_2)\Gamma(a_4+z_2)\Gamma(a_5+z_2)\Gamma(a_1+z_1+z_2)$$

$$\times \frac{\Gamma(2-\epsilon-a_{12}-z_1)\Gamma(4-2\epsilon+a_2-a-z_2)}{\Gamma(4-2\epsilon-a_{1235}-z_1)\Gamma(4-2\epsilon-a_{1246}-z_1)}$$

$$\times \Gamma(4-2\epsilon+a_3-a-z_1-z_2)\Gamma(4-2\epsilon+a_6-a-z_1-z_2) \; ,$$

where $a_{3456} = a_3 + a_4 + a_5 + a_6$, etc.