```
In[1]:= SetDirectory["c:/diskE/job2008/Zurich"];

In[2]:= << MB/MB.m

MB 1.1

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more info in hep-ph/0511200

last modified 06 Mar 08

In[3]:= SortByDimension[l_List] := Sort[l, Length[#1[[2, 2]]] > Length[#2[[2, 2]]] &];
        CoeffEps[X_, n_] := (X /. X[[1]] → Simplify[Coefficient[X[[1]], ep, n]]);
        MBDimension[int_MBint] := Length[int[[2, 2]]];

        (*   a 4fold MB representation for the on-shell 2loop
          non-planar vertex diagram derived loop by loop;
          sg=-1    *)

In[7]:= V2 = (sg^z4 Gamma[-1 - ep - z1 - z2] Gamma[-z2] Gamma[1 + z1 + z2] Gamma[-1 - ep - z1 - z3]
            Gamma[-z3] Gamma[1 + z1 + z3] Gamma[-1 - 2 ep - z2 - z4] Gamma[-1 - 2 ep - z3 - z4]
            Gamma[-z4] Gamma[2 + 2 ep + z4] Gamma[-z1 + z4] Gamma[2 + ep + z1 + z2 + z3 + z4]) /
          (Gamma[-3 ep] Gamma[-2 ep] Gamma[1 - z2] Gamma[1 - z3]) ;

In[8]:= V2rules = MBoptimizedRules[V2, ep → 0, {}, {ep}]

        MBrules::norules :  no rules could be found to regulate this integral

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        General::stop :  Further output of MBrules::norules will be suppressed during this calculation. ≫
```

Out[8]= $\left\{\left\{ep \to -\frac{5}{8}\right\}, \left\{z1 \to -\frac{1}{4}, z2 \to -\frac{1}{2}, z3 \to -\frac{5}{16}, z4 \to -\frac{1}{8}\right\}\right\}$

```
In[9]:= V2cont = MBcontinue[V2, ep → 0, V2rules];
```

```
Level 1
Taking -residue in z2 = -1 - ep - z1
Taking -residue in z3 = -1 - ep - z1
Taking -residue in z4 = -1 - 2 ep - z2
Taking -residue in z4 = -1 - 2 ep - z3
Level 2
Integral {1}
Taking -residue in z4 = -ep + z1
Integral {2}
Taking -residue in z2 = -1 - ep - z1
Taking -residue in z4 = -ep + z1
Taking -residue in z4 = -1 - 2 ep - z2
Integral {3}
Taking -residue in z2 = -1 - 2 ep - z1
Integral {4}
Taking -residue in z2 = -1 - ep - z1
Taking -residue in z3 = -1 - 2 ep - z1
Level 3
Integral {1, 1}
Integral {2, 1}
Taking -residue in z4 = -ep + z1
Integral {2, 2}
Integral {2, 3}
Taking -residue in z2 = -1 - 2 ep - z1
Integral {3, 1}
Integral {4, 1}
Taking -residue in z3 = -1 - 2 ep - z1
Integral {4, 2}
Taking -residue in z2 = -1 - 2 ep - z1
Level 4
Integral {2, 1, 1}
Integral {2, 3, 1}
Integral {4, 1, 1}
Integral {4, 2, 1}
16 integral(s) found
```

```
      (*    no    1/ep^4 poles???        *)
```

In[10]:= **V2select4 = MBpreselect[MBmerge[V2cont], {ep, 0, -4}]**

Out[10]= {}

In[11]:= **V2select0 = MBpreselect[MBmerge[V2cont], {ep, 0, 0}]**

In[12]:= **V2select0S = Simplify[SortByDimension[V2select0]]**

Out[12]=
$$\Big\{\text{MBint}\Big[\big(\text{sg}^{z4}\,\text{Gamma}[-ep]^2\,\text{Gamma}[1+ep+z1]^2\,\text{Gamma}[-ep+z1-z4]^2\,\text{Gamma}[-z4]\,\text{Gamma}[2+2\,ep+z4]\,\text{Gamma}[-z1+z4]\,\text{Gamma}[-ep-z1+z4]\big)\Big/\big(\text{Gamma}[-3\,ep]\,\text{Gamma}[-2\,ep]\,\text{Gamma}[2+ep+z1]^2\big),\ \Big\{\{ep\to 0\},\ \Big\{z1\to -\frac{1}{4},\ z4\to -\frac{1}{8}\Big\}\Big\}\Big],$$

$$\text{MBint}\Big[\Big(\text{sg}^{-1-2\,ep-z3}\,\text{Gamma}[-1-ep-z1-z3]\,\text{Gamma}[-z3]\,\text{Gamma}[1+z1+z3]\,\big(\text{sg}^{1+ep+z1+z3}\,\text{Gamma}[-ep]^2\,\text{Gamma}[ep-z1]\,\text{Gamma}[1+ep+z1]\,\text{Gamma}[2+ep+z1]\,\text{Gamma}[-1-ep-z1-z3]\,\text{Gamma}[1-ep+z1+z3]+\text{Gamma}[-2\,ep]\,\text{Gamma}[-1-2\,ep-z1-z3]\,\big(\text{sg}^{1+2\,ep+z1+z3}\,\text{Gamma}[ep]\,\text{Gamma}[-z1]\,\text{Gamma}[2+ep+z1]\,\text{Gamma}[1+2\,ep+z1]\,\text{Gamma}[1-ep+z1+z3]+\text{Gamma}[-ep]\,\text{Gamma}[1+ep+z1]\,\text{Gamma}[1-z3]\,\text{Gamma}[1+2\,ep+z3]\,\text{Gamma}[1+ep+z1+z3]\big)\big)\Big)\Big/\big(\text{Gamma}[-3\,ep]\,\text{Gamma}[-2\,ep]\,\text{Gamma}[2+ep+z1]\,\text{Gamma}[1-z3]\big),\ \Big\{\{ep\to 0\},\ \Big\{z1\to -\frac{1}{4},\ z3\to -\frac{5}{16}\Big\}\Big\}\Big],$$

$$\text{MBint}\Big[\Big(\text{sg}^{-1-2\,ep-z2}\,\text{Gamma}[-1-ep-z1-z2]\,\text{Gamma}[-z2]\,\text{Gamma}[1+z1+z2]\,\big(\text{sg}^{1+ep+z1+z2}\,\text{Gamma}[-ep]^2\,\text{Gamma}[ep-z1]\,\text{Gamma}[1+ep+z1]\,\text{Gamma}[2+ep+z1]\,\text{Gamma}[-1-ep-z1-z2]\,\text{Gamma}[1-ep+z1+z2]+\text{Gamma}[-2\,ep]\,\text{Gamma}[-1-2\,ep-z1-z2]\,\big(\text{sg}^{1+2\,ep+z1+z2}\,\text{Gamma}[ep]\,\text{Gamma}[-z1]\,\text{Gamma}[2+ep+z1]\,\text{Gamma}[1+2\,ep+z1]\,\text{Gamma}[1-ep+z1+z2]+\text{Gamma}[-ep]\,\text{Gamma}[1+ep+z1]\,\text{Gamma}[1-z2]\,\text{Gamma}[1+2\,ep+z2]\,\text{Gamma}[1+ep+z1+z2]\big)\big)\Big)\Big/\big(\text{Gamma}[-3\,ep]\,\text{Gamma}[-2\,ep]\,\text{Gamma}[2+ep+z1]\,\text{Gamma}[1-z2]\big),\ \Big\{\{ep\to 0\},\ \Big\{z1\to -\frac{1}{4},\ z2\to -\frac{1}{2}\Big\}\Big\}\Big],$$

$$\text{MBint}\Big[\Big(\text{sg}^{-ep+z1}\,\big(\text{sg}^{ep}\,\text{Gamma}[-3\,ep]\,\text{Gamma}[-2\,ep]\,\text{Gamma}[ep]^2\,\text{Gamma}[-z1]\,\text{Gamma}[2+ep+z1]\,\text{Gamma}[1+2\,ep+z1]^2-\text{Gamma}[-ep]^2\,\text{Gamma}[1+ep+z1]\,\text{Gamma}[2+2\,ep+z1]\,(-2\,\text{sg}^{ep}\,\text{Gamma}[-2\,ep]\,\text{Gamma}[ep]\,\text{Gamma}[-z1]\,\text{Gamma}[1+2\,ep+z1]+\text{Gamma}[-ep]\,\text{Gamma}[ep-z1]\,\text{Gamma}[1+ep+z1]\,(2\,\text{EulerGamma}+\text{Log}[sg]+\text{PolyGamma}[0,-2\,ep]+\text{PolyGamma}[0,-ep]-\text{PolyGamma}[0,ep-z1]+\text{PolyGamma}[0,2+ep+z1]))\big)\Big)\Big/\big(\text{Gamma}[-3\,ep]\,\text{Gamma}[2+ep+z1]\,\text{Gamma}[2+2\,ep+z1]\big),\ \Big\{\{ep\to 0\},\ \Big\{z1\to -\frac{1}{4}\Big\}\Big\}\Big]\Big\}$$

In[13]:= **MBDimension /@ V2select0S**

Out[13]= {2, 2, 2, 1}

**(*    One-dimensional contribution V2select0S[[4]]     *)**

In[14]:=
```
V2select0S[[4]]
```

Out[14]=
```
MBint[(sg^(-ep+z1) (sg^ep Gamma[-3 ep] Gamma[-2 ep] Gamma[ep]^2 Gamma[-z1] Gamma[2 + ep + z1]
      Gamma[1 + 2 ep + z1]^2 - Gamma[-ep]^2 Gamma[1 + ep + z1] Gamma[2 + 2 ep + z1]
      (-2 sg^ep Gamma[-2 ep] Gamma[ep] Gamma[-z1] Gamma[1 + 2 ep + z1] + Gamma[-ep]
        Gamma[ep - z1] Gamma[1 + ep + z1] (2 EulerGamma + Log[sg] + PolyGamma[0, -2 ep] +
          PolyGamma[0, -ep] - PolyGamma[0, ep - z1] + PolyGamma[0, 2 + ep + z1])))) /
   (Gamma[-3 ep] Gamma[2 + ep + z1] Gamma[2 + 2 ep + z1]),
  {{ep -> 0},
   {z1 -> -1/4}}]
```

```
(*          a piece of this:            *)
```

In[15]:=
```
V20 =
  (2 sg^z1 Gamma[-2 ep] Gamma[-ep]^2 Gamma[ep] Gamma[-z1] Gamma[1 + ep + z1] Gamma[1 + 2 ep + z1]) /
   (Gamma[-3 ep] Gamma[2 + ep + z1]);
```

```
(*    no 1/ep^4 poles here???   *)
```

In[16]:=
```
Series[V20 E^(2 EulerGamma ep), {ep, 0, -4}]
```

Out[16]=
$$\frac{1}{O[ep]^3}$$

In[17]:=
```
V20 /. sg^z1 -> E^(I Pi z1)
```

Out[17]=
```
(2 e^(i π z1) Gamma[-2 ep] Gamma[-ep]^2 Gamma[ep] Gamma[-z1] Gamma[1 + ep + z1] Gamma[1 + 2 ep + z1]) /
 (Gamma[-3 ep] Gamma[2 + ep + z1])
```

In[18]:=
```
% /. Gamma[2 + ep + z1] -> Gamma[1 + ep + z1] (1 + ep + z1)
```

Out[18]=
$$\frac{2\, e^{i \pi z1}\, \text{Gamma}[-2\, ep]\, \text{Gamma}[-ep]^2\, \text{Gamma}[ep]\, \text{Gamma}[-z1]\, \text{Gamma}[1+2\, ep+z1]}{(1+ep+z1)\, \text{Gamma}[-3\, ep]}$$

In[19]:=
```
(% /. Gamma[-z1] -> (-1)^n / n!) /. z1 -> n
```

Out[19]=
$$\frac{2\, (-1)^n\, e^{i n \pi}\, \text{Gamma}[-2\, ep]\, \text{Gamma}[-ep]^2\, \text{Gamma}[ep]\, \text{Gamma}[1+2\, ep+n]}{(1+ep+n)\, n!\, \text{Gamma}[-3\, ep]}$$

In[20]:=
```
% /. e^(i n π) -> (-1)^n
```

Out[20]=
$$\frac{2\, (-1)^{2 n}\, \text{Gamma}[-2\, ep]\, \text{Gamma}[-ep]^2\, \text{Gamma}[ep]\, \text{Gamma}[1+2\, ep+n]}{(1+ep+n)\, n!\, \text{Gamma}[-3\, ep]}$$

In[21]:=
```
% /. (-1)^(2 n) -> 1
```

Out[21]=
$$\frac{2\, \text{Gamma}[-2\, ep]\, \text{Gamma}[-ep]^2\, \text{Gamma}[ep]\, \text{Gamma}[1+2\, ep+n]}{(1+ep+n)\, n!\, \text{Gamma}[-3\, ep]}$$

In[22]:=
```
Sum[%, {n, 0, Infinity}]
```

Out[22]=
$$-\frac{2\, \pi\, \text{Csc}[2\, ep\, \pi]\, \text{Gamma}[-2\, ep]\, \text{Gamma}[-ep]^2\, \text{Gamma}[ep]\, \text{Gamma}[1+ep]}{\text{Gamma}[1-ep]\, \text{Gamma}[-3\, ep]}$$

In[23]:=
```
Simplify[Normal[Series[% E^(2 EulerGamma ep), {ep, 0, -4}]]]
```

Out[23]= $-\frac{3}{2\,ep^4}$

```
(*    a singularity in epsilon arises when integrating over large values of z *)

(*    do not use the loop-by-loop strategy of
  deriving MB representations for nonplanar diagrams  *)
```