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TWO-LOOP BETA-FUNCTION FOR NONLINEAR SIGMA-MODEL WITH AFFINE METRIC MANIFOLD

VASILY E. TARASOV
Theoretical High Energy Physics Department, Nuclear Physics Institute, Moscow State University, 119 899 Moscow, Russia

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Two-loop metric counterterms for nonlinear two-dimensional bosonic σ-model with affine metric target manifold are calculated. The correlation of the metric and affine connection is derived from conformal invariance condition for nonlinear σ-model which is considered as a dissipative system. Examples of non-flat non-Riemannian manifolds resulting in trivial metric beta-function are suggested.

1. Introduction

String theory in a curved space is a consistent quantum theory if the quantum nonlinear σ-model[1,2] defined on this manifold is conformally invariant,[3] i.e. the renormalization group β-functions are trivial.[3,4] Since the conformal anomaly of the nonlinear σ-model depends on geometrical structures of the target manifold, the requirement of conformal invariance of the σ-model imposes restrictions on consistent structures.

Different geometrical structures can be defined on a manifold. In the bosonic case a metric and a connection structures are used. Riemannian manifolds are usually considered as a target manifolds for nonlinear σ-model.[1,2] Connection of the Riemannian manifold is uniquely constructed from metric, i.e. the "strong" correlation between the connection and metric structures is postulated. In general these structures are not correlated.[5] Therefore it was suggested to obtain correlation between the metric and connection as a result of the uv finiteness (or conformal invariance) condition for nonlinear σ-model.[6]

The σ-model action depends only on the metric. Therefore it is surprising that the counterterms of the σ-model with affine metric manifold differ from that on a Riemannian manifold.[7] This difference cannot be reduced to the metric redefinition caused by infinitesimal coordinate transformation[2] or to the nonlinear renormalization of the quantum fields.[8] In order to resolve this paradox we briefly review

E-mail: tarasov@theory.npi.msu.su

the relationship between the geometrical structures of the manifold and the equation of motion.

Let us consider the equation of test particle motion on a generic manifold

$$du^i/dt - Q^i(q,u) = 0\,, \tag{1}$$

where q^i are the coordinates and $u^i = dq^i/dt$ $(i = 1,\ldots,n)$ are velocities. We suggest that these equations are invariant under general coordinate transformations and that for simplicity $Q^i(q,u)$ are homogeneous functions of second order in u. It is known that if the Helmholtz conditions are satisfied there exists local Lagrange function and Eq. (1) can be derived from the least action principle. In this case there are matrix multipliers[9,10] such that Eq. (1) becomes Euler–Lagrange equation. The special case is $Q^i(q,u) = -[^{\,i}_{kl}]u^k u^l$, where $[^{\,i}_{kl}]$ is a Christoffel symbol for matrix multipliers, the n-dimensional target space is Riemannian manifold and Eq. (1) defines the usual one-dimensional nonlinear σ-model. On the other hand, it is known that Lagrange function uniquely defines the metric structure on the $(n+1)$-dimensional configuration space.[11] Thus the equation of motion derived from the action principle is equivalent to the geodesic equation on a metric manifold. The connection on the metric manifold can be naturally defined as Christoffel symbol of the metric. As a result the motion of the system subjected to potential forces is equivalent to free motion of the test particle on the metric manifold, i.e. manifold with correlated connection and metric.

If the Helmholtz conditions are not satisfied, the equation of motion (1) can be represented as a motion of particle subjected to dissipative forces Q^i_d on the metric manifold with metric structure defined by the Lagrangian

$$\frac{du^i}{dt} - Q^i_p(q,u) - Q^i_d(q,u) = -(g^{-1})^{ij} D_j L(q,u) - Q^i_d(q,u) = 0\,, \tag{2}$$

where D_j is the Euler–Lagrange operator, $L(q,u)$ is the Lagrange function and $g_{ij}(q,u)$ is the matrix multiplier.[9] Dissipative force for the one-dimensional σ-model with affine metric target manifold is defined by the connection defect $Q^i_d = -D^i_{kl}(q)u^k u^l$. If the free motion of the test particle on the manifold is defined by Eq. (2) then this manifold is non-metric. This manifold is usually referred to as a generalized path space[12,13] and allows naturally to define connection with the coefficients $\Gamma^i_{kl}(q,u) = (-1/2)(\partial^2 Q^i/\partial u^k \partial u^l)$. In the generalized path space the connection is not correlated with the metric on this space. As a result, the motion of the systems subjected to dissipative forces on the metric manifold is equivalent to the free motion of the test particle on the non-metric (generalized path) manifold.

The affine metric manifold[5] is a simple example of the generalized path space with a metric structure. Thus consistent approach to the nonlinear σ-model with affine metric target manifold leads to a generalization of the usual σ-models which represents a particle subjected to dissipative forces. Analogously, the motion of a string in affine metric curved space is equivalent to the motion of the string subjected to dissipative forces on a Riemannian manifold.[14] For this reason the consistent theory of the bosonic string in the curved affine metric space is a quantum dissipative theory.

The equation of motion and the geodesic equation on a non-metric manifold can be derived from Sedov variational principle[15] which is the generalization of the action principle:

$$\delta S(q) + \delta\tilde{W}(q) = 0\,. \tag{3}$$

Here $S(q)$ is the holonomic functional usually referred to as the action and $\tilde{W}(q)$ is the nonholonomic functional (i.e. $\delta\delta'\tilde{W} \not\equiv \delta'\delta\tilde{W}$). From Eq. (2) the nonholonomic functional has the form

$$\delta\tilde{W} = \int dt\delta W = \int dt Q^i_d(q,u) g_{ij}\delta q^j\,, \tag{4}$$

i.e. nonholonomic functional is defined by the connection defect. Nonholonomic functional W is characterized by the following properties in the phase space: (1) $[W, p_k] = W^q_k$ *and* $[W, q^k] = -W^k_p$, i.e. the variation of the functional W is defined by $\delta W = W^q_k \delta q^k + W^k_p \delta p_k$. The brackets are the generalized (variational) Poisson brackets[16,14] which coincide with the usual Poisson brackets for holonomic functions. (2) $J[Z_k, W, Z_l] = J_{kl} \not\equiv 0$ if $k \not\equiv l$ where $J[A,B,C] = [A[BC]] + [B[CA]] + [C[AB]]$; $k = 1,\dots,2n$ and $Z_i = q^i$ and $Z_{n+i} = p_i$ if $i = 1,\dots,n$. The Jacobian J_{kl} characterizes deviation from the condition of integrability. W is the nonholonomic object if one of the Jacobians J_{kl} is not trivial. Note that the classical phase space equation of motion for dissipative systems has the form $dZ_k/dt = [Z_k, H - W]$ and Liouville equation for dissipative systems[17,16] is

$$\frac{d}{dt}\rho(q,p,t) = -\Omega(q,p)\rho(q,p,t), \qquad \text{where} \qquad \Omega(q,p) = \sum_{i=1}^{n} J[q^i, W, p_i]\,. \tag{5}$$

The quantum description of the dissipative systems without well known ambiguities,[19,9,10,21,14] without violation of the canonical commutation relations and outside the framework of quantum kinetics was suggested in Refs. 16 and 14. This approach *does not violate Heisenberg algebra* because it generalizes the canonical quantization by introducing the operator of the nonholonomic quantities in addition to the usual associative operators of momentum, coordinate and holonomic functions. Contrary to the usual heuristical and therefore ambiguous generalization[20,21] the generalization of von Neumann equation was derived from dissipative Liouville equation.[17,16]

In Ref. 16 the conformal anomaly of the trace of the energy momentum tensor for closed bosonic string on the affine metric manifold was considered and from the conformal invariance it was proved that the metric and dilaton β-functions of the σ-model on affine metric target manifold must be trivial as usual.

In this letter two-loop uv metric counterterms and the β-function for two-dimensional nonlinear σ-model on the affine metric field manifold are calculated. The correlation between the connection and the metric on the manifold are derived from the vanishing condition for the β-function.

2. Loop Calculations

Now let us consider the closed bosonic string theory in curved space-time. The worldsheet swept by the string is described by the map $X(x)$ from a two-dimensional parameter space N into n-dimensional space-time manifold M, i.e. $X(x) : N \to M$. The two-dimensional parameters are $x = (\tau, \sigma)$. The map $X(x)$ is given by space-time coordinates $X^k(x)$. The classical equation of motion for the closed bosonic string on the n-dimensional affine metric curved space-time has the form

$$\partial_\mu \sqrt{g}\, g^{\mu\nu} \partial_\nu X^i + \Gamma^i_{kl}(X) \partial_\mu X^k \sqrt{g}\, g^{\mu\nu} \partial_\nu X^l = 0\,, \tag{6}$$

where $g^{\mu\nu}(x)$ is the two-dimensional metric tensor; $\Gamma^i_{kl}(X)$ is the affine connection, which can be represented in the form $[{}^{\,i}_{kl}] + D^i_{kl}$; $[{}^{\,i}_{kl}]$ is the Christoffel symbol for the metric $G_{ij}(X)$; $D^i_{kl}(X)$ is the connection defect which can be written in the form[5]

$$D^i_{kl}(X) = (-1/2) G^{ij} (K_{jlk} + K_{jkl} - K_{klj}) + 2Q^i_{(kl)} + Q^i_{kl}\,, \tag{7}$$

where $K_{kli} = \nabla_i G_{kl}$ is non-metricity tensor and Q^i_{kl} is torsion. The equation of motion (9) is an equation of two-dimensional geodesic flow on the affine metric manifold (the two-dimensional analog of the geodesic). It is well known that this equation cannot be derived from the action principle. Note that the Riemannian geodesic flow ($D^i_{kl} = 0$) can be derived from this variational principle with action defined by

$$S(X) = \frac{1}{4\pi\alpha'} \int d^2x\, G_{kl}(X) \partial_\mu X^k \sqrt{g}\, g^{\mu\nu} \partial_\nu X^i\,. \tag{8}$$

The affine metric geodesic flow Eq. (9) can be derived from the Sedov variational principle if the variation of the nonholonomic functional has the form

$$\delta \tilde{W} = \int d^2x\, \delta W = -\frac{1}{2\pi\alpha'} \int d^2x\, D_{ikl}(X) \partial_\mu X^k \sqrt{g}\, g^{\mu\nu} \partial_\nu X^l \delta X^i\,. \tag{9}$$

The holonomic and nonholonomic functionals define a closed bosonic string propagating in the affine metric curved space-time.

In loop calculation we use the generating functional for connected Green functions in the phase space path-integral form for the non-Hamiltonian (dissipative) systems suggested in Refs. 16 and 14. This generating functional is written in the form

$$Z(J,g) = -i \ln \int DXDP \exp i \int d^2x \left(P_k \frac{dX^k}{d\tau} - H + W + \frac{i}{2}\Omega + K(J) \right),$$

where $K(J)$ is the source term and $\hbar = 1$. To perform the calculation of the on-shell uv counterterms in one- and two-loop order for the σ-model we use the affine metric covariant background field expansion in normal coordinates[18,7] and a new generating functional $Z(X_0, g, J)$. The covariant background field method[2,14] in the phase space is defined by the usual expansion of the coordinates $X^k(x)$. Note that the background field method can be considered as conservative model approximation for the quantum dissipative models. The generating functional $Z(X_0, g, J)$ is defined by

$$\exp\, iZ(X_0, g, J) = \int D\xi DP \exp\, i \int d^2x \left(P_k \frac{d}{d\tau} X^k - H + W + \frac{i}{2}\Omega + J_k \xi^k \right), \quad (10)$$

where $X = X(X_0, \xi)$; $X_0^i(x)$ is the solution of the classical equation of motion; $\xi^k(x)$ is a covariant field which is the tangent vector to the affine metric geodesic connecting X_0^k and X^k.

We obtain the Hamiltonian, nonholonomic functional and omega function in the conformal gauge as a power series in the field $\xi^k(x)$:

$$2\pi\alpha' H = -\frac{1}{2} G^{kl}(X) P_k P_l - \frac{1}{2} G_{kl}(X) X'^k X'^l\,, \quad (11)$$

$$2\pi\alpha' W = \frac{1}{2}\Delta_1^{kl} P_k P_l + \frac{1}{2}\Delta_{kl}^2 X'^k X'^l, \qquad \Omega = 2D^k(X) P_k\,, \quad (12)$$

where $X^i = X^i(X_0, \xi)$; $D^k(X) \equiv D_{ij}^k(X) G^{ij}(X)$; $X'^k \equiv (dX^k)/(d\sigma)$; P_k is the canonical momentum. The background field expansions of the Δ-operators are written as

$$\Delta_1^{kl} = 2D_i^{kl}(X_0)\xi^i + O(\xi^2); \qquad \Delta_{kl}^2 = -2D_{ikl}(X_0)\xi^i + O(\xi^2)\,. \quad (13)$$

To obtain all the one- and two-loop counterterms, we need to expand Hamiltonian, nonholonomic functional and omega function to the fourth order in the quantum fields $\xi^a(x)$. Integration over momentum P is Gaussian. It is easy to derive the path-integral form for the generating functional:

$$Z(X_0, g, J) = -i \ln \int D\xi \exp\, i \int d^2x A(X(X_0, \xi))\,, \quad (14)$$

where

$$2\pi\alpha' A(X) = -\frac{1}{2}[G^{-1} + \Delta_1]_{kl}^{-1} (\dot{X}^k + iD^k(X))(\dot{X}^l + iD^l(X))$$
$$+ \frac{1}{2}[G + \Delta^2]_{kl} X'^k X'^l + \frac{1}{2}\delta(0) \ln\det[(G^{-1} + \Delta_1)^{-1}] \quad (15)$$

and $\dot{X}^k = (dX^k)/(d\tau)$, G is the metric. Note that $D^k(X)$, Δ_1, Δ^2 are equal to zero for the usual nonlinear σ-model. The full expression for $A(X(X_0, \xi))$ and the metric β-function for general affine metric manifold are complicated. Therefore, let us consider the special form of the non-metricity tensor: $K_{ijl} = N_{ijl} = N_{(ijl)}$, where $N_{ij(l;k)} = N^n_{i(k}N_{l)jn}$ and $Q_{(ij)l} = 0$ and let us write the terms of $A(X)$ which give the non-trivial simple poles two-loop metric counterterms only:

$$A(X_0,\xi) = \frac{1}{2\pi\alpha'}[(1/2)\partial_\mu\xi^a\partial_\mu\xi^a + B_{abkl}\xi^a\xi^b\partial_\mu X_0^k\partial_\mu X_0^l + J_{abc}\xi^a\partial_\mu\xi^b\partial_\mu\xi^c + C_{abcl}\partial_\mu X_0^l\xi^a\xi^b\partial_\mu\xi^c + L_{abcd}\xi^a\xi^b\partial_\mu\xi^c\partial_\mu\xi^d + F_{abcd}\xi^a\xi^b\partial_\mu\xi^c\kappa^{\mu\nu}\partial_\nu\xi^d]\,, \tag{16}$$

where

$$J_{abc} = (1/3)N_{jki}e^i_a e^j_b e^k_c\,,$$

$$B_{abkl} = [(1/2)\hat{R}_{k(ij)l} - (1/8)N_{pl(i}N_{j)pk}]e^i_a e^j_b\,,$$

$$C_{abcl} = [(2/3)\hat{R}_{(k/ij/l)} - (1/6)N_{pi(l}N_{k)pj}]e^i_a e^j_b e^k_c\,,$$

$$L_{abcd} = [(1/6)\hat{R}_{(k/(ij)/l)} + (1/8)N_{p(i/(l}N_{k)/j)p}]e^i_a e^j_b e^k_c e^l_d\,,$$

$$F_{ijkl} = (1/2)N_{pil}N_{pjk}e^i_a e^j_b e^k_c e^l_d\,.$$

In the conformal gauge $\kappa^{\mu\nu}$ tensor has the form $\kappa^{\mu\nu} = (\kappa^{\tau\tau}, \kappa^{\tau\sigma}, \kappa^{\sigma\sigma}) = (-1,0,0)$. We use the following notations:

$$\hat{\nabla}_k\Lambda_i = \nabla_k\Lambda_i + Q^n_{ki}\Lambda_n = \partial_k\Lambda^i - \Gamma^n_{(ki)}\Lambda_n = \Lambda_{i;k}\,,$$

$$B_{(j/k/l)} = (1/2)(B_{jkl} + B_{lkj})\,,$$

$$\hat{R}^i_{jkl} = R^i_{jkl} + 2\hat{\nabla}_{[l/}Q^i_{j/k]} + 2Q^n_{j[k/}Q^i_{n/l]}\,, \qquad R^i_{jkl} = 2\partial_{[k/}\Gamma^i_{j/l]} + 2\Gamma^n_{j[l/}\Gamma^i_{n/k]}\,,$$

and $\Gamma^i_{(kl)}$ is the symmetric part of the affine connection.

Note that the expression for $A(X_0,\xi)$ accounts for the additional non-metric terms. It is known that propagator of the quantum fields $\xi^k(x)$ is not standard. Therefore, we introduce an n-bein $e^a_k(X)$ and define $\xi^a(x) = e^a_k\xi^k(x)$, where $\hat{\nabla}_k e^a_l = 0$. After this modification the kinetic terms become $\hat{\nabla}_\mu\xi^a\hat{\nabla}_\mu\xi^a$, where $\hat{\nabla}_\mu\xi^a = \partial_\mu\xi^a + \hat{\Lambda}^a_{bc}e^b_k\partial_\mu X^k_0\xi^c$. This mixed covariant derivative for the affine metric manifold M and the Minkowski space N involves the Schouten–Vranceanu connection $\hat{\Lambda}_{abc}$, which is equal to the Ricci rotation coefficient and the object $\omega^a_{kc} \equiv \hat{\Lambda}^a_{bc}e^b_k$ which is the spin connection[2] on the Riemannian manifold. Note that in addition to the diagrams in Ref. 7 we take into account the diagrams whose external background field lines involve the Schouten–Vranceanu connection. Contrary to the usual nonlinear σ-model[2] these diagrams must not cancel[16] and give the tensor contribution. This contribution appears because of the relation $\hat{\Lambda}_{(a/b/c)} = (-1/2)(K_{ijl} + 2Q_{(i/l/j)})e^i_a e^j_c e^l_b$.

The irreducible one-loop diagram (Fig. 1) produces the following divergence:

$$-(\mu^{2\varepsilon}/4\pi\varepsilon)B_{aakl}\partial_\mu X_0^k \partial_\mu X_0^l\,.$$

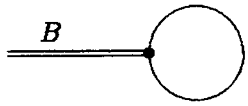


Fig. 1.

The non-trivial uv two-loop divergences of order $1/\varepsilon$ are caused by the graphs of Figs. 2–4. These divergences are

$$(\mu^{2\varepsilon}/16\pi^2\varepsilon)C_{(ab)ck}C_{a[bc]l}\partial_\mu X_0^k\partial_\mu X_0^l \text{ (Fig. 2)}$$

$$-(\mu^{2\varepsilon}/16\pi^2\varepsilon)(L_{cc(ab)}+L_{(ab)cc})B_{(ab)kl}\partial_\mu X_0^k\partial_\mu X_0^l \text{ (Fig. 3a)}$$

$$(\mu^{2\varepsilon}/16\pi^2\varepsilon)((-1/2)F_{cc(ab)}+(f_1+(1/2))F_{(ab)cc})B_{(ab)kl}\partial_\mu X_0^k\partial_\mu X_0^l \text{ (Fig. 3b)}$$

$$(\mu^{2\varepsilon}/16\pi^2\varepsilon)((3/2)J_{d(bc)}B_{adkl}+J_{a(bd)}B_{cdkl}-2J_{b(ad)}B_{cdkl}$$
$$+2J_{d(ac)}B_{bdkl})J_{a(bc)}\partial_\mu X_0^k\partial_\mu X_0^l \text{ (Fig. 4)}$$

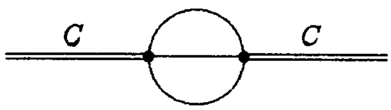
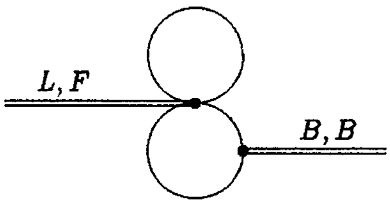


Fig. 2.



Figs. 3a–b

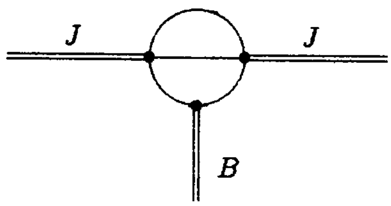


Fig. 4.

The divergent integrals are calculated using the dimensional regularization (in $n = 2 - 2\varepsilon$ dimensions) with the minimal subtraction and the general prescription for contraction of the two-dimensional $\kappa^{\mu\nu}$ tensor[16] $\kappa^{\mu\nu}\eta_{\mu\nu} = f(n)$ where $f(n) = 1 + f_1\varepsilon + O(\varepsilon^2)$ and $\eta_{\mu\nu}$ is the two-dimensional Minkowski metric. Different prescriptions may correspond to different renormalization schemes and thus the corresponding results should be related through redefinition of the couplings in analogy with the Riemannian two-dimensional nonlinear σ-model with the Wess–Zumino term.[22] To distinguish between ir and uv divergences we introduce an auxiliary mass term.

The two-loop pole divergences produced by the one-loop counterterms can be derived from

$$\Delta L^{(1)} = \frac{\mu^{2\varepsilon}}{4\pi\varepsilon}(P_{ab}\partial_\mu\xi^a\partial_\mu\xi^b + \mu^2 M_{ab}\xi^a\xi^b)\,, \tag{17}$$

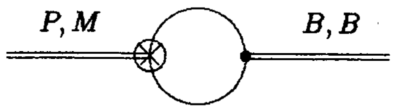
where

$$P_{ab} = -B_{ccij}e^i_a e^j_b\,, \qquad M_{ab} = [(-1/6)\hat{R}_{(i/nn/j)} - 2N_{nm(i}N_{j)nm}]e^i_a e^j_b\,.$$

The divergent parts of the graphs (Figs. 5a–b) are

$$-(\mu^{2\varepsilon}/16\pi^2\varepsilon)P_{(ab)}B_{(ab)kl}\partial_\mu X^k_0\partial_\mu X^l_0 \text{ (Fig. 5a)}$$

$$(\mu^{2\varepsilon}/16\pi^2\varepsilon)M_{(ab)}B_{(ab)kl}\partial_\mu X^k_0\partial_\mu X^l_0 \text{ (Fig. 5b)}$$



Figs. 5a–b

Then the two-loop metric β-function[1,2] for the bosonic nonlinear two-dimensional σ-model with affine metric target manifold has the form

$$\begin{aligned}\beta^G_{kl} = {}& (\alpha')[(1/8)N_{nm(k}N_{l)nm} - (1/2)\hat{R}_{(k/nn/l)}] + (\alpha')^2[(1/2)((2/3)\hat{R}_{(c/(ab)/k)} \\ & - (1/6)N_{n(c/(a}N_{b)/k)n})((2/3)\hat{R}_{(c/(ab)/l)} - (2/3)\hat{R}_{(b/(ac)/k)} \\ & + (1/6)N_{n(b/(a}N_{c)/k)n} - (1/6)N_{n(c/(a}N_{b)/l)n}) + ((1/2)\hat{R}_{(k/(ab)/l)} \\ & - (1/8)N_{n(a/(k}N_{l)/b)n})((1/6)\hat{R}_{(a/(nn)/b)} - (1/6)\hat{R}_{(n/(ab)/n)} \\ & - ((151/72) + (1/2)f_1)N_{nm(a}N_{b)nm})]\,. \end{aligned} \tag{18}$$

This metric β-function leads to the well known equation[1,2] on the Riemannian manifold ($N_{ijl} = 0$ and $Q^i_{kl} = 0$).

It is easy to see the condition which ensure the uv finiteness of the theory. The one- and two-loop parts of the metric β-function for two-dimensional nonlinear σ-model with affine metric target manifold M vanish if the correlation between the affine connection and the metric on the manifold M is given by:

$$\nabla_l G_{ij} = N_{ijl} = N_{(ijl)}\,, \qquad Q_{(ij)l} = 0\,,$$

$$\hat{\nabla}_{(l}N_{k)ij} = N^p_{i(k}N_{l)jp}\,, \qquad \hat{R}_{(k/(ij)/l)} = \frac{1}{4}N^p_{(k/(i}N_{j)/l)p}\,.$$

These conditions have no f_1 dependence and define non-flat space, i.e. Riemannian curvature tensor is nonzero. Note that the contribution to the metric β-function from the σ-model action (8) is zero in all loops if the affine metric manifold with the non-metricity tensor K_{ijl} and torsion tensor Q^i_{kl} is defined by

$$\hat{R}_{kijl} \equiv R_{kijl} - 2\hat{\nabla}_{[j/}Q_{ki/l]} - 2Q^n_{i[l/}Q_{kn/j]} = 0, \qquad \hat{\nabla}_k G_{ij} = K_{ijk} - 2Q_{(ij)k} = 0\,.$$

These equations define the affine metric manifold which is not flat.

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