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 TARASOV V.E. [1]

QUANTIZATION, GENERATING FUNCTIONAL AND CONFORMAL ANOMALY FOR NON-LINEAR AFFINE-METRIC SIGMA-MODEL

NUCLEAR PHYSICS INSTITUTE, MOSCOW STATE UNIVERSITY, RUSSIA

The Sedov variational principle, which is the generalization of the least action principle for the dissipative processes and the classical dissipative mechanics in the phase space are used. The main points of the quantum dissipative dynamics suggested in the recent paper are considered. As an example of the dissipative quantum theory we discuss the non-linear two-dimensional affine-metric sigma-model. The conformal anomaly of the energy momentum tensor trace for the closed bosonic string on the affine-metric manifold is investigated. The two-loop metric beta-function for non-linear dissipative sigma-model was calculated.

Variational Lagrangian and Hamiltonian mechanics describes the systems subjected to the potential forces only [1]. The dissipative forces are beyond the sphere of the variational principles [2-5]. For this reason the statistical mechanics does not describe the irreversible and dissipative processes. It is caused by the absence of the Liapunov function in the phase space in the Hamiltonian mechanics (Poincare-Misra theorem [6]). To describe the dissipative and irreversible processes we must introduce the additional postulate in statistical mechanics (for example, the Bogolubov principle of weakening (relaxation) correlation [7] and the hypothesis of the relaxation time hierarchy [8]). It is known that the initial point of the quantum mechanics formalism is Hamiltonian mechanics [9]. Therefore the quantum mechanics and statistics describes the physical objects in the potential force fields only. The Sedov L.I. suggests the variational principle [2-5] which is the generalization of the least actional principle for the dissipative and irreversible processes. The Sedov variational principle was used to consider dissipative systems in the phase space [10] and to generalize the quantum dynamics for the dissipative and irreversible processes [10]. In this paper we consider the sigma-model approach [11,12] to the quantum string theory [13] as an example of the dissipative quantum theory. The conformal anomaly of the energy momentum tensor trace [11] for closed bosonic string on the curved affine-metric manifold (i.e. in dissipative and nondissipative background fields) is discussed. The two-loop metric ultraviolet renormalization group beta-function [14] for two-dimensional non-linear dissipative bosonic sigma-model suggested in [10] is obtained. The results are compared with the ultraviolet two-loop metric counterterms for affine-metric sigma-model suggested in the papers [15,16].

The classical equation of motion for the n-dimension nonlinear affine-metric sigma-model has the form

$$\partial^{\mu}\partial_{\mu}X^{i} + \left(\left[{}^{\,i}_{kl}\right] + D^{i}_{kl} \right)\partial_{\mu}X^{k}\sqrt{g}g^{\mu\nu}\partial_{\nu}X^{l} = 0. \qquad (1)$$

[1] E-mail address: tarasov@compnet.msu.su.

where $[^i_{kl}]$ is Christoffel symbol for the metric $G_{ij}(q)$; $D_{ikl}(X)$ is a connection defect tensor [15,16]; $g^{\mu\nu}(x)$ is the n-dimensional metric tensor. The equation of motion (1) is an equation of the n-dimensional geodesic flow on the affine-metric manifold (the n-dimensional analogue of the geodesic line). It is well known that this equation can not be derived from the least action principle if the connection defect is other then null. Note that the Riemannian geodesic flow can be derived from this variational principle with the Lagrangian density defined by

$$L(X) = \frac{1}{2} G_{kl}(X)\partial_\mu X^k \sqrt{g} g^{\mu\nu} \partial_\nu X^l. \qquad (2)$$

The Sedov variational principle has the form: $\delta S(X)+\delta\tilde{W}(X) = 0$, where S(X) is the holonomic functional called action and $\tilde{W}(X)$ is the nonholonomic functional. The nonholonomic functional is defined by the nonholonomic equation. Let us choose the variation of the nonholonomic functional in the form

$$\delta\tilde{W} = -\int d^n x \; D_{ikl}(X) \; \partial_\mu X^k \sqrt{g} g^{\mu\nu} \partial_\nu X^l \; \delta X^i \qquad (3)$$

Then the geodesic flow equation (1) can be derived from the Sedov variational principle [17].The initial point of the dissipative quantum mechanics formalism [10] is the classical dissipative mechanics in the phase space. Let us consider main points of the dissipative analogue of the Hamiltonian dynamics

1. If the coordinates z^k $(k = 1, ..., 2m)$, where $z^i = q^i$ and z^{m+i} $i = 1, ...m$, w, t of the (2m+2)-dimensional extended phase space are connected by the equations $\delta w - w_k(z,t)\delta z^k = 0$, where w_k is the vector function in phase space, we call the dependence w on the coordinate q and momentum p the holonomic-nonholonomic function and denote $w = w(q,p) \in \Phi$. If the vector functions satisfy the integrability condition $(\partial w_k(z))/(\partial z^l) = (\partial w_l(z))/(\partial z^k)$, the coordinate w is the holonomic function ($w \in F$). If these vector functions don't satisfy this condition, we call the object $w(q,p)$ the nonholonomic function or the Sedovian ($w \in \tilde{F}$).

2. Let us define the variational Poisson brackets for $\forall a, b \in \Phi$ in the form:

$$[a, b] \equiv \frac{\delta a}{\delta q^i}\frac{\delta b}{\delta p_i} - \frac{\delta a}{\delta p_i}\frac{\delta b}{\delta q^i} \,. \qquad (4)$$

The basic properties of these Poisson brackets are following

1) $\forall a, b \in \Phi \quad [a, b] = -[b, a] \in F$,
2) $\forall a \wedge b \wedge c \in F \quad AS[a, b, c] = 0$,
3) $\forall a, b, c, \in \Phi : \; a \vee b \vee c \in \tilde{F} \quad AS[a, b, c,] \not\equiv 0$,

where $AS[a, b, c] \equiv [a, [b, c]] + [b, [c, a]] + [c, [a, b]]$. These properties for the holonomic functions coincide with the definition of the usual Poisson brackets [1]. The characteristic properties of the physical quantities are following

1) $[p_i, p_j] = [q^i, q^j] = 0$ *and* $[q^i, p_j] = \delta^i_j$,
2) $[w, p_i] = w^q_i$ *and* $[w, q^i] = -w^i_p$,
3) $[[w, p_i], p_j] \not\equiv [[w, p_j], p_i]$ *and* $[[w, q^i], q^j] \not\equiv [[w, q^j], q^i]$,
4) $[q^i, [w, p_j]] \not\equiv [p_j, [w, q^i]]$ *or* $AS[q^i, w, p_j] = \Omega^i_j \not\equiv 0$.

The object Ω^i_j characterizes the deviation from the condition of integrability (5).

3. The dissipative analogues of the Hamiltonian equations of motion are

$$\frac{dq^i}{dt} = [q^i, h - w] \quad \frac{dp_i}{dt} = [p_i, h - w] \,, \; \frac{dA(t)}{dt} = \frac{\partial A(t)}{\partial t} + [A(t), h - w]. \qquad (5)$$

4. The dissipative analogue of the Liouville equation [6] is

$$\frac{d\rho}{dt} = -\,\Omega\,\rho, \; where \;\; \Omega = \sum_{i=1}^{n} \Omega^i_i = \sum_{i=1}^{n} AS[q^i, w, p_i]. \qquad (6)$$

5. The additional statement for the Poincare-Misra theorem [6] is following: "The Liapunov function of the coordinate and momentum exists in the dissipative Hamiltonian mechanics". Let us define the function $\eta(q,p,t) \equiv -ln\rho(q,p,t)$ and assume $\Omega > 0$. The equation (6) shows that $d\eta/dt = \Omega$. and the function η satisfies the relation $d\eta/dt > 0$. In the general case, any function $f(q,p,t)$ which is the composite function $f(q,p,t) = g(\rho(q,p,t))$ and satisfies the relation $\Omega\ (\partial g(\rho))/(\partial\rho) < 0\ (\forall t)$ is the Liapunov function, that is $(df)/(dt) > 0$.

The quantum dynamics for the systems defined by the holonomic and nonholonomic functionals was suggested in the recent paper [10]. The main points of the dissipative quantum mechanics are following

1. The operators of physical quantities are defined by the following relations:

1) $[Q^i, Q^j] = [P_i, P_j] = 0 \quad and \quad [Q^i, P_j] = i\hbar\delta^i_j$,

2) $[W, P_i] = i\hbar W^q_i \quad and \quad [W, Q^i] = -i\hbar W^i_p$

3) $[[W, P_i], P_j] \not\equiv [[W, P_j], P_i] \quad and \quad [[W, Q^i], Q^j] \not\equiv [[W, Q^j], Q^i]$, $i \not\equiv j$

4) $[Q^i, [W, P_j]] \not\equiv [P_j, [W, Q^i]] \quad or \quad AS[Q^i, W, P_j] = \Omega^i_j \not\equiv 0$,

where $AS[A,B,C] = -\ (\ [A[BC]] + [B[CA]] + [C[AB]]\ )/(\hbar^2)$ and $Q^\dagger = Q$; $P^\dagger = P$; $W^\dagger = W$; $\Omega^\dagger = \Omega$; $[A,B] \equiv AB - BA$. To satisfy these commutation rules the operators of the nonholonomic quantities must be nonassociative. It is sufficient to require that the operator W of the Sedovian satisfies the following conditions:

a) $((WA^i)B^j) = (W(A^iB^j)) \quad and \quad (A^i(B^jW)) = ((A^iB^j)W)$,

b) $((A^iW)A^j) \not\equiv (A^i(WA^j))\ if\ i \not\equiv j; \quad ((A^iW)B^j) \not\equiv (A^i(WB^j))\ if\ A^i \not\equiv B^i$,

where A and B are P or Q operators.

2. The state in the quantum dissipative mechanics can be represented by the density operator $\rho(t)$ as usual. This operator is defined by the usual conditions: $\rho^\dagger(t) = \rho(t)$ and $Sp(\rho(t)) = 1$ for fixed time point. The average of the physical quantity A is defined by $< A >= Sp(A(t)\rho(t))$ for any fixed time moment. The dissipative analogue of the Heisenberg equation for the operator of physical quantity $A(t) \equiv A(Q,P,t)$ and the dissipative analogue of the Neumann (quantum Liouville) equation for the "density matrix" operator $\rho(t)$ are written in the form

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \frac{i}{\hbar}[H - W, A] \quad and \quad \frac{d\rho}{dt} = -\frac{1}{2}[\,\rho\,,\,\Omega\,]_+\,, \qquad (7)$$

where anticommutator $[\ ,\ ]_+$ is the consequence of the hermiticity for the density operator ρ and for the operator Ω. The solution of the first equation may be written in the form

$$A(t) = S(t,t_0)\ A(t_0)\ S^\dagger(t,t_0), \quad where\ S(t,t_0) = Texp - \frac{i}{\hbar}\int_{t_0}^{t} d\tau\ (H - W)(\tau). \ (8)$$

T-exponent is defined as usual [18], but with the following flow chart $exp\ A = 1 + A + \frac{1}{2}(AA) + \frac{1}{6}((AA)A) + \frac{1}{24}(((AA)A)A) + \ldots$.

3. The time evolution of the physical quantity operator is unitary and the evolution of the density operator is nonunitary. Therefore the pure state at the moment $t = t_0$ ($\rho^2(t_0) = \rho(t_0)$) is not a pure state at the next time moment. We can define the entropy operator of the state [6] $s \equiv -\ ln\rho$, which satisfies the equation $(ds)/(dt) = \Omega$.

4. The dissipative analogue of the Schroedinger equation has the form

$$\frac{d}{dt}\rho_S(t,t_0) = \frac{i}{\hbar}[\rho_S,\ (H - W)_S] - \frac{1}{2}[\Omega_S,\ \rho_S]_+\ , \qquad (9)$$

where the operator $\rho_S(t,t_0)$ is the Schroedinger representation of operator $\rho_H(t) = \rho(t)$ i.e. $\rho_S(t,t_0) = S^\dagger(t,t_0)\rho_H(t_0)S(t,t_0)$ and $\rho_S(t_0,t_0) = \rho_H(t_0)$. The solution of the equation (9) can be written in the form $\rho_S(t,t_0) = U_S^\dagger(t,t')\rho_S(t',t_0)U_S(t,t')$, where

$$U_S^\dagger(t,t') = Texp - \frac{i}{\hbar}\int_{t'}^{t} d\tau\ (H - W - \frac{i\hbar}{2}\Omega)_S(\tau,t_0). \tag{10}$$

5. The important feature of the basis vectors [9] in the dissipative quantum mechanics is the definition of these vectors at fixed time moment. It is caused by the time dependence of the density operator and of the wave vectors in the Heisenberg representation. That is $[q,t_1>_H \not\equiv [q,t_2>_H$ contrary to usual quantum mechanics [9]. It is easy to prove the following statements: " 1. The basis vector unitary transformed is a basis vector ; 2. There exists a unitary transformation for any two basis vectors defined at the non equal time points." Thus, Schroedinger representation of the basis vector $[q,t,t_0>_S \equiv S^\dagger(t-t_0)[q,t>_H$ might be considered as the unitary transformation of the basis vector, i.e. $[q,t_0>_H = S^\dagger(t-t_0)[q,t>_H$. Note, that the trace of the operator can be defined in fixed time point only.

6. The Green's function for the wave vector Schroedinger equation

$$G_S(q,q',t-t') \equiv <q,t]_S\ U_S^\dagger(t,t')\ [q',t'>_S\ \theta(t-t'), \tag{11}$$

using Faddeev method [19] is written in the Feynman representation as

$$G_S(q,q',t-t') = \int Dq\ Dp\ exp\frac{i}{\hbar}\int_{t'}^{t} d\tau\ (p\frac{dq}{d\tau} - h(q,p,\tau) + w(q,p,\tau) + \frac{i\hbar}{2}\Omega_{cl}).$$

7. The usual Hamiltonian formulation of the damped harmonic oscillator, so called Bateman-Morse-Feshbach damped oscillator, needs at least two deegrees of freedom [20-21]. We consider [10] the one dimensional harmonic oscillator with friction ($\delta w = \gamma m p \delta q$). The eigenvalues E_n for the dissipative analogue of the stationary Schroedinger equation for this oscillator in QP-ordering and background field approximation are written for $\Delta E_n = E_{n+1} - E_n$ in the form

$$\Delta E_n(\omega) = (\hbar\sqrt{\omega^2 - \gamma^2}\ when\ \omega^2 > 2\gamma^2)\ and\ (0\ when\ \omega^2 < 2\gamma^2). \tag{12}$$

The life time for the state derived from $Im(E_n) = -\gamma$ is $T = \frac{\hbar}{2\gamma} < \infty$. Note that the jump in the point $\omega = \sqrt{2}\gamma$ is the purely quantum dissipative effect.

Let us consider now the closed bosonic string theory [13] in curved space-time [11,12] or more exactly the two-dimensional nonlinear sigma-model [14-16] and the sigma-model approach to the string theory [11,12,26,10]. The world sheet swept out by the string is described by map $X(x)$ from two-dimensional parameter space N into m-dinensional space-time manifold M, i.e., $X(x) : N \to M$. The two-dimensional parameters are $x = (\tau,\sigma)$ and the map $X(x)$ is given by space-time coordinates $X^k(x)$. Let us choose the holonomic functional in the form

$$S(X) = S(G,\Phi,g) = \int d^2x\ (\ L(X) + \frac{\alpha'}{2}\ \sqrt{g}R^{(2)}(g)\ \Phi(X)) \tag{13}$$

and the nonholonomic functional in the form (3), where α' is the inverse of the string tension; $\Phi(X)$ is the dilaton field. The Lagrangian and Sedovian define a closed bosonic string propagating in the presence of dissipative and nondissipative background fields or in curved affine-metric space-time. Let us choose a parametrization for two-dimensional metric tensor $g^{\mu\nu}$ in the form

$$g_{\mu\nu}(x)dx^\mu dx^\nu = c(x)\ (\ n^2(x)(d\tau)^2 - (\ d\sigma + m(x)d\tau)^2\ ), \tag{14}$$

In this case the densities of the Hamiltonian without the dilaton field term, Sedovian and Omega are rewritten in the form

$$h = -\frac{n}{2} G^{kl}(X)\Pi_k\Pi_l + m\,\Pi_k X'^k - \frac{n}{2} G_{kl}(X)X'^k X'^l, \qquad (15)$$

$$w = \frac{n}{2}(\Delta_1^{kl}\Pi_k\Pi_l + \Delta_{kl}^2 X'^k X'^l), \quad \Omega = 2n\, D^k(X)\Pi_k, \qquad (16)$$

where $D^k(X) \equiv D^k_{ij}(X)\, G^{ij}(X)$, $X'^k \equiv (dX^k)/(d\sigma)$, Π_k is the canonical momentum , Δ are tensorial integral operators, which can be written in the conditional form of indefinite multiple integral δX^k :

$$\Delta_1^{kl} = 2\int \delta X^i D_i^{kl}(X), \quad \Delta_{kl}^2 = -2\int \delta X^i D_{ikl}(X). \qquad (17)$$

Unfortunately we have no correct mathematical definition of these operators. This difficulty can be removed by expressing the nonholonomic functional as a power series in a covariant field $\xi^k(x)$ which is the tangent vector to the geodesic line containing X_0^k and $X^k = X_0^k + f^k(X_0,\xi)$. The background field expansions of the Δ - operators are written in the form

$$\Delta_1^{kl} = 2\, D_i^{kl}(X_0)\,\xi^i + O(\xi^2), \quad \Delta_{kl}^2 = -2\, D_{ikl}(X_0)\,\xi^i + O(\xi^2). \qquad (18)$$

The covariant background field method [40,44,21] in the phase space is defined by the usual expansion of the coordinates $X^k(x)$ only. Note that the model defined by (13) (and (3)) in the conformal gauge $n = 1, m = 0$ called two-dimensional nonlinear (dissipative) sigma-model. Let us define the generating functional for connected Green functions [18,19] in the form

$$W(J,g) = -i\hbar\, ln \int DXD\Pi\; exp\frac{i}{\hbar}\int d^2x\, (Z_1(X,\Pi,g) + Z_2(X,J)), \qquad (19)$$

$$where \quad Z_1(X,\Pi,g) \equiv \Pi_k \frac{d}{d\tau}X^k - h + w + \frac{i\hbar}{2}\Omega + \frac{\alpha'}{2}\sqrt{g}R^{(2)}(g)\Phi(X). \qquad (20)$$

$Z_2(X,J)$ is the source term discussed in [23-25]. We derive the covariant background field expansion of Z_1, Z_2 and define a new generating functional $W(X_0,g,J)$:

$$exp\frac{i}{\hbar}(W(X_0,g,J) + \tilde{W}(X_0)) = \int D\xi D\Pi\; exp\frac{i}{\hbar}\int d^2x (Z_1(X(X_0,\xi),\Pi,g) + J_k\xi^k).$$

The functional integral over momentum Π is Gaussian integral. It is easy to derive the path integral form for the generating functional:

$$W(X_0,g,J) = -i\hbar\, ln\int D\xi\; exp\frac{i}{\hbar}(\, A(X(X_0,\xi)) + M(X(X_0,\xi))\,). \qquad (21)$$

The effective action $A(X)$ is written in the following form [10]

$$A(X) = S(G,\Phi,g) + S(D,g)\,, \quad where\; S(D,g) = \sum\nolimits_{K=1}^{3} S_K(D,g), \qquad (22)$$

$$S_1 = -\frac{1}{2}\int d^2x \Delta_{kl}^2 \partial_\mu X^k \sqrt{g} g^{\mu\nu}\partial_\nu X^l = -\tilde{W}(X), \qquad (23)$$

$$S_2 = \frac{1}{2}\int d^2x\, F_{kl}\partial_\mu X^k \sqrt{g}\kappa^{\mu\nu}\partial_\nu X^l, \quad S_3 = \int d^2x\sqrt{g}(V_{k\mu}g^{\mu\nu}\partial_\nu X^k + B(X)), \qquad (24)$$

$$F_{kl} = [G^{-1} + \Delta_1]^{-1}{}_{kl} - [G + \Delta^2]_{kl} = 4D_i{}^n{}_k D_{jnl}\xi^i\xi^j + O(\xi^3), \qquad (25)$$

$$V_{k\mu} \equiv \frac{1}{2}g_{\mu\nu}k^\nu [G^{-1} + \Delta^1]^{-1}{}_{kl}D^l(X), \quad B(X) \equiv \frac{1}{2}c^{-1}(x)[G^{-1} + \Delta^1]^{-1}_{kl}D^k(X)D^l(X)$$

$$M(X) = \int d^2x \, \frac{i\hbar}{2} \, \delta(0) \, lndet(G^{-1}(X) + \Delta_1(X))^{-1} \,), \qquad (26)$$

$$k^\mu = (k^\tau, k^\sigma) = (-2ic^{-1}, 2imc^{-1}), \qquad (27)$$

$$\kappa^{\mu\nu} = (\kappa^{\tau\tau}, \kappa^{\tau\sigma}, \kappa^{\sigma\sigma}) = (-n^{-2}c^{-1}, mn^{-2}c^{-1}, -m^2n^{-2}c^{-1}) \qquad (28)$$

and $D^l(X) = G^{lk}G^{ij}D_{kij}(X)$. Note that the effective action is conformally invariant, because has no $c(x)$ dependence. Account is to be taken of the parametrization of the two-dimensional tensors $\kappa^{\mu\nu}$ and k^μ which are connected with the parametrization of two-dimensional metric tensor $g^{\mu\nu}$, i.e. $\kappa^{\mu\nu} = \kappa^{\mu\nu}(g)$ and $k^\mu = k^\nu(g)$.

The energy-momentum tensor for the closed bosonic string is defined as usualy [11,12,26] by $T^{\mu\nu}(x) = (-2/\sqrt{g})\delta S(G,\Phi,g)/(\delta g_{\mu\nu})$ In the general case, the objects in $S(D,g)$ are the nonholonomic objects. We use the covariant background field method [22,15]. Therefore these objects are the power series in quantum fields $\xi^k(x)$. Note that the background field expansion of the nonholonomic functionals and the two-dimensional metric variation of it are not commutative operations, i.e. $(\delta\tilde{W}(X))/(\delta g^{\mu\nu}) = 0$ and $(\delta\tilde{W}(X_0,\xi))/(\delta g^{\mu\nu}) \not\equiv 0$. Therefore the vacuum expectation value of the energy-momentum tensor [26]

$$< T^{\mu\nu}(x) > \equiv Nexp(-\frac{i}{\hbar}W(J,g)) \int D\xi \; T^{\mu\nu}(x) \; exp\frac{i}{\hbar}(A(X) + M(X)) \qquad (29)$$

can not be written in the form $(-2/\sqrt{g})(\delta W(J,g))/(\delta g_{\mu\nu})$ because we consider the nonholonomic functionals as the background field power series. (In the opposite case, we must prove the Gaussian momentum integration formulas for the tensorial indefinite integral operator Δ in functional integral beyond the background field method.) Let us define the two-metrical generating functional [10] in the form

$$W(g^{\mu\nu}, a^{\mu\nu}, X_0, J) \equiv -i\hbar \; ln \int D\xi exp\frac{i}{\hbar}(S(G,\Phi,g^{\mu\nu}) + S(D,a^{\mu\nu}) + M(X) + J_k\xi^k).$$

The usual functional $W(g,X_0,J)$ is derived by $W(g,X_0,J) = W(g,a,X_0,J)_{g=a}$. The vacuum expectation value (29) can be written in the form

$$< T^{\mu\nu}(x) > = -\frac{2}{\sqrt{g}}[\frac{\delta}{\delta g_{\mu\nu}}W(g,a,X_0,J)]_{g=a}. \qquad (30)$$

It is easy to derive the conformal anomaly of the trace of the energy-momentum tensor [11] for dissipative nonlinear sigma model in the form [10]

$$< T^\mu_\mu > = \frac{1}{2}\,\bar{\beta}^G_{kl}\;\partial^\mu X^k_0\partial_\mu X^l_0 \;+\; \frac{\alpha'}{2}\;R^{(2)}(g)\;\bar{\beta}^\Phi, \qquad (31)$$

$$where \quad \bar{\beta}^G_{kl} = \bar{\beta}^G_{kl} + \dots, \quad \bar{\beta}^\Phi = \beta^\Phi + \alpha'\hat{\nabla}_k\Phi\hat{\nabla}^k\Phi - \frac{1}{4}\bar{\beta}^G_{ij}\,(\,G^{ij} + \dots) \qquad (32)$$

$$and \quad \bar{\beta}^G_{kl} = \beta^G_{kl} + 2\alpha'\,\hat{\nabla}_k\hat{\nabla}_l\Phi\,, \quad \hat{\nabla}_kV_l \equiv \partial_kV_l + (\,[{}^i_{kl}] + D^i_{(kl)}\,)\,V_i \qquad (33)$$

Let us take into account the background field expansion [22,15] of $\partial_\mu X^k = C^k_l(X_0,\xi)\;\partial_\mu X^l_0$ and choose the following solution of the classical equation (1) of motion $X_0^k(x) = const$. The vacuum expectation value for the law of energy-momentum tensor change can be written in the usual form [12,26] $< \nabla^\mu T_{\mu\nu} >= 0$. It is easy to derive that the central charge of the Virasoro algebra [27] is proportional to the dilaton β -function as usualy [12]. The sufficient condition for the validity of this relation [12] is $\tilde{\beta}^\Phi = const$ and $\tilde{\beta}^G_{kl} = 0$. Where $\tilde{\beta}^G$ is defined by the metric beta -function of the two-dimensional nonlinear dissipative sigma-model. In the two-loop metric beta-function calculation we use affine-metric background field method [15,16], introduce an auxilliary mass term [28], the dimensional regularization $2 \to n = 2 - 2\epsilon$ and the minimal subtraction with the general prescription for contraction for the two-dimensional $\kappa^{\mu\nu}$ tensor $\kappa^{\mu\nu}\eta_{\mu\nu} = f(n)$ where $f(n) = 1 + f_1\epsilon + O(\epsilon^2)$ and $\eta_{\mu\nu}$ is two-dimensional Minkowski metric. Different prescriptions may correspond to different renormalization shemes and thus their results should be related through redefenition of the couplings G_{kl} and F_{kl} by analogy to Riemannian two-dimensional non-linear sigma-model with the Wess-Zumino term [29]. It is known that propagator of the quantum fields $\xi^k(x)$ is not standard. Therefore we introduce an m-bein $e_k^a(X)$ and define $\xi^a(x) = e_k^a \xi^k(x)$, where $\hat{\nabla}_k e_l^a = 0$. After this modification the kinetic terms become $\hat{\nabla}_\mu \xi^a \hat{\nabla}_\nu \xi^a$, where $\hat{\nabla}_\mu \xi^a = \partial_\mu \xi^a + \hat{\Lambda}^a_{bc}\ e_k^b\ \partial_\mu X_0^k\ \xi^c$ This mixed covariant derivative for affine-metric manifold M and the Minkowski space N involves the Schouten-Vranceanu connection [30,31] $\Lambda_{abc} = \hat{\Lambda}_{abc} + 2Q_{(ij)l}\ e_a^i e_c^j e_b^l$, (where Q^k_{ij} is the torsion tensor of the affine-metric manifold) which is equal to the Ricci rotation coefficient [32] and the object $\omega^a_{kc} \equiv \Lambda^a_{bc} e_k^b$ is spin connection [22] on the Riemannian manifold. Note, in addition to [15,16] we take into account the diagrams whose external background field lines involve the Schouten-Vranceanu connection. This diagrams must not cancel [17] in contrary to the usual non-linear sigma-model [22]. It is caused by the relation $\Lambda_{(a/b/c)} = (-1/2)K_{ijl}\ e_a^i e_c^j e_b^l$, where K_{ijl} is nonmetricity tensor of affine-metric manifold. The two-loop metric beta-function of the dissipative sigma-model is $\beta^G = \beta^G_{AM} + \beta^G_1 + \beta^G_2$, where β^G_{AM} is the metric beta-function [14,22] of the affine-metric sigma-model defined in [15,16] , i.e. the part of the metric beta-function from the action $S(G, \Phi, g)$ only, where two-dimensional metric is the Minkowski metric; β^G_K $K = 1, 2$ is the part of metric beta-function from $S_K(D, g)$ in equations (22)-(24). Note that the two-loop metric beta-function β^G_3 is equal to zero. This is analogue of the results for nonlinear sigma-model considered in [33-35]. The full expression of the two-loop ultraviolet counterterms is very complicated , but it is easy to see the following ultraviolet finiteness conditions. The one and two loop counterterms for two-dimensional non-linear dissipative sigma-model vanish if the correlation between the affine connection and the metric structures on the manifold M is given by [10]:

$$\hat{\nabla}_k G_{ij} = N_{ijk} = N_{(ijk)}\ ;\ \hat{\nabla}_{(l} N_{k)ij} = N^p_{i(k} N_{l)jp}\ ;\ \hat{R}_{(k/(ij)/l)} = \frac{3}{4}\ N^p_{(k/(i} N_{j)/l)p} \cdot (34)$$

It is easy to see that the ultraviolet finiteness conditions have not the f_1 dependence. Note that the affine-metric beta-function is zero in all loops if the affine-metric manifold with the nonmetrisity tensor K_{ijl} and torsion tensor Q^i_{kl} is defined [17,10] by

$$\hat{R}_{kijl} \equiv R_{kijl} - 2\hat{\nabla}_{[j/} Q_{ki/l]} - 2Q^n_{i[l/} Q_{kn/j]} = 0 \quad \hat{\nabla}_k G_{ij} = K_{ijk} - 2Q_{(ij)k} = 0. \ (35)$$

It is easy to see that this affine-metric manifold is not flat space.

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