In[1]:=
```
SetDirectory["c:/diskE/job2008/Zurich"];
```

In[2]:=
```
<< MB/MB.m;
<< MB/MBresolve.m
```

```
MB 1.1

by Michal Czakon

more info in hep-ph/0511200

last modified 06 Mar 08

MBresolve 1.0

by Alexander Smirnov

last modified 22 Oct 08
```

```
(*   The end of derivation of MB
  representation for the planar massless double box diagram at
  p1^2=p2^2=p3^2=p4^2=0.

     Notation:   S=-s=-(p1+p2)^2, T=-t=-(p1+p3)^2 *)

(*  This is MB representation for the box   *)
```

In[4]:=
```
Box1[a1_, a2_, a3_, a4_] :=
  (S^(2-a1-a2-a3-a4-ep-z1) T^z1 Gamma[a1 + a2 + a3 + a4 - 2 + ep + z1] Gamma[a2 + z1] Gamma[a4 + z1]
     Gamma[2 - a1 - a2 - a4 - ep - z1] Gamma[2 - a2 - a3 - a4 - ep - z1] Gamma[-z1] ) /
   (Gamma[a1] Gamma[a2] Gamma[a3] Gamma[a4] Gamma[4 - a1 - a2 - a3 - a4 - 2 ep]);
```

In[5]:=
```
( 1 / S^(a4 + a5 + a6 + a7 + ep - 2 + z2 + z3 + z4) /
    (Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7] Gamma[4 - a4 - a5 - a6 - a7 - 2 ep])
  Gamma[a4 + a5 + a6 + a7 - 2 + ep + z2 + z3 + z4] Gamma[a7 + z2 + z3 + z4] Gamma[a5 + z4]
  Gamma[2 - a4 - a5 - a7 - ep - z3 - z4] Gamma[2 - a5 - a6 - a7 - ep - z2 - z4]
  Gamma[-z2] Gamma[-z3] Gamma[-z4] ) Box1[a1 - z2, a2, a3 - z3, a8 - z4];
```

In[6]:=
```
Simplify[%]
```

Out[6]=
```
(S^(4-a1-a2-a3-a4-a5-a6-a7-a8-2 ep-z1) T^z1 Gamma[-z1] Gamma[a2 + z1] Gamma[-z2] Gamma[-z3]
   Gamma[a8 + z1 - z4] Gamma[2 - a5 - a6 - a7 - ep - z2 - z4] Gamma[2 - a4 - a5 - a7 - ep - z3 - z4]
   Gamma[-2 + a1 + a2 + a3 + a8 + ep + z1 - z2 - z3 - z4] Gamma[-z4] Gamma[a5 + z4]
   Gamma[2 - a1 - a2 - a8 - ep - z1 + z2 + z4] Gamma[2 - a2 - a3 - a8 - ep - z1 + z3 + z4]
   Gamma[a7 + z2 + z3 + z4] Gamma[-2 + a4 + a5 + a6 + a7 + ep + z2 + z3 + z4]) /
 (Gamma[a2] Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7] Gamma[4 - a4 - a5 - a6 - a7 - 2 ep]
   Gamma[a1 - z2] Gamma[a3 - z3] Gamma[a8 - z4] Gamma[4 - a1 - a2 - a3 - a8 - 2 ep + z2 + z3 + z4])
```

```
(*   Changing variables    *)
```

In[7]:=
```
% /.  {z2 → z2 - z4, z3 → z3 - z4};
```

In[8]:=
```
% /. {z3 → z3 + z1};
```

In[9]:=
```
% /. z4 → z4 + z1;
```

In[10]:=
```
% /. z2 → z2 + z1;
```

In[11]:=
```
Simplify[% /. z4 → z4 + z2 + z3]
```

Out[11]=
```
(S^(4-a1-a2-a3-a4-a5-a6-a7-a8-2 ep-z1) T^z1 Gamma[-z1] Gamma[a2 + z1] Gamma[2 - a5 - a6 - a7 - ep - z1 - z2]
   Gamma[2 - a1 - a2 - a8 - ep + z2] Gamma[2 - a4 - a5 - a7 - ep - z1 - z3]
   Gamma[2 - a2 - a3 - a8 - ep + z3] Gamma[a7 + z1 - z4] Gamma[-2 + a4 + a5 + a6 + a7 + ep + z1 - z4]
   Gamma[a8 - z2 - z3 - z4] Gamma[-z1 - z2 - z3 - z4] Gamma[-2 + a1 + a2 + a3 + a8 + ep + z4]
   Gamma[z2 + z4] Gamma[z3 + z4] Gamma[a5 + z1 + z2 + z3 + z4]) /
 (Gamma[a2] Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7]
   Gamma[4 - a4 - a5 - a6 - a7 - 2 ep] Gamma[4 - a1 - a2 - a3 - a8 - 2 ep + z1 - z4]
   Gamma[a8 - z1 - z2 - z3 - z4] Gamma[a3 + z2 + z4] Gamma[a1 + z3 + z4])
```

In[12]:=
```
B2[a1_, a2_, a3_, a4_, a5_, a6_, a7_, a8_] :=
 (S^(4-a1-a2-a3-a4-a5-a6-a7-a8-2 ep-z1) T^z1 Gamma[-z1] Gamma[a2 + z1] Gamma[2 - a5 - a6 - a7 - ep - z1 - z2]
    Gamma[2 - a1 - a2 - a8 - ep + z2] Gamma[2 - a4 - a5 - a7 - ep - z1 - z3]
    Gamma[2 - a2 - a3 - a8 - ep + z3] Gamma[a7 + z1 - z4] Gamma[-2 + a4 + a5 + a6 + a7 + ep + z1 - z4]
    Gamma[a8 - z2 - z3 - z4] Gamma[-z1 - z2 - z3 - z4] Gamma[-2 + a1 + a2 + a3 + a8 + ep + z4]
    Gamma[z2 + z4] Gamma[z3 + z4] Gamma[a5 + z1 + z2 + z3 + z4]) /
  (Gamma[a2] Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7] Gamma[4 - a4 - a5 - a6 - a7 - 2 ep]
    Gamma[4 - a1 - a2 - a3 - a8 - 2 ep + z1 - z4]
    Gamma[a8 - z1 - z2 - z3 - z4] Gamma[a3 + z2 + z4] Gamma[a1 + z3 + z4])

(*   the function in the one-loop integration formula  *)
```

In[13]:=
```
G[a1_, a2_] := Gamma[a1 + a2 + ep - 2] Gamma[2 - ep - a1]
   Gamma[2 - ep - a2] / Gamma[a1] / Gamma[a2] / Gamma[4 - 2 ep - a1 - a2];

(*     a vertical check: shrink vertical lines,
a2,a5,a7→0
                                                  *)
```

In[14]:=
```
B2[a1, a2, a3, a4, a5, a6, a7, 0]
```

Out[14]=
```
(S^(4-a1-a2-a3-a4-a5-a6-a7-2 ep-z1) T^z1 Gamma[-z1] Gamma[a2 + z1] Gamma[2 - a5 - a6 - a7 - ep - z1 - z2]
   Gamma[2 - a1 - a2 - ep + z2] Gamma[2 - a4 - a5 - a7 - ep - z1 - z3] Gamma[2 - a2 - a3 - ep + z3]
   Gamma[a7 + z1 - z4] Gamma[-2 + a4 + a5 + a6 + a7 + ep + z1 - z4] Gamma[-z2 - z3 - z4]
   Gamma[-2 + a1 + a2 + a3 + ep + z4] Gamma[z2 + z4] Gamma[z3 + z4] Gamma[a5 + z1 + z2 + z3 + z4]) /
 (Gamma[a2] Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7] Gamma[4 - a4 - a5 - a6 - a7 - 2 ep]
   Gamma[4 - a1 - a2 - a3 - 2 ep + z1 - z4] Gamma[a3 + z2 + z4] Gamma[a1 + z3 + z4])
```

```
(*    a2→0; Gamma[-z1] Gamma[a2+z1]   *)
```

In[15]:=
```
-Residue[B2[a1, a2, a3, a4, a5, a6, a7, 0], {z1, 0}]
```

Out[15]=
```
(S^(4-a1-a2-a3-a4-a5-a6-a7-2 ep) Gamma[2 - a5 - a6 - a7 - ep - z2]
   Gamma[2 - a1 - a2 - ep + z2] Gamma[2 - a4 - a5 - a7 - ep - z3] Gamma[2 - a2 - a3 - ep + z3]
   Gamma[a7 - z4] Gamma[-2 + a4 + a5 + a6 + a7 + ep - z4] Gamma[-z2 - z3 - z4]
   Gamma[-2 + a1 + a2 + a3 + ep + z4] Gamma[z2 + z4] Gamma[z3 + z4] Gamma[a5 + z2 + z3 + z4]) /
 (Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7] Gamma[4 - a4 - a5 - a6 - a7 - 2 ep]
   Gamma[4 - a1 - a2 - a3 - 2 ep - z4] Gamma[a3 + z2 + z4] Gamma[a1 + z3 + z4])
```

In[16]:=
```
% /. a2 → 0
```

Out[16]=
```
(S^(4-a1-a3-a4-a5-a6-a7-2 ep) Gamma[2 - a5 - a6 - a7 - ep - z2]
   Gamma[2 - a1 - ep + z2] Gamma[2 - a4 - a5 - a7 - ep - z3] Gamma[2 - a3 - ep + z3]
   Gamma[a7 - z4] Gamma[-2 + a4 + a5 + a6 + a7 + ep - z4] Gamma[-z2 - z3 - z4]
   Gamma[-2 + a1 + a3 + ep + z4] Gamma[z2 + z4] Gamma[z3 + z4] Gamma[a5 + z2 + z3 + z4]) /
 (Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7] Gamma[4 - a4 - a5 - a6 - a7 - 2 ep]
   Gamma[4 - a1 - a3 - 2 ep - z4] Gamma[a3 + z2 + z4] Gamma[a1 + z3 + z4])
```

```
(*    a5→0; Gamma[a5+z2+z3+z4] Gamma[-z2-z3-z4]   *)
```

In[17]:=
```
-Residue[%, {z4, -z2 - z3}]
```

Out[17]=
$$\left(S^{4-a1-a3-a4-a5-a6-a7-2\,ep}\,\Gamma[2-a5-a6-a7-ep-z2]\,\Gamma[-z2]\,\Gamma[2-a1-ep+z2]\right.$$
$$\Gamma[2-a4-a5-a7-ep-z3]\,\Gamma[-2+a1+a3+ep-z2-z3]\,\Gamma[-z3]$$
$$\left.\Gamma[2-a3-ep+z3]\,\Gamma[a7+z2+z3]\,\Gamma[-2+a4+a5+a6+a7+ep+z2+z3]\right)\Big/$$
$$(\Gamma[a4]\,\Gamma[a6]\,\Gamma[a7]\,\Gamma[4-a4-a5-a6-a7-2\,ep]$$
$$\Gamma[a1-z2]\,\Gamma[a3-z3]\,\Gamma[4-a1-a3-2\,ep+z2+z3])$$

In[18]:=
```
% /. a5 → 0
```

Out[18]=
$$\left(S^{4-a1-a3-a4-a6-a7-2\,ep}\,\Gamma[2-a6-a7-ep-z2]\,\Gamma[-z2]\,\Gamma[2-a1-ep+z2]\right.$$
$$\Gamma[2-a4-a7-ep-z3]\,\Gamma[-2+a1+a3+ep-z2-z3]\,\Gamma[-z3]$$
$$\left.\Gamma[2-a3-ep+z3]\,\Gamma[a7+z2+z3]\,\Gamma[-2+a4+a6+a7+ep+z2+z3]\right)\Big/$$
$$(\Gamma[a4]\,\Gamma[a6]\,\Gamma[a7]\,\Gamma[4-a4-a6-a7-2\,ep]$$
$$\Gamma[a1-z2]\,\Gamma[a3-z3]\,\Gamma[4-a1-a3-2\,ep+z2+z3])$$

In[19]:=
```
-Residue[%, {z3, 0}]
```

Out[19]=
$$\left(S^{4-a1-a3-a4-a6-a7-2\,ep}\,\Gamma[2-a3-ep]\,\Gamma[2-a4-a7-ep]\right.$$
$$\Gamma[2-a6-a7-ep-z2]\,\Gamma[-2+a1+a3+ep-z2]\,\Gamma[-z2]$$
$$\left.\Gamma[a7+z2]\,\Gamma[2-a1-ep+z2]\,\Gamma[-2+a4+a6+a7+ep+z2]\right)\Big/$$
$$(\Gamma[a3]\,\Gamma[a4]\,\Gamma[a6]\,\Gamma[a7]\,\Gamma[4-a4-a6-a7-2\,ep]$$
$$\Gamma[a1-z2]\,\Gamma[4-a1-a3-2\,ep+z2])$$

In[20]:=
```
-Residue[%, {z2, 0}]
```

Out[20]=
$$\left(S^{4-a1-a3-a4-a6-a7-2\,ep}\,\Gamma[2-a1-ep]\,\Gamma[2-a3-ep]\,\Gamma[2-a4-a7-ep]\right.$$
$$\left.\Gamma[2-a6-a7-ep]\,\Gamma[-2+a1+a3+ep]\,\Gamma[-2+a4+a6+a7+ep]\right)\Big/$$
$$(\Gamma[a1]\,\Gamma[a3]\,\Gamma[a4]\,\Gamma[a6]\,\Gamma[4-a1-a3-2\,ep]\,\Gamma[4-a4-a6-a7-2\,ep])$$

In[21]:=
```
% /. a7 → 0
```

Out[21]=
$$\left(S^{4-a1-a3-a4-a6-2\,ep}\,\Gamma[2-a1-ep]\,\Gamma[2-a3-ep]\,\Gamma[2-a4-ep]\right.$$
$$\left.\Gamma[2-a6-ep]\,\Gamma[-2+a1+a3+ep]\,\Gamma[-2+a4+a6+ep]\right)\Big/$$
$$(\Gamma[a1]\,\Gamma[a3]\,\Gamma[a4]\,\Gamma[a6]\,\Gamma[4-a1-a3-2\,ep]\,\Gamma[4-a4-a6-2\,ep])$$

In[22]:=
```
G[a1, a3] G[a4, a6] / %
```

Out[22]=
$$S^{-4+a1+a3+a4+a6+2\,ep}$$

```
(*    a7→0; Gamma[a7+z2+z3] Gamma[-z2]Gamma[-z3]   *)

(*     a horizontal check: shrink horizontal lines,
a1,a3,a4,a6→0
                                                  *)
```

In[23]:=
```
B2[a1, a2, a3, a4, a5, a6, a7, 0]
```

Out[23]=
$$\left(S^{4-a1-a2-a3-a4-a5-a6-a7-2\,ep-z1}\,T^{z1}\,\Gamma[-z1]\,\Gamma[a2+z1]\,\Gamma[2-a5-a6-a7-ep-z1-z2]\right.$$
$$\Gamma[2-a1-a2-ep+z2]\,\Gamma[2-a4-a5-a7-ep-z1-z3]\,\Gamma[2-a2-a3-ep+z3]$$
$$\Gamma[a7+z1-z4]\,\Gamma[-2+a4+a5+a6+a7+ep+z1-z4]\,\Gamma[-z2-z3-z4]$$
$$\left.\Gamma[-2+a1+a2+a3+ep+z4]\,\Gamma[z2+z4]\,\Gamma[z3+z4]\,\Gamma[a5+z1+z2+z3+z4]\right)\Big/$$
$$(\Gamma[a2]\,\Gamma[a4]\,\Gamma[a5]\,\Gamma[a6]\,\Gamma[a7]\,\Gamma[4-a4-a5-a6-a7-2\,ep]$$
$$\Gamma[4-a1-a2-a3-2\,ep+z1-z4]\,\Gamma[a3+z2+z4]\,\Gamma[a1+z3+z4])$$

```
Gamma[-2 + a4 + a5 + a6 + a7 + ep + z1 - z4] Gamma[2 - a4 - a5 - a7 - ep - z1 - z3] Gamma[z3 + z4]

 -2 + a4 + a5 + a6 + a7 + ep + z1 - z4 + 2 - a4 - a5 - a7 - ep - z1 - z3 + z3 + z4

a6
```

In[24]:=
```
Residue[B2[a1, a2, a3, a4, a5, a6, a7, 0], {z3, -z4}]
```

Out[24]=
$$\left(S^{4-a1-a2-a3-a4-a5-a6-a7-2\,ep-z1}\,T^{z1}\,\Gamma(-z1)\,\Gamma(a2+z1)\right.$$
$$\Gamma(2-a5-a6-a7-ep-z1-z2)\,\Gamma(-z2)\,\Gamma(2-a1-a2-ep+z2)\,\Gamma(a5+z1+z2)$$
$$\Gamma(2-a2-a3-ep-z4)\,\Gamma(a7+z1-z4)\,\Gamma(-2+a4+a5+a6+a7+ep+z1-z4)$$
$$\left.\Gamma(-2+a1+a2+a3+ep+z4)\,\Gamma(2-a4-a5-a7-ep-z1+z4)\,\Gamma(z2+z4)\right)\Big/$$
$$(\Gamma(a1)\,\Gamma(a2)\,\Gamma(a4)\,\Gamma(a5)\,\Gamma(a6)\,\Gamma(a7)$$
$$\Gamma(4-a4-a5-a6-a7-2\,ep)\,\Gamma(4-a1-a2-a3-2\,ep+z1-z4)\,\Gamma(a3+z2+z4))$$

In[25]:=
```
-Residue[ %, {z4, -2 + a4 + a5 + a6 + a7 + ep + z1}]
```

Out[25]=
$$\left(S^{4-a1-a2-a3-a4-a5-a6-a7-2\,ep-z1}\,T^{z1}\,\Gamma(2-a4-a5-a6-ep)\right.$$
$$\Gamma(4-a2-a3-a4-a5-a6-a7-2\,ep-z1)\,\Gamma(-z1)$$
$$\Gamma(a2+z1)\,\Gamma(-4+a1+a2+a3+a4+a5+a6+a7+2\,ep+z1)$$
$$\Gamma(2-a5-a6-a7-ep-z1-z2)\,\Gamma(-z2)\,\Gamma(2-a1-a2-ep+z2)$$
$$\left.\Gamma(a5+z1+z2)\,\Gamma(-2+a4+a5+a6+a7+ep+z1+z2)\right)\Big/$$
$$(\Gamma(a1)\,\Gamma(a2)\,\Gamma(a4)\,\Gamma(a5)\,\Gamma(a7)$$
$$\Gamma(6-a1-a2-a3-a4-a5-a6-a7-3\,ep)\,\Gamma(4-a4-a5-a6-a7-2\,ep)$$
$$\Gamma(-2+a3+a4+a5+a6+a7+ep+z1+z2))$$

In[26]:=
```
% /. a6 → 0
```

Out[26]=
$$\left(S^{4-a1-a2-a3-a4-a5-a7-2\,ep-z1}\,T^{z1}\,\Gamma(2-a4-a5-ep)\right.$$
$$\Gamma(4-a2-a3-a4-a5-a7-2\,ep-z1)\,\Gamma(-z1)\,\Gamma(a2+z1)$$
$$\Gamma(-4+a1+a2+a3+a4+a5+a7+2\,ep+z1)\,\Gamma(2-a5-a7-ep-z1-z2)\,\Gamma(-z2)$$
$$\left.\Gamma(2-a1-a2-ep+z2)\,\Gamma(a5+z1+z2)\,\Gamma(-2+a4+a5+a7+ep+z1+z2)\right)\Big/$$
$$(\Gamma(a1)\,\Gamma(a2)\,\Gamma(a4)\,\Gamma(a5)\,\Gamma(a7)\,\Gamma(6-a1-a2-a3-a4-a5-a7-3\,ep)$$
$$\Gamma(4-a4-a5-a7-2\,ep)\,\Gamma(-2+a3+a4+a5+a7+ep+z1+z2))$$

```
(* a1,a3,a4→0  *)

(*    let a4→0    *)

Gamma[-2 + a4 + a5 + a7 + ep + z1 + z2] Gamma[2 - a5 - a7 - ep - z1 - z2]

-2 + a4 + a5 + a7 + ep + z1 + z2 + 2 - a5 - a7 - ep - z1 - z2

a4
```

In[27]:=
```
-Residue[ %, {z2, 2 - a5 - a7 - ep - z1}]
```

Out[27]=
$$\left(S^{4-a1-a2-a3-a4-a5-a7-2\,ep-z1}\,T^{z1}\,\Gamma(2-a4-a5-ep)\,\Gamma(2-a7-ep)\right.$$
$$\Gamma(4-a1-a2-a5-a7-2\,ep-z1)\,\Gamma(4-a2-a3-a4-a5-a7-2\,ep-z1)\,\Gamma(-z1)$$
$$\left.\Gamma(a2+z1)\,\Gamma(-2+a5+a7+ep+z1)\,\Gamma(-4+a1+a2+a3+a4+a5+a7+2\,ep+z1)\right)\Big/$$
$$(\Gamma(a1)\,\Gamma(a2)\,\Gamma(a3+a4)\,\Gamma(a5)\,\Gamma(a7)$$
$$\Gamma(6-a1-a2-a3-a4-a5-a7-3\,ep)\,\Gamma(4-a4-a5-a7-2\,ep))$$

In[28]:=
```
% /. a4 → 0
```

Out[28]=
$$\left(S^{4-a1-a2-a3-a5-a7-2\,ep-z1}\,T^{z1}\,\Gamma(2-a5-ep)\,\Gamma(2-a7-ep)\right.$$
$$\Gamma(4-a1-a2-a5-a7-2\,ep-z1)\,\Gamma(4-a2-a3-a5-a7-2\,ep-z1)\,\Gamma(-z1)$$
$$\left.\Gamma(a2+z1)\,\Gamma(-2+a5+a7+ep+z1)\,\Gamma(-4+a1+a2+a3+a5+a7+2\,ep+z1)\right)\Big/$$
$$(\Gamma(a1)\,\Gamma(a2)\,\Gamma(a3)\,\Gamma(a5)\,\Gamma(a7)$$
$$\Gamma(6-a1-a2-a3-a5-a7-3\,ep)\,\Gamma(4-a5-a7-2\,ep))$$

```
(* a1,a3→0  *)

(*    let a1→0    *)

Gamma[-4 + a1 + a2 + a3 + a5 + a7 + 2 ep + z1] Gamma[4 - a2 - a3 - a5 - a7 - 2 ep - z1]
```

```
-4 + a1 + a2 + a3 + a5 + a7 + 2 ep + z1 + 4 - a2 - a3 - a5 - a7 - 2 ep - z1
```

```
a1
```

In[29]:=
```
-Residue[ %, {z1, 4 - a2 - a3 - a5 - a7 - 2 ep}]
```

Out[29]=
```
(S^-a1 T^(4-a2-a3-a5-a7-2 ep) Gamma[-a1 + a3] Gamma[4 - a3 - a5 - a7 - 2 ep] Gamma[2 - a2 - a3 - ep]
   Gamma[2 - a5 - ep] Gamma[2 - a7 - ep] Gamma[-4 + a2 + a3 + a5 + a7 + 2 ep]) / (Gamma[a2]
   Gamma[a3] Gamma[a5] Gamma[a7] Gamma[6 - a1 - a2 - a3 - a5 - a7 - 3 ep] Gamma[4 - a5 - a7 - 2 ep])
```

In[30]:=
```
% /. a1 → 0
```

Out[30]=
```
(T^(4-a2-a3-a5-a7-2 ep) Gamma[4 - a3 - a5 - a7 - 2 ep] Gamma[2 - a2 - a3 - ep]
   Gamma[2 - a5 - ep] Gamma[2 - a7 - ep] Gamma[-4 + a2 + a3 + a5 + a7 + 2 ep]) /
 (Gamma[a2] Gamma[a5] Gamma[a7] Gamma[6 - a2 - a3 - a5 - a7 - 3 ep] Gamma[4 - a5 - a7 - 2 ep])
```

In[31]:=
```
% /. a3 → 0
```

Out[31]=
```
(T^(4-a2-a5-a7-2 ep) Gamma[2 - a2 - ep]
   Gamma[2 - a5 - ep] Gamma[2 - a7 - ep] Gamma[-4 + a2 + a5 + a7 + 2 ep]) /
 (Gamma[a2] Gamma[a5] Gamma[a7] Gamma[6 - a2 - a5 - a7 - 3 ep])
```

In[32]:=
```
G[a2, a7] G[a2 + a7 + ep - 2, a5] / %
```

Out[32]=
```
T^(-4+a2+a5+a7+2 ep)
```

```
(*  In addition to the usual factor  (I Pi^(d/2))^2,
let us pull out the factor

  S^(4-a1-a2-a3-a4-a5-a6-a7-a8-2 ep).

   Let us turn to the variable x=T/S = t/s      *)
```

In[33]:=
```
K2[a1_, a2_, a3_, a4_, a5_, a6_, a7_, a8_] :=
 ( x^z1 Gamma[-z1] Gamma[a2 + z1] Gamma[2 - a5 - a6 - a7 - ep - z1 - z2]
    Gamma[2 - a1 - a2 - a8 - ep + z2] Gamma[2 - a4 - a5 - a7 - ep - z1 - z3]
    Gamma[2 - a2 - a3 - a8 - ep + z3] Gamma[a7 + z1 - z4] Gamma[-2 + a4 + a5 + a6 + a7 + ep + z1 - z4]
    Gamma[a8 - z2 - z3 - z4] Gamma[-z1 - z2 - z3 - z4] Gamma[-2 + a1 + a2 + a3 + a8 + ep + z4]
    Gamma[z2 + z4] Gamma[z3 + z4] Gamma[a5 + z1 + z2 + z3 + z4]) /
  (Gamma[a2] Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7] Gamma[4 - a4 - a5 - a6 - a7 - 2 ep]
    Gamma[4 - a1 - a2 - a3 - a8 - 2 ep + z1 - z4]
    Gamma[a8 - z1 - z2 - z3 - z4] Gamma[a3 + z2 + z4] Gamma[a1 + z3 + z4])
```

```
(*  The 2-box with the powers of the propagators equal to one *)
```

In[34]:=
```
B2 = x K2[1, 1, 1, 1, 1, 1, 1, 0]
```

Out[34]=
```
(x^(1+z1) Gamma[-z1] Gamma[1 + z1] Gamma[-1 - ep - z1 - z2] Gamma[-ep + z2] Gamma[-1 - ep - z1 - z3]
   Gamma[-ep + z3] Gamma[1 + z1 - z4] Gamma[2 + ep + z1 - z4] Gamma[-z2 - z3 - z4]
   Gamma[1 + ep + z4] Gamma[z2 + z4] Gamma[z3 + z4] Gamma[1 + z1 + z2 + z3 + z4]) /
 (Gamma[-2 ep] Gamma[1 - 2 ep + z1 - z4] Gamma[1 + z2 + z4] Gamma[1 + z3 + z4])
```

```
(*       auxiliary functions         *)
```

In[35]:=
```
SortByDimension[l_List] := Sort[l, Length[#1[[2, 2]]] > Length[#2[[2, 2]]] &];
CoeffEps[X_, n_] := (X /. X[[1]] → Simplify[Coefficient[X[[1]], ep, n]]);
MBDimension[int_MBint] := Length[int[[2, 2]]];
```

In[38]:= **B2rules = MBoptimizedRules[B2, ep → 0, {}, {ep}]**

Out[38]= $\left\{\left\{\text{ep} \to -\frac{9}{16}\right\}, \left\{\text{z1} \to -\frac{1}{2}, \text{z2} \to -\frac{5}{16}, \text{z3} \to -\frac{3}{8}, \text{z4} \to \frac{7}{16}\right\}\right\}$

In[39]:= **B2cont = MBcontinue[B2, ep → 0, B2rules];**

```
Level 1
Taking -residue in z2 = -1 - ep - z1
Taking +residue in z2 = ep
Taking -residue in z3 = -1 - ep - z1
Taking +residue in z3 = ep
Level 2
Integral {1}
Taking -residue in z3 = -1 - ep - z1
Taking +residue in z4 = 1 + ep + z1
Integral {2}
Taking -residue in z1 = -1 - 2 ep
Taking -residue in z3 = -1 - ep - z1
Taking -residue in z4 = -ep - z3
Integral {3}
Taking +residue in z4 = 1 + ep + z1
Integral {4}
Taking -residue in z1 = -1 - 2 ep
Taking -residue in z2 = -1 - ep - z1
Taking +residue in z2 = ep
Taking -residue in z4 = -ep - z2
Level 3
Integral {1, 1}
Taking +residue in z4 = 1 + 2 ep + z1
Taking +residue in z4 = 1 + ep + z1
Integral {1, 2}
Integral {2, 1}
Taking -residue in z4 = -2 ep
Taking -residue in z4 = -ep - z3
Integral {2, 2}
Taking +residue in z4 = 1 + ep + z1
Integral {2, 3}
Integral {3, 1}
Integral {4, 1}
Taking -residue in z4 = -2 ep
```

```
Taking -residue in z4 = -ep - z2
Integral {4, 2}
Taking +residue in z4 = 1 + ep + z1
Integral {4, 3}
Taking -residue in z1 = -1 - 2 ep
Taking -residue in z4 = -2 ep
Integral {4, 4}
Level 4
Integral {1, 1, 1}
Integral {1, 1, 2}
Integral {2, 1, 1}
Taking +residue in z3 = 3 ep
Taking +residue in z3 = 2 ep
Integral {2, 1, 2}
Integral {2, 2, 1}
Integral {4, 1, 1}
Taking +residue in z2 = 3 ep
Taking +residue in z2 = 2 ep
Integral {4, 1, 2}
Integral {4, 2, 1}
Integral {4, 3, 1}
Taking -residue in z4 = -4 ep
...no contribution
Taking -residue in z4 = -2 ep
Integral {4, 3, 2}
Level 5
Integral {2, 1, 1, 1}
Integral {2, 1, 1, 2}
Integral {4, 1, 1, 1}
Integral {4, 1, 1, 2}
Integral {4, 3, 1, 1}
30 integral(s) found
```

In[40]:=
```
B2select = MBpreselect[B2cont, {ep, 0, 0}]
```

In[41]:=
```
B2exp = Simplify[MBexpand[B2select, E^(2 EulerGamma ep), {ep, 0, 0}]]
```

In[42]:=
```
B2expS = SortByDimension[MBmerge[B2exp]]
```

Out[42]=
$$\left\{\text{MBint}\left[\frac{\text{Gamma}[-z3]\,\text{Gamma}[z3]\,\text{Gamma}[1-z4]\,\text{Gamma}[-z3-z4]\,\text{Gamma}[z4]\,\text{Gamma}[z3+z4]^2}{\text{Gamma}[1+z3+z4]},\ \left\{\{\text{ep}\to 0\},\ \left\{z3\to -\frac{3}{8},\ z4\to \frac{7}{16}\right\}\right\}\right],\right.$$

$$\text{MBint}\left[\frac{\text{Gamma}[-z2]\,\text{Gamma}[z2]\,\text{Gamma}[1-z4]\,\text{Gamma}[-z2-z4]\,\text{Gamma}[z4]\,\text{Gamma}[z2+z4]^2}{\text{Gamma}[1+z2+z4]},\ \left\{\{\text{ep}\to 0\},\ \left\{z2\to -\frac{5}{16},\ z4\to \frac{7}{16}\right\}\right\}\right],$$

$$\text{MBint}\left[2\,x^{1+z1}\,\text{Gamma}[-1-z1]\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^2\,\text{Gamma}[-1-z1-z3]\,\text{Gamma}[-z3]\,\text{Gamma}[z3]\,\text{Gamma}[2+z1+z3],\ \left\{\{\text{ep}\to 0\},\ \left\{z1\to -\frac{1}{2},\ z3\to -\frac{3}{8}\right\}\right\}\right],$$

$$\text{MBint}\left[2\,x^{1+z1}\,\text{Gamma}[-1-z1]\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^2\,\text{Gamma}[-1-z1-z2]\,\text{Gamma}[-z2]\,\text{Gamma}[z2]\,\text{Gamma}[2+z1+z2],\ \left\{\{\text{ep}\to 0\},\ \left\{z1\to -\frac{1}{2},\ z2\to -\frac{5}{16}\right\}\right\}\right],$$

$$\text{MBint}\left[-\frac{1}{2\,\text{ep}\,\text{Gamma}[1+z4]}\text{Gamma}[1-z4]\,\text{Gamma}[-z4]\,\text{Gamma}[z4]^3\,(1+4\,\text{ep}\,\text{EulerGamma}+\text{ep}\,\text{PolyGamma}[0,1-z4]+4\,\text{ep}\,\text{PolyGamma}[0,z4]-\text{ep}\,\text{PolyGamma}[0,1+z4]),\ \left\{\{\text{ep}\to 0\},\ \left\{z4\to \frac{7}{16}\right\}\right\}\right],$$

$$\text{MBint}\left[\frac{1}{12\,\text{Gamma}[1+z3]}\text{Gamma}[-z3]^2\,\text{Gamma}[z3]\left(24\,\text{Gamma}[z3]^2+\frac{1}{\text{ep}^2}\text{Gamma}[1+z3]^2\left(6+12\,\text{ep}\,\text{EulerGamma}+12\,\text{ep}^2\,\text{EulerGamma}^2+4\,\text{ep}^2\,\pi^2-12\,\text{ep}\,\text{Log}[x]-24\,\text{ep}^2\,\text{EulerGamma}\,\text{Log}[x]+12\,\text{ep}^2\,\text{Log}[x]^2+3\,\text{ep}^2\,\text{PolyGamma}[0,-z3]^2+6\,\text{ep}\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x])\,\text{PolyGamma}[0,z3]+3\,\text{ep}^2\,\text{PolyGamma}[0,z3]^2+6\,\text{ep}\,\text{PolyGamma}[0,-z3]\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x]+\text{ep}\,\text{PolyGamma}[0,z3])+3\,\text{ep}^2\,\text{PolyGamma}[1,-z3]-21\,\text{ep}^2\,\text{PolyGamma}[1,z3]\right)\right),\ \left\{\{\text{ep}\to 0\},\ \left\{z3\to -\frac{3}{8}\right\}\right\}\right],$$

$$\text{MBint}\left[\frac{1}{12\,\text{Gamma}[1+z2]}\text{Gamma}[-z2]^2\,\text{Gamma}[z2]\left(24\,\text{Gamma}[z2]^2+\frac{1}{\text{ep}^2}\text{Gamma}[1+z2]^2\left(6+12\,\text{ep}\,\text{EulerGamma}+12\,\text{ep}^2\,\text{EulerGamma}^2+4\,\text{ep}^2\,\pi^2-12\,\text{ep}\,\text{Log}[x]-24\,\text{ep}^2\,\text{EulerGamma}\,\text{Log}[x]+12\,\text{ep}^2\,\text{Log}[x]^2+3\,\text{ep}^2\,\text{PolyGamma}[0,-z2]^2+6\,\text{ep}\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x])\,\text{PolyGamma}[0,z2]+3\,\text{ep}^2\,\text{PolyGamma}[0,z2]^2+6\,\text{ep}\,\text{PolyGamma}[0,-z2]\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x]+\text{ep}\,\text{PolyGamma}[0,z2])+3\,\text{ep}^2\,\text{PolyGamma}[1,-z2]-21\,\text{ep}^2\,\text{PolyGamma}[1,z2]\right)\right),\ \left\{\{\text{ep}\to 0\},\ \left\{z2\to -\frac{5}{16}\right\}\right\}\right],$$

$$\text{MBint}\left[\frac{1}{\text{ep}}4\,x^{1+z1}\,\text{Gamma}[-1-z1]^2\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^2\,(3\,\text{ep}\,\text{Gamma}[1+z1]+\text{Gamma}[2+z1](-1+4\,\text{ep}\,\text{PolyGamma}[0,-1-z1]-2\,\text{ep}\,\text{PolyGamma}[0,1+z1]-2\,\text{ep}\,\text{PolyGamma}[0,2+z1])),\ \left\{\{\text{ep}\to 0\},\ \left\{z1\to -\frac{1}{2}\right\}\right\}\right],$$

$$\left.\text{MBint}\left[-\frac{1}{18\,\text{ep}^4}\left(-72+42\,\text{ep}^2\,\pi^2+20\,\text{ep}^4\,\pi^4+6\,\text{ep}^2\left(-6+17\,\text{ep}^2\,\pi^2\right)\text{Log}[x]^2-12\,\text{ep}^3\,\text{Log}[x]^3+24\,\text{ep}^4\,\text{Log}[x]^4-213\,\text{ep}^3\,\text{PolyGamma}[2,1]+3\,\text{ep}\,\text{Log}[x]\left(30-31\,\text{ep}^2\,\pi^2+94\,\text{ep}^3\,\text{PolyGamma}[2,1]\right)\right),\ \{\{\text{ep}\to 0\},\ \{\}\}\right]\right\}$$

In[43]:=
```
Length[B2expS]
```

Out[43]= 9

In[44]:= **MBDimension /@ B2expS**

Out[44]= {2, 2, 2, 2, 1, 1, 1, 1, 0}

In[45]:= **B2expS[[1]]**

Out[45]= $\text{MBint}\left[\frac{\text{Gamma}[-z3]\,\text{Gamma}[z3]\,\text{Gamma}[1-z4]\,\text{Gamma}[-z3-z4]\,\text{Gamma}[z4]\,\text{Gamma}[z3+z4]^2}{\text{Gamma}[1+z3+z4]}, \left\{\{\text{ep}\to 0\}, \left\{z3\to -\frac{3}{8}, z4\to \frac{7}{16}\right\}\right\}\right]$

In[46]:= **Barnes2[B2expS[[1]], z4]**

Out[46]= $\text{MBint}\left[\frac{1}{6}\pi^2\,\text{Gamma}[-z3]^2\,\text{Gamma}[z3]\,\text{Gamma}[1+z3] - \text{Gamma}[-z3]^2\,\text{Gamma}[z3]\,\text{Gamma}[1+z3]\,\text{PolyGamma}[1, 1+z3], \left\{\{\text{ep}\to 0\}, \left\{z3\to -\frac{3}{8}\right\}\right\}\right]$

In[47]:= **res01 = $\frac{17}{4}$ Zeta[4]**

Out[47]= $\frac{17\,\pi^4}{360}$

In[48]:= **% // N**

Out[48]= 4.59987

In[49]:= **NIntegrate[Barnes2[B2expS[[1]], z4] [[1]] / (2 Pi) /. {z3 → -3/8 + I * y1}, {y1, -Infinity, Infinity}]**

Out[49]= 4.59987 + 0. i

In[50]:= **B2expS[[2]]**

Out[50]= $\text{MBint}\left[\frac{\text{Gamma}[-z2]\,\text{Gamma}[z2]\,\text{Gamma}[1-z4]\,\text{Gamma}[-z2-z4]\,\text{Gamma}[z4]\,\text{Gamma}[z2+z4]^2}{\text{Gamma}[1+z2+z4]}, \left\{\{\text{ep}\to 0\}, \left\{z2\to -\frac{5}{16}, z4\to \frac{7}{16}\right\}\right\}\right]$

In[51]:= **Barnes2[B2expS[[2]], z4]**

Out[51]= $\text{MBint}\left[\frac{1}{6}\pi^2\,\text{Gamma}[-z2]^2\,\text{Gamma}[z2]\,\text{Gamma}[1+z2] - \text{Gamma}[-z2]^2\,\text{Gamma}[z2]\,\text{Gamma}[1+z2]\,\text{PolyGamma}[1, 1+z2], \left\{\{\text{ep}\to 0\}, \left\{z2\to -\frac{5}{16}\right\}\right\}\right]$

In[52]:= **res02 = $\frac{17}{4}$ Zeta[4]**

Out[52]= $\frac{17\,\pi^4}{360}$

In[53]:= **% // N**

Out[53]= 4.59987

In[54]:=
```
NIntegrate[Barnes2[B2expS[[2]], z4] [[1]] / (2 Pi)  /. {z2 → -5/16 + I * y1},
 {y1, -Infinity, Infinity} ]
```

Out[54]= $4.59987 - 2.88547 \times 10^{-13}\, i$

In[55]:=
```
B2expS[[3]]
```

Out[55]=
$$\text{MBint}\left[2\, x^{1+z1}\, \text{Gamma}[-1-z1]\, \text{Gamma}[-z1]\, \text{Gamma}[1+z1]^2\, \text{Gamma}[-1-z1-z3]\right.$$
$$\left.\text{Gamma}[-z3]\, \text{Gamma}[z3]\, \text{Gamma}[2+z1+z3],\ \left\{\{ep \to 0\},\ \left\{z1 \to -\frac{1}{2},\ z3 \to -\frac{3}{8}\right\}\right\}\right]$$

In[56]:=
```
Barnes1[B2expS[[3]], z3]
```

Out[56]=
$$\text{MBint}\left[-2\, x^{1+z1}\, \text{Gamma}[-1-z1]^2\, \text{Gamma}[-z1]\, \text{Gamma}[1+z1]^3 + \right.$$
$$2\, \text{EulerGamma}\, x^{1+z1}\, \text{Gamma}[-1-z1]^2\, \text{Gamma}[-z1]\, \text{Gamma}[1+z1]^2\, \text{Gamma}[2+z1] +$$
$$\left.2\, x^{1+z1}\, \text{Gamma}[-1-z1]^2\, \text{Gamma}[-z1]\, \text{Gamma}[1+z1]^2\, \text{Gamma}[2+z1]\, \text{PolyGamma}[0, 2+z1],\right.$$
$$\left.\left\{\{ep \to 0\},\ \left\{z1 \to -\frac{1}{2}\right\}\right\}\right]$$

In[57]:=
```
B2expS[[4]]
```

Out[57]=
$$\text{MBint}\left[2\, x^{1+z1}\, \text{Gamma}[-1-z1]\, \text{Gamma}[-z1]\, \text{Gamma}[1+z1]^2\, \text{Gamma}[-1-z1-z2]\right.$$
$$\left.\text{Gamma}[-z2]\, \text{Gamma}[z2]\, \text{Gamma}[2+z1+z2],\ \left\{\{ep \to 0\},\ \left\{z1 \to -\frac{1}{2},\ z2 \to -\frac{5}{16}\right\}\right\}\right]$$

In[58]:=
```
Barnes1[B2expS[[4]], z2]
```

Out[58]=
$$\text{MBint}\left[-2\, x^{1+z1}\, \text{Gamma}[-1-z1]^2\, \text{Gamma}[-z1]\, \text{Gamma}[1+z1]^3 + \right.$$
$$2\, \text{EulerGamma}\, x^{1+z1}\, \text{Gamma}[-1-z1]^2\, \text{Gamma}[-z1]\, \text{Gamma}[1+z1]^2\, \text{Gamma}[2+z1] +$$
$$\left.2\, x^{1+z1}\, \text{Gamma}[-1-z1]^2\, \text{Gamma}[-z1]\, \text{Gamma}[1+z1]^2\, \text{Gamma}[2+z1]\, \text{PolyGamma}[0, 2+z1],\right.$$
$$\left.\left\{\{ep \to 0\},\ \left\{z1 \to -\frac{1}{2}\right\}\right\}\right]$$

In[59]:=
```
B2expS[[5]]
```

Out[59]=
$$\text{MBint}\left[-\frac{1}{2\, ep\, \text{Gamma}[1+z4]}\right.$$
$$\text{Gamma}[1-z4]\, \text{Gamma}[-z4]\, \text{Gamma}[z4]^3\, (1 + 4\, ep\, \text{EulerGamma} + ep\, \text{PolyGamma}[0, 1-z4] +$$
$$\left.4\, ep\, \text{PolyGamma}[0, z4] - ep\, \text{PolyGamma}[0, 1+z4]),\ \left\{\{ep \to 0\},\ \left\{z4 \to \frac{7}{16}\right\}\right\}\right]$$

In[60]:=
```
CoeffEps[B2expS[[5]], -1]
```

Out[60]=
$$\text{MBint}\left[-\frac{\text{Gamma}[1-z4]\, \text{Gamma}[-z4]\, \text{Gamma}[z4]^3}{2\, \text{Gamma}[1+z4]},\ \left\{\{ep \to 0\},\ \left\{z4 \to \frac{7}{16}\right\}\right\}\right]$$

In[61]:=
```
CoeffEps[B2expS[[5]], -1] /. {Gamma[1 - z4] → -z4 Gamma[-z4], Gamma[1 + z4] → z4 Gamma[z4]}
```

Out[61]=
$$\text{MBint}\left[\frac{1}{2}\, \text{Gamma}[-z4]^2\, \text{Gamma}[z4]^2,\ \left\{\{ep \to 0\},\ \left\{z4 \to \frac{7}{16}\right\}\right\}\right]$$

In[62]:=
```
res11 = Barnes1[%, z4][[1]]
```

Out[62]=
$$-\frac{1}{2}\, \text{PolyGamma}[2, 1]$$

In[63]:=
```
CoeffEps[B2expS[[5]], 0]
```

Out[63]=
$$\text{MBint}\left[-\frac{1}{2\,\text{Gamma}[1+z4]}\text{Gamma}[1-z4]\,\text{Gamma}[-z4]\,\text{Gamma}[z4]^3\right.$$
$$(4\,\text{EulerGamma}+\text{PolyGamma}[0,1-z4]+4\,\text{PolyGamma}[0,z4]-\text{PolyGamma}[0,1+z4]),$$
$$\left.\left\{\{\text{ep}\to 0\},\left\{\text{z4}\to\frac{7}{16}\right\}\right\}\right]$$

In[64]:=
```
res03 = -11/2 Zeta[4]
```

Out[64]=
$$-\frac{11\,\pi^4}{180}$$

In[65]:=
```
% // N
```

Out[65]=
$$-5.95278$$

In[66]:=
```
NIntegrate[CoeffEps[B2expS[[5]], 0][[1]] / (2 Pi) /. {z4 -> 7/16 + I * y1},
 {y1, -Infinity, Infinity}]
```

Out[66]=
$$-5.95278+0.\,i$$

In[67]:=
```
B2expS[[6]]
```

Out[67]=
$$\text{MBint}\left[\frac{1}{12\,\text{Gamma}[1+z3]}\text{Gamma}[-z3]^2\,\text{Gamma}[z3]\right.$$
$$\left(24\,\text{Gamma}[z3]^2+\frac{1}{\text{ep}^2}\text{Gamma}[1+z3]^2\,(6+12\,\text{ep}\,\text{EulerGamma}+12\,\text{ep}^2\,\text{EulerGamma}^2+4\,\text{ep}^2\,\pi^2-\right.$$
$$12\,\text{ep}\,\text{Log}[x]-24\,\text{ep}^2\,\text{EulerGamma}\,\text{Log}[x]+12\,\text{ep}^2\,\text{Log}[x]^2+3\,\text{ep}^2\,\text{PolyGamma}[0,-z3]^2+$$
$$6\,\text{ep}\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x])\,\text{PolyGamma}[0,z3]+3\,\text{ep}^2\,\text{PolyGamma}[0,z3]^2+$$
$$6\,\text{ep}\,\text{PolyGamma}[0,-z3]\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x]+\text{ep}\,\text{PolyGamma}[0,z3])+$$
$$\left.3\,\text{ep}^2\,\text{PolyGamma}[1,-z3]-21\,\text{ep}^2\,\text{PolyGamma}[1,z3])\right),\left\{\{\text{ep}\to 0\},\left\{\text{z3}\to-\frac{3}{8}\right\}\right\}\Bigg]$$

In[68]:=
```
B2expS[[7]]
```

Out[68]=
$$\text{MBint}\left[\frac{1}{12\,\text{Gamma}[1+z2]}\text{Gamma}[-z2]^2\,\text{Gamma}[z2]\right.$$
$$\left(24\,\text{Gamma}[z2]^2+\frac{1}{\text{ep}^2}\text{Gamma}[1+z2]^2\,(6+12\,\text{ep}\,\text{EulerGamma}+12\,\text{ep}^2\,\text{EulerGamma}^2+4\,\text{ep}^2\,\pi^2-\right.$$
$$12\,\text{ep}\,\text{Log}[x]-24\,\text{ep}^2\,\text{EulerGamma}\,\text{Log}[x]+12\,\text{ep}^2\,\text{Log}[x]^2+3\,\text{ep}^2\,\text{PolyGamma}[0,-z2]^2+$$
$$6\,\text{ep}\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x])\,\text{PolyGamma}[0,z2]+3\,\text{ep}^2\,\text{PolyGamma}[0,z2]^2+$$
$$6\,\text{ep}\,\text{PolyGamma}[0,-z2]\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x]+\text{ep}\,\text{PolyGamma}[0,z2])+$$
$$\left.3\,\text{ep}^2\,\text{PolyGamma}[1,-z2]-21\,\text{ep}^2\,\text{PolyGamma}[1,z2])\right),\left\{\{\text{ep}\to 0\},\left\{\text{z2}\to-\frac{5}{16}\right\}\right\}\Bigg]$$

In[69]:=
```
F67 =
  MBint[Simplify[(B2expS[[6]][[1]] /. z3 → z2) + B2expS[[7]][[1]]], {{ep → 0}, {z2 → -1/2}}]
```

Out[69]=
$$\text{MBint}\left[\frac{1}{6\,\text{Gamma}[1+z2]}\text{Gamma}[-z2]^2\,\text{Gamma}[z2]\right.$$
$$\left(24\,\text{Gamma}[z2]^2+\frac{1}{\text{ep}^2}\text{Gamma}[1+z2]^2\,\left(6+12\,\text{ep}\,\text{EulerGamma}+12\,\text{ep}^2\,\text{EulerGamma}^2+4\,\text{ep}^2\,\pi^2-\right.\right.$$
$$12\,\text{ep}\,\text{Log}[x]-24\,\text{ep}^2\,\text{EulerGamma}\,\text{Log}[x]+12\,\text{ep}^2\,\text{Log}[x]^2+3\,\text{ep}^2\,\text{PolyGamma}[0,-z2]^2+$$
$$6\,\text{ep}\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x])\,\text{PolyGamma}[0,z2]+3\,\text{ep}^2\,\text{PolyGamma}[0,z2]^2+$$
$$\left.\left.6\,\text{ep}\,\text{PolyGamma}[0,-z2]\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x]+\text{ep}\,\text{PolyGamma}[0,z2])+\right.\right.$$
$$\left.\left.3\,\text{ep}^2\,\text{PolyGamma}[1,-z2]-21\,\text{ep}^2\,\text{PolyGamma}[1,z2]\right)\right),\left\{\{\text{ep}\to 0\},\left\{z2\to -\frac{1}{2}\right\}\right\}\Bigg]$$

In[70]:=
```
res04 = -Pi^2/(6 ep^2) + 1/ep (Pi^2/3 Log[x] + Zeta[3]) + (-Pi^2/3 Log[x]^2 - 2 Zeta[3] Log[x] + Pi^4/9);
```

In[71]:=
```
res04 /. {ep → 0.3, x → 0.1}
```

Out[71]=
$$-40.6045$$

In[72]:=
```
NIntegrate[F67[[1]]/(2 Pi) /. {ep → 0.3, x → 0.1, z2 → -1/2 + I * y1},
 {y1, -Infinity, Infinity}]
```

Out[72]=
$$-40.6045+0.\,i$$

In[73]:=
```
B2expS[[8]]
```

Out[73]=
$$\text{MBint}\left[\frac{1}{\text{ep}}4\,x^{1+z1}\,\text{Gamma}[-1-z1]^2\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^2\right.$$
$$(3\,\text{ep}\,\text{Gamma}[1+z1]+\text{Gamma}[2+z1]\,(-1+4\,\text{ep}\,\text{PolyGamma}[0,-1-z1]-$$
$$\left.2\,\text{ep}\,\text{PolyGamma}[0,1+z1]-2\,\text{ep}\,\text{PolyGamma}[0,2+z1])),\left\{\{\text{ep}\to 0\},\left\{z1\to -\frac{1}{2}\right\}\right\}\right]$$

In[74]:=
```
B2expS[[9]][[1]]
```

Out[74]=
$$-\frac{1}{18\,\text{ep}^4}\left(-72+42\,\text{ep}^2\,\pi^2+20\,\text{ep}^4\,\pi^4+6\,\text{ep}^2\,\left(-6+17\,\text{ep}^2\,\pi^2\right)\,\text{Log}[x]^2-12\,\text{ep}^3\,\text{Log}[x]^3+24\,\text{ep}^4\,\text{Log}[x]^4-\right.$$
$$\left.213\,\text{ep}^3\,\text{PolyGamma}[2,1]+3\,\text{ep}\,\text{Log}[x]\,\left(30-31\,\text{ep}^2\,\pi^2+94\,\text{ep}^3\,\text{PolyGamma}[2,1]\right)\right)$$

```
(* Collecting result for terms with trivial dependence on x  *)
```

In[75]:=
```
restr = Apart[FullSimplify[res01 + res02 + res11/ep + res03 + res04 + B2expS[[9]][[1]]], ep]
```

Out[75]=
$$\frac{4}{\text{ep}^4}-\frac{5\,\text{Log}[x]}{\text{ep}^3}+\frac{-5\,\pi^2+4\,\text{Log}[x]^2}{2\,\text{ep}^2}+\frac{33\,\pi^2\,\text{Log}[x]+4\,\text{Log}[x]^3-130\,\text{Zeta}[3]}{6\,\text{ep}}+$$
$$\frac{1}{30}\left(-29\,\pi^4-180\,\pi^2\,\text{Log}[x]^2-40\,\text{Log}[x]^4+880\,\text{Log}[x]\,\text{Zeta}[3]\right)$$

```
(* Collecting terms with nontrivial dependence on x  *)
```

In[76]:=
```
MBmerge[{Barnes1[B2expS[[3]], z3] , Barnes1[B2expS[[4]], z2]  , B2expS[[8]]}][[1]]
```

Out[76]=
$$\text{MBint}\left[\frac{1}{\text{ep}}4\,x^{1+z1}\,\text{Gamma}[-1-z1]^2\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^2\right.$$
$$(2\,\text{ep}\,\text{Gamma}[1+z1]+\text{Gamma}[2+z1]\,(-1+\text{ep}\,\text{EulerGamma}+4\,\text{ep}\,\text{PolyGamma}[0,-1-z1]-$$
$$\left.2\,\text{ep}\,\text{PolyGamma}[0,1+z1]-\text{ep}\,\text{PolyGamma}[0,2+z1])),\left\{\{\text{ep}\to 0\},\left\{z1\to -\frac{1}{2}\right\}\right\}\right]$$

In[77]:=
```
NIntegrate[%[[1]] / (2 Pi)  /. {ep -> 0.3, x -> 0.1, z1 -> -1/2 + I * y1}, {y1, -Infinity, Infinity} ]
```

Out[77]=
$$-7.68026+0.\,i$$

In[78]:=
$$\text{resnontr}=-\frac{2\,\pi^2\,\text{Log}[1+x]}{\text{ep}}+\frac{10}{3}\,\pi^2\,\text{Log}[x]\,\text{Log}[1+x]-\frac{2\,\text{Log}[x]^2\,\text{Log}[1+x]}{\text{ep}}+\frac{8}{3}\,\text{Log}[x]^3\,\text{Log}[1+x]-$$
$$\pi^2\,\text{Log}[1+x]^2-\text{Log}[x]^2\,\text{Log}[1+x]^2-\frac{20}{3}\,\pi^2\,\text{PolyLog}[2,-x]-\frac{4\,\text{Log}[x]\,\text{PolyLog}[2,-x]}{\text{ep}}-$$
$$2\,\text{Log}[x]^2\,\text{PolyLog}[2,-x]+\frac{4\,\text{PolyLog}[3,-x]}{\text{ep}}+24\,\text{Log}[x]\,\text{PolyLog}[3,-x]-$$
$$4\left(-\frac{1}{2}\,\text{PolyLog}[2,-x]^2-\text{Log}[1+x]\,\text{PolyLog}[3,-x]\right)-44\,\text{PolyLog}[4,-x]+$$
$$4\,\text{Log}[x]\,(-\text{Log}[1+x]\,\text{PolyLog}[2,-x]-2\,\text{PolyLog}[1,2,-x])+4\,\text{Log}[x]\,\text{PolyLog}[1,2,-x]-$$
$$4\left(\frac{1}{2}\,\text{PolyLog}[2,-x]^2-2\,\text{PolyLog}[2,2,-x]\right)-4\,\text{PolyLog}[2,2,-x]-4\,\text{Log}[1+x]\,\text{Zeta}[3];$$

In[79]:=
```
% /. {ep -> 0.3, x -> 0.1}
```

Out[79]=
$$-7.68026-3.59491\times 10^{-15}\,i$$

In[80]:=
```
resnontr + restr;
```

```
Apart[%, ep]
```

$$\frac{4}{ep^4}-\frac{5\,\text{Log}[x]}{ep^3}+\frac{-5\,\pi^2+4\,\text{Log}[x]^2}{2\,ep^2}+$$
$$\frac{1}{6\,ep}\left(33\,\pi^2\,\text{Log}[x]+4\,\text{Log}[x]^3-12\,\pi^2\,\text{Log}[1+x]-12\,\text{Log}[x]^2\,\text{Log}[1+x]-\right.$$
$$\left.24\,\text{Log}[x]\,\text{PolyLog}[2,-x]+24\,\text{PolyLog}[3,-x]-130\,\text{Zeta}[3]\right)+$$
$$\frac{1}{30}\left(-29\,\pi^4-180\,\pi^2\,\text{Log}[x]^2-40\,\text{Log}[x]^4+100\,\pi^2\,\text{Log}[x]\,\text{Log}[1+x]+80\,\text{Log}[x]^3\,\text{Log}[1+x]-\right.$$
$$30\,\pi^2\,\text{Log}[1+x]^2-30\,\text{Log}[x]^2\,\text{Log}[1+x]^2-200\,\pi^2\,\text{PolyLog}[2,-x]-60\,\text{Log}[x]^2\,\text{PolyLog}[2,-x]-$$
$$120\,\text{Log}[x]\,\text{Log}[1+x]\,\text{PolyLog}[2,-x]+720\,\text{Log}[x]\,\text{PolyLog}[3,-x]+$$
$$120\,\text{Log}[1+x]\,\text{PolyLog}[3,-x]-1320\,\text{PolyLog}[4,-x]-120\,\text{Log}[x]\,\text{PolyLog}[1,2,-x]+$$
$$\left.120\,\text{PolyLog}[2,2,-x]+880\,\text{Log}[x]\,\text{Zeta}[3]-120\,\text{Log}[1+x]\,\text{Zeta}[3]\right)$$

```
(*              the result              *)
```

$$\mathbf{1/x}\left(\frac{4}{ep^4}-\frac{5\,\text{Log}[x]}{ep^3}+\frac{-5\,\pi^2+4\,\text{Log}[x]^2}{2\,ep^2}+\right.$$
$$\frac{1}{6\,ep}\left(33\,\pi^2\,\text{Log}[x]+4\,\text{Log}[x]^3-12\,\pi^2\,\text{Log}[1+x]-12\,\text{Log}[x]^2\,\text{Log}[1+x]-\right.$$
$$\left.24\,\text{Log}[x]\,\text{PolyLog}[2,-x]+24\,\text{PolyLog}[3,-x]-130\,\text{Zeta}[3]\right)+$$
$$\frac{1}{30}\left(-29\,\pi^4-180\,\pi^2\,\text{Log}[x]^2-40\,\text{Log}[x]^4+100\,\pi^2\,\text{Log}[x]\,\text{Log}[1+x]+80\,\text{Log}[x]^3\,\text{Log}[1+x]-\right.$$
$$30\,\pi^2\,\text{Log}[1+x]^2-30\,\text{Log}[x]^2\,\text{Log}[1+x]^2-200\,\pi^2\,\text{PolyLog}[2,-x]-60\,\text{Log}[x]^2$$
$$\text{PolyLog}[2,-x]-120\,\text{Log}[x]\,\text{Log}[1+x]\,\text{PolyLog}[2,-x]+720\,\text{Log}[x]\,\text{PolyLog}[3,-x]+$$
$$120\,\text{Log}[1+x]\,\text{PolyLog}[3,-x]-1320\,\text{PolyLog}[4,-x]-120\,\text{Log}[x]\,\text{PolyLog}[1,2,-x]+$$
$$\left.\left.120\,\text{PolyLog}[2,2,-x]+880\,\text{Log}[x]\,\text{Zeta}[3]-120\,\text{Log}[1+x]\,\text{Zeta}[3]\right)\right)$$