```
In[2]:= << MB/MB.m

MB 1.1

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more info in hep-ph/0511200

last modified 06 Mar 08

(*  2fold MB representation for the non-planar vertex massless diagram.
    The factor QQ^(4-a1-a2-a3-a4-a5-a6-2 ep) is omitted.
    QQ=-(p1-p2)^2.
  The factor (I Pi^(d/2))^2 is also omitted as usually.  *)

In[3]:= NPMB[a1_, a2_, a3_, a4_, a5_, a6_] := ((-1)^(a1 + a2 + a3 + a4 + a5 + a6) /
      (Gamma[a1] Gamma[a2] Gamma[a3] Gamma[a4] Gamma[a5] Gamma[a6])
    Gamma[2 - ep - a3 - a5] Gamma[2 - ep - a4 - a6] / Gamma[4 - 2 ep - a3 - a4 - a5 - a6] /
     Gamma[6 - 3 ep - a1 - a2 - a3 - a4 - a5 - a6]
    Gamma[a1 + a2 + a3 + a4 + a5 + a6 + 2 ep - 4 + z1 + z2] Gamma[-z1] Gamma[-z2]
    Gamma[2 - ep - a1 - a2 - z1] Gamma[a4 + z2] Gamma[a1 + z1 + z2]
    Gamma[4 - 2 ep - a1 - a3 - a4 - a5 - a6 - z2] Gamma[4 - 2 ep - a1 - a2 - a4 - a5 - a6 - z1 - z2]
    Gamma[a5 + z2] Gamma[4 - 2 ep - a1 - a2 - a3 - a4 - a5 - z1 - z2] /
      Gamma[4 - 2 ep - a1 - a2 - a4 - a6 - z1] / Gamma[4 - 2 ep - a1 - a2 - a3 - a5 - z1]);

(*  The diagram with all powers of the propagators equal
 to one. We shall evaluate it in expansion in ep up to ep^0. *)

In[4]:= V2 = NPMB[1, 1, 1, 1, 1, 1]

Out[4]= (Gamma[-ep]^2 Gamma[-ep - z1] Gamma[-z1] Gamma[-1 - 2 ep - z2] Gamma[-1 - 2 ep - z1 - z2]^2
    Gamma[-z2] Gamma[1 + z2]^2 Gamma[1 + z1 + z2] Gamma[2 + 2 ep + z1 + z2]) /
  (Gamma[-3 ep] Gamma[-2 ep] Gamma[-2 ep - z1]^2)

In[5]:= V2rules = MBoptimizedRules[V2, ep → 0, {}, {ep}]

Out[5]= {{ep → -5/8}, {z1 → -1/4, z2 → -1/4}}

In[6]:= V2cont = MBcontinue[V2, ep → 0, V2rules];

Level 1

Taking -residue in z2 = -1 - 2 ep

Taking -residue in z2 = -1 - 2 ep - z1

Level 2

Integral {1}

Taking +residue in z1 = 2 ep

Integral {2}

Level 3

Integral {1, 1}

4 integral(s) found

In[7]:= V2select = MBpreselect[MBmerge[V2cont], {ep, 0, 0}]
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In[8]:= **V2exp = Simplify[MBexpand[V2select, Exp[2 ep EulerGamma], {ep, 0, 0}]]**

Out[8]= $\{\text{MBint}[\frac{1}{ep^4} - \frac{\pi^2}{2\,ep^2} - \frac{41\,\pi^4}{40} + \frac{55\,\text{PolyGamma}[2,1]}{3\,ep}, \{\{ep \to 0\}, \{\}\}],$
$\text{MBint}[\frac{1}{4\,ep^2}\text{Gamma}[-z1]^2\,\text{Gamma}[z1]\,\text{Gamma}[1+z1]\,(12 + 12\,ep\,\text{EulerGamma} + 6\,ep^2\,\text{EulerGamma}^2 -$
$7\,ep^2\,\pi^2 + 6\,ep^2\,\text{PolyGamma}[0,-z1]^2 + 12\,ep^2\,\text{PolyGamma}[0,z1]^2 - 12\,ep^2\,\text{PolyGamma}[0,1+z1]^2 -$
$24\,ep\,\text{PolyGamma}[0,z1]\,(1 + ep\,\text{EulerGamma} + ep\,\text{PolyGamma}[0,1+z1]) + 12\,ep$
$\text{PolyGamma}[0,-z1]\,(3 + 3\,ep\,\text{EulerGamma} - 2\,ep\,\text{PolyGamma}[0,z1] + 4\,ep\,\text{PolyGamma}[0,1+z1]) -$
$66\,ep^2\,\text{PolyGamma}[1,-z1] + 12\,ep^2\,\text{PolyGamma}[1,z1] - 12\,ep^2\,\text{PolyGamma}[1,1+z1]),$
$\{\{ep \to 0\}, \{z1 \to -\frac{1}{4}\}\}], \text{MBint}[6\,\text{Gamma}[-1-z2]\,\text{Gamma}[-1-z1-z2]^2\,\text{Gamma}[-z2]$
$\text{Gamma}[1+z2]^2\,\text{Gamma}[1+z1+z2]\,\text{Gamma}[2+z1+z2], \{\{ep \to 0\}, \{z1 \to -\frac{1}{4}, z2 \to -\frac{1}{4}\}\}]\}$

In[9]:= **Length[V2exp]**

Out[9]= 3

In[10]:= **res1 = V2exp[[1]][[1]]**

Out[10]= $\frac{1}{ep^4} - \frac{\pi^2}{2\,ep^2} - \frac{41\,\pi^4}{40} + \frac{55\,\text{PolyGamma}[2,1]}{3\,ep}$

In[11]:= **V2exp[[3]]**

Out[11]= $\text{MBint}[6\,\text{Gamma}[-1-z2]\,\text{Gamma}[-1-z1-z2]^2\,\text{Gamma}[-z2]\,\text{Gamma}[1+z2]^2$
$\text{Gamma}[1+z1+z2]\,\text{Gamma}[2+z1+z2], \{\{ep \to 0\}, \{z1 \to -\frac{1}{4}, z2 \to -\frac{1}{4}\}\}]$

In[12]:= **Barnes1[V2exp[[3]], z1]**

Out[12]= $\text{MBint}[\pi^2\,\text{Gamma}[-1-z2]\,\text{Gamma}[-z2]\,\text{Gamma}[1+z2]^2, \{\{ep \to 0\}, \{z2 \to -\frac{1}{4}\}\}]$

In[13]:= **res2 = Barnes1[%, z2][[1]]**

Out[13]= $-\frac{\pi^4}{6}$

In[14]:= **V2exp[[2]]**

Out[14]= $\text{MBint}[\frac{1}{4\,ep^2}$
$\text{Gamma}[-z1]^2\,\text{Gamma}[z1]\,\text{Gamma}[1+z1]\,(12 + 12\,ep\,\text{EulerGamma} + 6\,ep^2\,\text{EulerGamma}^2 - 7\,ep^2\,\pi^2 +$
$6\,ep^2\,\text{PolyGamma}[0,-z1]^2 + 12\,ep^2\,\text{PolyGamma}[0,z1]^2 - 12\,ep^2\,\text{PolyGamma}[0,1+z1]^2 -$
$24\,ep\,\text{PolyGamma}[0,z1]\,(1 + ep\,\text{EulerGamma} + ep\,\text{PolyGamma}[0,1+z1]) + 12\,ep$
$\text{PolyGamma}[0,-z1]\,(3 + 3\,ep\,\text{EulerGamma} - 2\,ep\,\text{PolyGamma}[0,z1] + 4\,ep\,\text{PolyGamma}[0,1+z1]) -$
$66\,ep^2\,\text{PolyGamma}[1,-z1] + 12\,ep^2\,\text{PolyGamma}[1,z1] -$
$12\,ep^2\,\text{PolyGamma}[1,1+z1]), \{\{ep \to 0\}, \{z1 \to -\frac{1}{4}\}\}]$

In[15]:= **CoeffEps[X_, n_] := (X /. X[[1]] → Simplify[Coefficient[X[[1]], ep, n]]);**

In[16]:= **CoeffEps[V2exp[[2]], -2]**

Out[16]= $\text{MBint}[3\,\text{Gamma}[-z1]^2\,\text{Gamma}[z1]\,\text{Gamma}[1+z1], \{\{ep \to 0\}, \{z1 \to -\frac{1}{4}\}\}]$

In[17]:= **res32 = Barnes1[CoeffEps[V2exp[[2]], -2], z1][[1]]**

Out[17]= $-\frac{\pi^2}{2}$

In[18]:= **CoeffEps[V2exp[[2]], -1]**

Out[18]= MBint[3 Gamma[-z1]^2 Gamma[z1] Gamma[1 + z1] (EulerGamma + 3 PolyGamma[0, -z1] - 2 PolyGamma[0, z1]), {{ep → 0}, {z1 → $-\frac{1}{4}$}}]

In[19]:= **res31 = 9 Zeta[3];**

In[20]:= **res31 // N**

Out[20]= 10.8185

In[21]:= **NIntegrate[CoeffEps[V2exp[[2]], -1][[1]] / (2 Pi) /. {z1 → $-\frac{1}{4}$ + I * y1}, {y1, -Infinity, Infinity}]**

Out[21]= $10.8185 + 2.13163 \times 10^{-14}\, i$

In[22]:= **CoeffEps[V2exp[[2]], 0]**

Out[22]= MBint[$\frac{1}{4}$ Gamma[-z1]^2 Gamma[z1] Gamma[1 + z1] (6 EulerGamma^2 - 7 π^2 + 6 PolyGamma[0, -z1]^2 + 12 PolyGamma[0, z1]^2 - 12 PolyGamma[0, 1 + z1]^2 - 24 PolyGamma[0, z1] (EulerGamma + PolyGamma[0, 1 + z1]) + 12 PolyGamma[0, -z1] (3 EulerGamma - 2 PolyGamma[0, z1] + 4 PolyGamma[0, 1 + z1]) - 66 PolyGamma[1, -z1] + 12 PolyGamma[1, z1] - 12 PolyGamma[1, 1 + z1]), {{ep → 0}, {z1 → $-\frac{1}{4}$}}]

In[23]:= **res30 = $\frac{7\pi^4}{10}$;**

In[24]:= **res30 // N**

Out[24]= 68.1864

In[25]:= **NIntegrate[CoeffEps[V2exp[[2]], 0][[1]] / (2 Pi) /. {z1 → $-\frac{1}{4}$ + I * y1}, {y1, -Infinity, Infinity}]**

Out[25]= $68.1864 + 0.\, i$

(*   result   *)

In[26]:= **FullSimplify[res1 + res2 + res32 / ep^2 + res31 / ep + res30]**

Out[26]= $\frac{1}{ep^4} - \frac{\pi^2}{ep^2} - \frac{59\pi^4}{120} - \frac{83\,\mathrm{Zeta}[3]}{3\,ep}$