Summing up series with nested sums

$$S_i(n) = \sum_{j=1}^{n} \frac{1}{j^i}\ , \quad S_{ik}(n) = \sum_{j=1}^{n} \frac{S_k(j)}{j^i}\ ,$$

$$S_{ikl}(n) = \sum_{j=1}^{n} \frac{S_{kl}(j)}{j^i}\ , \quad S_{iklm}(n) = \sum_{j=1}^{n} \frac{S_{klm}(j)}{j^i}$$

E.g., with one index:

$$\begin{aligned} \psi(n) &= S_1(n-1) - \gamma_{\mathrm{E}}\ , \\ \psi^{(k)}(n) &= (-1)^k k!\,(S_{k+1}(n-1) - \zeta(k+1))\ , \quad k = 1, 2, \ldots\ , \end{aligned}$$

`SUMMER` [J.A.M. Vermaseren'00]

`XSummer` [S. Moch and P. Uwer'00]

Harmonic polylogarithms (HPL)

$H_{a_1,a_2,\ldots,a_n}(x) \equiv H(a_1,a_2,\ldots,a_n;x)$, with $a_i = 1,0,-1$

[E. Remiddi & J.A.M. Vermaseren'00]

are generalizations of the usual polylogarithms $\mathrm{Li}_a(z)$ and Nielsen polylogarithms $S_{a,b}(z)$

$$H(a_1,a_2,\ldots,a_n;x) = \int_0^x f(a_1;t)H(a_2,\ldots,a_n;t)\mathrm{d}t\,,$$

where $f(\pm 1;t) = 1/(1\mp t)$, $f(0;t) = 1/t$,

$$H(\pm 1;x) = \mp\ln(1\mp x),\quad H(0;x) = \ln x\,,$$

with $a_i = 1,0,-1$.

HPL are implemented in `Mathematica` [D. Maitre'06]