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QUANTUM NANOTECHNOLOGY

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Nanotechnology is based on manipulations of individual atoms and molecules to build complex atomic structures. Quantum nanotechnology is a broad concept that deals with a manipulation of individual quantum states of atoms and molecules. Quantum nanotechnology differs from nanotechnology as a quantum computer differs from a classical molecular computer. The nanotechnology deals with a manipulation of quantum states in bulk rather than individually. In this paper, we define the main notions of quantum nanotechnology. Quantum analogs of assemblers, replicators and self-reproducing machines are discussed. We prove the possibility of realizing these analogs. A self-cloning (self-reproducing) quantum machine is a quantum machine which can make a copy of itself. The impossibility of ideally cloning an unknown quantum state is one of the basic rules of quantum theory. We prove that quantum machines cannot be self-cloning if they are Hamiltonian. There exist quantum non-Hamiltonian machines that are self-cloning machines. Quantum nanotechnology allows us to build quantum nanomachines. These nanomachines are not only small machines of nanosize. Quantum nanomachines should use new (quantum) principles of work.

Keywords: Nanomachines; nanotechnology; assemblers; replicators; self-reproducing machines; quantum cloning; self-reproducing quantum nanomachine.

1. Introduction

Nanotechnology was first recognized after Richard Feynman presented his talk titled “There’s Plenty of Room at the Bottom” to the American Physical Society in 1959, when discussing the changing nature of the field of physics: “The principles of physics, as far as I can see, do not speak against the possibility of maneuvering things atom by atom”. In 1988, Eric Drexler taught the first course in nanotechnology while a visiting scholar at Stanford University. He suggested[1] the possibility of nanosized objects that were self-replicating nanomachines.

Nanotechnology deals with a manipulation of individual atoms and molecules to build complex atomic structures. We think that the development of nanotechnology allows us to realize manipulations of individual quantum states of atoms and molecules. The technology for manipulating individual quantum states can be called “quantum nanotechnology”. Note that quantum nanotechnology differs from nanotechnology as a quantum computer differs from a classical molecular computer. Nanotechnology deals with a manipulation of atoms and molecules, which are definitely quantum machines. The technology is realized by the molecules not in their quantum states. This technology is classical, because one cannot find a superposition of “being hydrogen” and “being carbon”. The nanotechnology is based on manipulations of quantum states in bulk rather than individually. Manipulations of individual quantum states give us new possibilities.

In this paper, we prove the possibility of building a quantum nanotechnology. Quantum analogs of assemblers, replicators and self-reproducing machines are discussed. The following quantum machines are considered in this paper:

(a) A self-cloning (self-reproducing) quantum machine is a quantum machine which can make a copy of itself.
(b) A quantum replicator is a quantum machine which can realize a sequence of self-cloning (self-reproducing) quantum operations.
(c) A quantum assembler is defined as a quantum machine which can be used to build a quantum state structure from given states. Quantum assemblers can be considered as quantum factories.

In this paper, we concentrate on a model of self-reproducing quantum machines. A theory of self-reproducing classical automata has been suggested by von Neumann.[2] With regard to kinematic self-replicating classical machines, see Ref. 3. Theoretical approaches to the problem of self-reproduction of molecular machines were attempted by considering the origin of life as an organization of molecules through some catalytic actions, on the analogy of physical interaction. Such pioneering work was done by Eigen, Schuster, and Dyson.[4–7] The discovery of the polymerase activity of the self-splicing ribosomal RNA (ribonucleic acid) intervening sequence of *Tetrahymena thermophila* told us that life started from self-replicative RNA sequences called replicases.[8,9] Although the replicase is a hypothetical RNA molecule, the presence of both the "information" and "function" of self-replication in the same RNA molecule simplifies the problems of self-reproduction of molecular machines.

The well-known self-cloning processes are realized in nature by self-reproducing molecular machines. Information is encoded by molecules, which are definitely quantum machines, but it is encoded in the nature of the molecules not in their quantum state. Such an encoding is classical, because one cannot find a molecule that is a superposition of "being hydrogen" and "being carbon". If information is represented by sequences of molecules, it can be self-reproducing and this process can be called self-cloning.

Wigner was probably the first, to consider the problem of self-reproduction within the quantum formalism.[10] The impossibility of ideally copying (or cloning) an unknown quantum state is one of the basic rules of quantum mechanics.[11] The no-cloning theorem of Wootters and Zurek[12] says that there is no quantum copy machine which can copy any unknown quantum pure state. They have proven the no-cloning theorem for pure states and for just unitary transformations. The result of the Wootters–Zurek no-cloning theorem has been extended to mixed states by Barnum, Caves, Fuchs, Jozsa, and Schumacher.[14] The theorem for mixed states[14] proves that quantum machines that realize broadcasting of two noncommuting mixed states are impossible. The no-cloning theorem tells us that cloning quantum machines cannot work ideally. There is a problem of how well they can copy quantum states, i.e., how close the copy state can be to the original state. This problem was solved by Buzek and Hillery in Ref. 13, where an approximate cloning system was presented. They suggested[13] a quantum cloning machine which is input-state-independent (universal). The probabilistic cloning quantum machine was proposed by Duan and Guo.[15,16] Note that a quantum model of a replicator dynamics of populations[6] by using the game theory for evolution of mixed states of a quantum system is considered in Ref. 17. A population is represented by a quantum system in which each subpopulation is represented by a pure state with some probability.

In some sense, a self-cloning quantum machine is a quantum analog of a simple mathematical model of ribosomes, i.e., molecular machines which build proteins molecules according to the instruction (program) read from DNA molecules. In this paper, we concentrate on a model of self-reproducing quantum machines, when information is encoded in states ρ of the quantum machines. Each self-cloning quantum machine is defined by a state ρ of the machine and a transformation (quantum operation) such that $\rho \otimes \rho' \to \rho \otimes \rho$, where ρ' is an unknown state and ρ is a "proper" state of the machine. The proper state is a state of the machine. In our laboratory we do not know this quantum state. Therefore this state cannot be cloning by an external copying device. An ideally copying device cannot be constructed for the state that is unknown to us. This is the no-cloning theorem. Note that this theorem cannot describe the possibility of self-cloning of this state. A quantum self-cloning process is a copying of the quantum state of a machine by the machine instead of a copying by an external device. In order to ideally copy this state by our device, the state must be known to us. We prove that there exist quantum machines that can be self-reproducing machines, replicators and assemblers. These quantum machines are non-Hamiltonian.

In Sec. 2, a brief review of quantum states and operations of quantum machines is made to fix notations and provide a convenient reference. In Sec. 3, a model of self-cloning quantum machines is suggested. In Sec. 4, a quantum replicator is considered as a quantum machine that realizes a sequence of self-reproducing quantum operations. In Sec. 5, quantum assemblers, quantum disassemblers, and quantum viruses are discussed. Finally, a short conclusion is given in Sec. 6.

2. States and Operations of Quantum Machines

Let us give a brief review of quantum states and operations of quantum machines to fix notations and provide a convenient reference (see for example Refs. 18 and 19).

In general, states of quantum machines are described by density operators. A density operator is a self-adjoint ($\rho^* = \rho$), nonnegative ($\rho \geq 0$) operator with unit trace ($\mathrm{Tr}\,\rho = 1$). Pure states can be characterized by the condition $\rho^2 = \rho$.

The state $|\rho(t))$ of an n-qubit machine can be represented by

$$|\rho(t)) = \sum_{\mu=0}^{N-1} |\mu)\rho_\mu(t), \tag{1}$$

where $N = 4^n$ and $\rho_\mu(t) = (\mu|\rho(t))$ are real-valued functions. Here, we denote an element A of a Liouville space by a ket vector $|A)$. The inner product of two elements $|A)$ and $|B)$ of the Liouville space is defined as $(A|B) = \mathrm{Tr}[A^*B]$. Regarding the concepts of Liouville space and superoperators, see for example Refs. 18–21. The basis for a Liouville space $\overline{\mathcal{H}}^{(n)}$ of an n-qubit machine is defined by

$$\begin{aligned}|\mu) = |\mu_1, \ldots, \mu_n) &= \frac{1}{\sqrt{2^n}}|\sigma_\mu) \\ &= \frac{1}{\sqrt{2^n}}|\sigma_{\mu_1} \otimes \cdots \otimes \sigma_{\mu_n}),\end{aligned} \tag{2}$$

where we use μ in the representation $\mu = \mu_1 4^{n-1} + \cdots + \mu_{n-1}4 + \mu_n$, $\mu_i \in \{0, 1, 2, 3\}$, $\otimes$ is a tensor product, and $(\mu|\mu') = \delta_{\mu\mu'}$. Here σ_{μ_k} are Pauli matrices. The element $|\mu)$ is called the generalized computational basis.[18,19]

Using the fact that $\rho(t)$ is a self-adjoint, nonnegative operator of unit trace, we obtain

$$\begin{aligned}\rho_0(t) = (0|\rho(t)) = \frac{1}{\sqrt{2^n}}\mathrm{Tr}[\rho(t)] = \frac{1}{\sqrt{2^n}}, \\ \frac{1}{2^n} \leq \sum_{\mu=0}^{N-1} \rho_\mu^2(t) \leq 1,\end{aligned} \tag{3}$$

and $\rho_\mu^*(t) = \rho_\mu(t)$.

The most general change of quantum state is a quantum operation (see for example Refs. 18–21). A quantum operation is a map (superoperator) of a set of density operators. A quantum operation is a transformation $\hat{\mathcal{E}}$ that maps a density operator $|\rho)$ of a quantum machine into a density operator $|\rho')$ of the machine. If $|\rho)$ is a density operator, then $\hat{\mathcal{E}}|\rho)$ should also be a density operator. Therefore we have the following requirements for $\hat{\mathcal{E}}$. A general quantum operation is a real positive (or completely positive) trace-preserving superoperator $\hat{\mathcal{E}}$ on a Liouville space $\overline{\mathcal{H}}^{(n)}$. A linear quantum operation $\hat{\mathcal{E}}$ can be represented[18,19] by the equation

$$\hat{\mathcal{E}} = \sum_{\mu=0}^{N-1}\sum_{\nu=0}^{N-1} \mathcal{E}_{\mu\nu}\, |\mu)(\nu|, \tag{4}$$

where $N = 4^n$. The matrix $\mathcal{E}_{\mu\nu}$ has the form

$$\mathcal{E}_{\mu\nu} = \left(\frac{1}{2^n}\right) \mathrm{Tr}[\sigma_\mu \hat{\mathcal{E}}(\sigma_\nu)],$$

where $\sigma_\mu = \sigma_{\mu_1} \otimes \cdots \otimes \sigma_{\mu_n}$. As a result, quantum states and quantum operations can be described by matrices ρ_μ and $\mathcal{E}_{\mu\nu}$.

3. Self-Cloning Quantum Machines

A self-cloning (self-reproducing) machine is considered as a system which can make a copy of itself.[1] Let us define self-cloning quantum machines.[10]

Definition. A self-cloning quantum machine (SCQM) is the pair

$$\mathrm{SCQM} = \{\hat{\mathcal{E}}_\rho; |\rho)\},$$

where the quantum operation $\hat{\mathcal{E}}_\rho$ transforms the input unknown data $|\rho')$ according to some given program $|\rho)$ such that

$$\hat{\mathcal{E}}_\rho|\rho) \otimes |\rho') = |\rho) \otimes |\rho). \tag{5}$$

In general, $|\rho)$ can be a pure or mixed state. Let us define the following two types of quantum machines.

Definition. A quantum machine is Hamiltonian if the quantum operation $\hat{\mathcal{E}}$ can be represented in the form $\hat{\mathcal{E}}\rho = U^*\rho U$, where $U^*U = UU^* = I$ for all states ρ of the machine, otherwise it is said to be non-Hamiltonian.

A self-cloning quantum machine is defined by a state $|\rho)$ of the machine and a transformation (quantum operation) $\hat{\mathcal{E}}$ such that $\hat{\mathcal{E}}|\rho) \otimes |\rho') = |\rho) \otimes |\rho)$, where $|\rho')$ is an unknown state, and $|\rho)$ is a "proper" state of the machine. The proper state is a fixed state of the machine. In our laboratory we do not know this quantum state. Therefore this state cannot be cloning by the external copying machines. The ideally copying device cannot be constructed for the state that is unknown to us.[11,12] This is the no-cloning theorem. Note that this theorem cannot describe the possibility of self-cloning of this state. A quantum self-cloning process is a copying of a quantum state of a machine by the machine itself instead of a copying by an external device. In order to ideally copy this state by our device, the state must be known to us. It is not hard to prove that no Hamiltonian self-cloning quantum machines exist.

Theorem. *There exist non-Hamiltonian quantum machines which can make copies of themselves.*

This statement means that the transformation (5), where ρ' is an unknown quantum state, can be realized by non-Hamiltonian quantum machines. The ideally self-reproducing non-Hamiltonian quantum machine can be constructed. This theorem states that self-reproducing transformations exist. To prove this theorem, a self-cloning quantum operation will be presented.

Let ρ be a state of a quantum machine. The transformation (5), where ρ' is an unknown quantum state, can be realized if the self-reproducing quantum operation $\hat{\mathcal{E}}_\rho$ of the quantum machine $\mathrm{SCQM} = \{\hat{\mathcal{E}}_\rho, \rho\}$ is defined by

$$\hat{\mathcal{E}}_\rho = \hat{I}^{(n)} \otimes \hat{R}^{(n)}, \tag{6}$$

where

$$\hat{R}^{(n)} = \sqrt{2^n}|\rho)(0|, \quad \hat{I}^{(n)} = \sum_{\mu=0}^{4^n-1} |\mu)(\mu|.$$

Note that the matrix $\mathcal{E}_{\mu\nu}$ is a tensor product of the matrices $R_{\mu\nu} = \sqrt{2^n}\rho_\mu \delta_{\nu 0}$ and $I_{\mu\nu} = \delta_{\mu\nu}$. Here we can assume that a basis to which ρ belongs is known and $\rho, \rho' \in \overline{\mathcal{H}}^{(n)}$, where $\dim(\overline{\mathcal{H}}^{(n)}) = 4^n$, i.e. the representation (1) is used. Note that the quantum operation $\hat{R}^{(n)}$ gives $\hat{R}^{(n)}|\rho') = |\rho)$. This is the self-cloning quantum machine that makes a copy of itself, not copying any changes in time.

Let us consider a self-cloning quantum machine that can give itself a copy including any changes in time. This type of self-cloning quantum machine is defined by a state $\rho(t)$ of the machine at $t > 0$ and the time-dependent quantum operation $\hat{\mathcal{E}}_\rho(t, t')$ such that

$$\hat{\mathcal{E}}_\rho(t, t')|\rho(t')) \otimes |\rho'(t')) = |\rho(t' + \Delta t)) \otimes |\rho(t')), \tag{7}$$

where ρ' is an unknown state, and $\Delta t = t - t'$ is a time of cloning. Here t' is an instant of the beginning of cloning, and t is an instant of finishing of quantum cloning. The original state [source $\rho(t')$] of the machine is changed during the time of copying.

The process (7) can be realized by the self-cloning quantum machine

$$\mathrm{SCQM}_t = \{\hat{\mathcal{E}}_\rho(t, t'), \rho\},$$

where the self-reproducing quantum operation is

$$\hat{\mathcal{E}}_\rho(t, t') = \hat{S}^{(n)}(t, t') \otimes \hat{R}^{(n)}(t, t').$$

Here

$$\hat{R}^{(n)}(t, t') = \sqrt{2^n}|\rho(t'))(0|,$$

and $\hat{S}^{(n)}(t, t')$ is a quantum operation that describes the change of a quantum state of the quantum machine such that

$$\hat{S}^{(n)}(t, t')|\rho(t')) = |\rho(t)).$$

The state $\rho(t')$ on the right hand side of Eq. (7) can be considered as the next "young" generation of the state $\rho(t)$.

4. Quantum Replicator

In the terminology of Dawkins,[22] machines that give rise to copies of themselves are called replicators. In this environment, RNA molecules qualify: a single molecule soon becomes 2, then 4, 8, 16, 32, and so forth, multiplying exponentially. Drexler defined a replicator as a nanomachine which can get itself copied,[1] including any changes it may have undergone. In a broader sense, a replicator is a machine which can make a copy of itself, not necessarily copying any changes it may have undergone.

We consider a quantum replicator as a quantum machine that realizes a sequence of self-reproducing quantum operations.

Definition. A quantum replicator (QR) is the pair

$$\mathrm{QR} = \{\hat{\mathcal{E}}_\rho(t_N, t_0);\ \rho(t_0)\},$$

where the quantum operation

$$\hat{\mathcal{E}}_\rho(t_N, t_0) = \hat{\mathcal{E}}_N(t_N, t_{N-1}) \circ \hat{\mathcal{E}}_{N-1}(t_{N-1}, t_{N-2}) \circ \cdots \circ \hat{\mathcal{E}}_1(t_1, t_0) \quad (8)$$

transforms the input unknown data $|\rho')$ according to a given program, $\rho_0(t_0) = \rho(t_0)$. The operators $\hat{\mathcal{E}}_k(t_k, t_{k-1})$ are defined by

$$\hat{\mathcal{E}}_k(t_k, t_{k-1})|\rho_{k-1}(t_{k-1})) \otimes |\rho'_{k-1}(t_{k-1})) = |\rho_{k-1}(t_k)) \otimes |\rho_0^{(k)}(t_0)),$$

where we use the notation

$$|\rho_k(t_k)) = |\rho_{k-1}(t_k)) \otimes |\rho_0^{(k-1)}(t_{k-1})),$$
$$|\rho_0^{(k)}(t)) = \otimes^{2^k} \rho_0(t) \quad (k = 0, \ldots, N).$$

Let us consider the quantum replicator with $t_N = NT$, where T is a time of creation of the duplicate, and N is a positive integer number (the generation number).

A set of self-reproducing quantum operations $\hat{\mathcal{E}}_\rho(t_k, t_{k-1})$ can be called a quantum life. These operations are defined by

$$\hat{\mathcal{E}}_1(t_1, t_0) = \hat{S}_T \otimes \hat{R},$$
$$\hat{\mathcal{E}}_2(t_2, t_1) = \hat{S}_T \otimes \hat{S}_T \otimes \hat{R} \otimes \hat{R},$$
$$\hat{\mathcal{E}}_3(t_3, t_2) = \hat{S}_T \otimes \hat{S}_T \otimes \hat{S}_T \otimes \hat{S}_T \otimes \hat{R} \otimes \hat{R} \otimes \hat{R} \otimes \hat{R}, \quad (9)$$

and so on. The self-reproducing operation for the $k+1$ generation is

$$\hat{\mathcal{E}}_{k+1}(t_{k+1}, t_k) = \left(\otimes^{2^k}\ \hat{S}_T\right) \otimes \left(\otimes^{2^k}\ \hat{R}\right).$$

Here $\hat{S}_T$ is a quantum operation that describes the change of a quantum state of the quantum machine such that

$$\hat{S}_T|\rho(t)) = |\rho(t+T)).$$

As a special case, we can consider the identity quantum operation $\hat{S}_T = \hat{I}^{(n)}$ for $|\rho_0) \in \overline{\mathcal{H}}^{(n)}$.

If $|\rho(t)) \in \overline{\mathcal{H}}^{(n)}$, then $\hat{\mathcal{E}}_k(t_k, t_{k-1})$ is a superoperator on $\overline{\mathcal{H}}^{(M(k))}$, where $M(k) = n^{2^k}$. In the definition, we use the composition $\circ$, which means that

$$\hat{\mathcal{E}}_{k+1} \circ \hat{\mathcal{E}}_k = \hat{\mathcal{E}}_{k+1}(\hat{\mathcal{E}}_k \otimes \hat{I}_k),$$

where $\hat{I}_k$ is an identity superoperator on $\overline{\mathcal{H}}^{(M(k))}$, i.e.,

$$\hat{I}_k = \otimes^{2^k} \hat{I}^{(n)}.$$

For example, we use

$$\hat{\mathcal{E}}_2(t_2, t_1) \circ \hat{\mathcal{E}}_1(t_1, t_0) = \hat{\mathcal{E}}_2(t_2, t_1)\ (\hat{\mathcal{E}}_1(t_1, t_0) \otimes \hat{I}^{(n)} \otimes \hat{I}^{(n)}).$$

Substitution of Eq. (9) into this equation gives

$$\begin{aligned} \hat{\mathcal{E}}_2(t_2, t_1) \circ \hat{\mathcal{E}}_1(t_1, t_0) &= (\hat{S}_T \otimes \hat{S}_T \otimes \hat{R} \otimes \hat{R})\,(\hat{S}_T \otimes \hat{R} \otimes \hat{I}^{(n)} \otimes \hat{I}^{(n)}) \\ &= \hat{S}_T\hat{S}_T \otimes \hat{S}_T\hat{R} \otimes \hat{R}\hat{I}^{(n)} \otimes \hat{R}\hat{I}^{(n)} \\ &= \hat{S}_T^2 \otimes \hat{S}_T\hat{R} \otimes \hat{R} \otimes \hat{R}. \end{aligned}$$

As a result, we obtain

$$\hat{\mathcal{E}}_\rho(t_2, t_0) = \hat{S}_T^2 \otimes \hat{S}_T\hat{R} \otimes \hat{R} \otimes \hat{R}.$$

Using Eq. (8) and the definition of multiplication $\circ$, we obtain

$$\hat{\mathcal{E}}_\rho(t_1, t_0) = \hat{S}_T \otimes \hat{R},$$
$$\hat{\mathcal{E}}_\rho(t_2, t_0) = \hat{S}_T^2 \otimes \hat{S}_T\hat{R} \otimes \hat{R} \otimes \hat{R},$$
$$\hat{\mathcal{E}}_\rho(t_3, t_0) = \hat{S}_T^3 \otimes \hat{S}_T^2\hat{R} \otimes \hat{S}_T\hat{R} \otimes \hat{S}_T\hat{R} \otimes \hat{R} \otimes \hat{R} \otimes \hat{R} \otimes \hat{R},$$

and so on. The self-reproducing operation for quantum replication is

$$\hat{\mathcal{E}}_\rho(t_{N+1}, t_0) = \hat{S}_T^{N+1} \otimes \left(\otimes^2\ \hat{S}_T^N\hat{R}\right) \otimes \cdots \otimes \left(\otimes^{2^N}\ \hat{R}\right).$$

As a result, we have

$$\hat{\mathcal{E}}_\rho(t_{N+1}, t_0) = \otimes_{k=0}^{N+1}\left\{\otimes^{2^k}\ \hat{S}_T^{N-k}\hat{R}\right\}.$$

Here we use the notation

$$\otimes_{k=0}^{N+1} A_k = A_0 \otimes A_1 \otimes \cdots \otimes A_{N+1},$$

and $\otimes^{2^k} S$ means the tensor product 2^k times.

Self-reproduction is a fundamental feature of all known life. A life can be considered as phenomena which are non-Hamiltonian (open) systems able to get themselves copied. A quantum life can be considered as an evolution of non-Hamiltonian quantum machines that can be considered as self-cloning machines. Quantum self-cloning processes can be considered as an analog of some reproduction that is a process by which a machine creates an identical copy of itself without a contribution of information from other states. Note that quantum machines that reproduce through this reproduction tend to grow in number exponentially.

Wigner found[10] that quantum mechanics leads to a practically zero probability for the existence of self-cloning machines. If we assume the possibility of the existence of one quantum replicator, then we obtain an exponential growth in the number of these machines. As a result, the probability for the existence of self-reproducing machines grows exponentially also.

5. Quantum Assembler, Quantum Disassembler and Quantum Virus

5.1. *Quantum assembler*

An assembler[1] is a molecular nanomachine that can be programmed to build a molecular structure or device from simpler chemical building blocks. We can define a quantum assembler as a quantum machine that can clone a fixed state or a sequence of states.

Definition. A quantum assembler (QA) is the pair

$$\mathrm{QA} = \{\hat{\mathcal{E}}_{\rho_f};\, |\rho);\ |\rho_f)\},$$

where $|\rho) \in \overline{\mathcal{H}}^{(n)}$ is a state of the quantum machine and $|\rho_f) \in \overline{\mathcal{H}}^{(m)}$ is a fixed state to be copied. A QA can be described as a quantum operation $\hat{\mathcal{E}}_{\rho_f}$ that transforms the input unknown data $|\rho') \in \overline{\mathcal{H}}^{(m)}$ according to a given program $|\rho_f)$:

$$\hat{\mathcal{E}}_{\rho_f}|\rho) \otimes |\rho') = |\rho) \otimes |\rho_f).$$

No quantum Hamiltonian assembler exists. A QA is a non-Hamiltonian quantum machine. It is represented by the quantum operation

$$\hat{\mathcal{E}}_{\rho_f} = \hat{I}^{(n)} \otimes \hat{A}^{(m)}, \tag{10}$$

where

$$\hat{A}^{(m)} = \sqrt{2^n}|\rho_f)(0|,$$

and $|0) \in \overline{\mathcal{H}}^{(m)}$.

In general, the fixed state ρ_f can be a sequence of m quantum states:

$$|\rho_f) = \otimes_{k=1}^{s}|\rho_{f_k}) = |\rho_{f_1}) \otimes |\rho_{f_2}) \otimes \cdots \otimes |\rho_{f_m}).$$

We can consider a QA such that $m = 2$ and $\rho_{f_1} = \rho_{f_2}$. This assembler can be called a quantum cloning assembler.

Definition. A quantum cloning assembler (QCA) is the pair

$$\mathrm{QCA} = \{\hat{\mathcal{E}}_{\rho_f};\, |\rho);\, |\rho_f)\},$$

where $|\rho) \in \overline{\mathcal{H}}^{(n)}$ is a state of the quantum machine and $|\rho_f) \in \overline{\mathcal{H}}^{(m)}$ is a fixed state to be copied. A QCA is described as a quantum operation $\hat{\mathcal{E}}_{\rho_f}$ that transforms the input unknown data $|\rho') \in \overline{\mathcal{H}}^{(m)} \otimes \overline{\mathcal{H}}^{(m)}$ according to a given program $|\rho_f)$ such that

$$\hat{\mathcal{E}}_{\rho_f}|\rho) \otimes |\rho') = |\rho) \otimes |\rho_f) \otimes |\rho_f).$$

There exists a non-Hamiltonian QA that ideally duplicates a fixed state ρ_f. A QA as a non-Hamiltonian quantum machine is represented by the quantum operation

$$\hat{\mathcal{E}}_{\rho_f} = \hat{I}^{(n)} \otimes \hat{A}^{(m^2)}, \tag{11}$$

where

$$\hat{A}^{(m^2)} = \sqrt{2^{m^2}}\,|\rho_f \otimes \rho_f)(0|,$$

and $|0) \in \overline{\mathcal{H}}^{(m)} \otimes \overline{\mathcal{H}}^{(m)}$. Note that the operation $\hat{A}^{(m^2)}$ gives

$$\hat{A}^{(m^2)}|\rho') = |\rho_f \otimes \rho_f).$$

As a result, the quantum operation (11) defines a QCA.

5.2. *Quantum disassembler*

A disassembler[1] is a system of nanomachines able to take an object apart a few atoms at a time, while recording its structure at the molecular level. We can define a quantum analog of a disassembler in the following way:

Definition. A quantum disassembler (QDA) is a quantum machine able to take an unknown sequence,

$$|\rho_{k_1}) \otimes |\rho_{k_2}) \otimes \cdots \otimes |\rho_{k_m}), \tag{12}$$

of known quantum states from a set $\{\rho_k : k = 1, \ldots, M\}$ and then record (represent) its structure [the sequence $(k_1, k_2, \ldots, k_m)$] by the generalized computational states $|k]$:

$$\begin{aligned} |0] &= |0)\frac{1}{\sqrt{2^n}}, \\ |k] &= |0)\frac{1}{\sqrt{2^n}} + |\mu)C_k \quad (k = 1, \ldots, 4^n - 1), \end{aligned} \tag{13}$$

where

$$0 < C_k \leq \sqrt{1 - \frac{1}{2^n}}. \tag{14}$$

A QDA is the pair

$$\text{QDA} = \{\hat{\mathcal{E}};\, \hat{\mathcal{E}}_c;\, |\rho_c);\, |\rho_k);\, k = 1, \ldots, M\},$$

where $|\rho_k) \in \overline{\mathcal{H}}^{(n_k)}$ are states of the quantum machine. A QDA consists of a quantum operation $\hat{\mathcal{E}}$ which transforms the input unknown sequence (12) of known states into the sequence of states $|k]$:

$$\hat{\mathcal{E}}\,|\rho_{k_1}) \otimes |\rho_{k_2}) \otimes \cdots \otimes |\rho_{k_m}) = |k_1] \otimes |k_2] \otimes \cdots \otimes |k_m].$$

The main part of a QDA should be a quantum computer $\{\hat{\mathcal{E}}_c; |\rho_c)\}$ which can realize a quantum computation by quantum operation on mixed states.[19] This computer allows one to identify a state $|\rho_{k_i})$ from a set of known states, i.e. it allows one to define the number k_i. Note that a quantum algorithm of this identification is an open question at this moment.

5.3. *Quantum virus*

The suggested theorem about self-cloning quantum machines means that there exist quantum analogs of RNA. Another possible corollary of the theorem is the existence of a quantum analog of viruses. A virus is a molecular machine that is unable to grow or reproduce outside a big molecule that is a "host cell". We can assume the existence of a quantum analog of the virus. A quantum virus is a self-reproducing quantum machine that is unable to grow or reproduce outside a big quantum system.

A viral quantum machine, or quantum virion, consists of "genetic material", the quantum replicator (analog of RNA), within a quantum protective coat that can be called a quantum capsid. The shape of a quantum capsid can be varied from simple forms to more complex structures with an envelope. Functionally, quantum viral envelopes are used to help quantum viruses enter big quantum systems (quantum host cells).

The no-cloning theorem has a direct application to secret communications and quantum cryptography. A striking feature of quantum mechanics represented in the no-cloning theorem is that one cannot freely and ideally read out information of a system without affecting the state of the system. It is known that the information can be approximate and probability copying.[13,15,16] We assume that a quantum machine (such as a quantum computer) with a quantum virus can ideally copy this information (by the self-cloning operation) without affecting itself.

6. Conclusion

Quantum nanotechnology allows us to build quantum nanomachines. Quantum nanomachines cannot be considered only as molecular nanomachines,[23–27] just as quantum computers are not only molecular computers. Quantum nanomachines are not only machines of nanosize. They should use new (quantum) principles of work. A quantum computer is an example of a quantum machine for computations. Quantum replicators and quantum assemblers, which are considered in this paper, are other examples of quantum machines. Quantum machines can be used for creation of quantum states, and complex structures of quantum states. For example, they can be used for self-cloning of quantum states. Quantum cloning machines can create (by self-cloning operations) superconducting states of molecular nanowires,[28–30] superfluiding states[31–33] of nanomachines motion, or superradiance states[34–37] of nanomachines that are molecular nanoantennas.

References

1. K. E. Drexler, *Engines of Creation*: *The Coming Era of Nanotechnology* (Oxford University Press, 1990), see also http://www.e-drexler.com/d/06/00/EOC/EOC_Table_of_Contents.html.
2. J. von Neumann, *Theory of Self-Reproducing Automata Source* (University of Illinois, 1966).
3. R. A. Freitas, Jr. and R. C. Merkle, *Kinematic Self-Replicating Machines* (Landes Bioscience, 2004), see also http://www.molecularassembler.com/KSRM.htm.
4. M. Eigen, *Die Naturwissenschaften* **47**, 3465 (1971).
5. M. Eigen and P. Schuster, *Die Naturwissenschaften* **64**, 541 (1977).
6. M. Eigen and P. Schuster, *The Hypercycle. A Principle of Natural Self-Organization* (Springer, Berlin, New York, 1979).
7. F. J. Dyson, *J. Mol. Evol.* **18**, 344 (1982).
8. T. R. Cech, *Proc. Natl. Acad. Sci. USA* **83**, 4360 (1986).
9. J. D. Watson, N. H. Hopkins, J. W. Roberts, J. A. Steitz and A. M. Weiner, *Molecular Biology of the Gene*, 3rd edn., Vol. II (Benjamin Cummings, California, 1987), pp. 1103–1124.
10. E. P. Wigner, *The Logic of Personal Knowledge*: *Essays Presented to Michael Polanyi* (Routledge and Paul, London, 1961), pp. 231–238; also *Symmetries and Reflections* (Indian University Press, Bloomington, London, 1970).
11. V. Scarani, S. Iblisdir and N. Gisin, *Rev. Mod. Phys.* **77**, 1225 (2005).

12. W. K. Wootters and W. H. Zurek, *Nature* (*London*) **299**, 802 (1982).
13. V. Buzek and M. Hillery, *Phys. Rev. A* **54**, 1844 (1996).
14. H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa and B. Schumacher, *Phys. Rev. Lett.* **76**, 2818 (1996) [arXiv:quant-ph/9511010].
15. L. M. Duan and G. C. Guo, *Phys. Lett. A* **243**, 261 (1998).
16. L. M. Duan and G. C. Guo, *Phys. Rev. Lett.* **80**, 4999 (1998) [arXiv:quant-ph/9804064].
17. E. G. Hidalgo, *Physica A* **369**, 393 (2006) [arXiv: quant-ph/0510238].
18. V. E. Tarasov, *Quantum Mechanics of Non-Hamiltonian and Dissipative Systems* (Elsevier, Amsterdam, Oxford, 2008).
19. V. E. Tarasov, *J. Phys. A* **35**, 5207 (2002) [arXiv: quant-ph/0312131].
20. V. E. Tarasov, *Phys. Rev. E* **66**, 056116 (2002) [arXiv:quant-ph/0311177].
21. V. E. Tarasov, *J. Phys. A* **37**, 3241 (2004) [arXiv: 0706.2142].
22. R. Dawkins, *The Selfish Gene* (Oxford University Press, 1976).
23. K. E. Drexler, *Ann. Rev. Biophys. Biomol. Struct.* **23**, 377 (1994).
24. G. A. Ozin, I. Manners, S. Fournier-Bidoz and A. Arsenault, *Adv. Mater.* **17**, 3011 (2005).
25. *Molecular Nanomachines*, special volume of *J. Phys. Condens. Matter* **18**, 1777 (2006), see also http://www.iop.org/EJ/toc/0953-8984/18/33.
26. Y. E. Lozovik, A. Minogin and A. M. Popov, *Phys. Lett. A* **313**, 112 (2003).
27. M. Blencowe, *Phys. Rep.* **395**, 159 (2004).
28. S. Dubois, A. Michel, J. P. Eymery, J. L. Duvail and L. Piraux, *J. Mater. Res.* **14**, 665 (1999).
29. G. Schon, *Nature* **404**, 948 (2000).
30. D. S. Golubev and A. D. Zaikin, *Phys. Rev. B* **64**, 014504 (2001) [arXiv:cond-mat/0012104].
31. P. Sindzingre, D. M. Ceperley and M. L. Klein, *Phys. Rev. Lett.* **67**, 1871 (1991).
32. C. H. Mak, S. Zakharov and D. B. Spry, *J. Chem. Phys.* **122**, 104301 (2005).
33. Y. Kwon and K. B. Whaley, *Phys. Rev. Lett.* **89** 273401 (2002).
34. R. H. Dicke, *Phys. Rev.* **93**, 99 (1954).
35. D. Dialetis, *Phys. Rev. A* **2**, 599 (1970).
36. R. Bonifacio, P. Schwendimann and F. Haake, *Phys. Rev. A* **4**, 302, 854 (1971).
37. N. Lambert, C. Emary and T. Brandes, *Phys. Rev. Lett.* **92**, 073602 (2004) [arXiv:quant-ph/0309027].