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POSSIBLE MANIFESTATIONS OF MULTIDIMENSIONALITY OF SPACE-TIME IN A SIMPLE MODEL

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Spontaneous breaking of supersymmetry at scales much smaller than the Plank mass can be related to early compactification of extra dimensions of the space-time. Possible manifestations of extra dimensions in scattering amplitudes are discussed on the example of a scalar model in 2+2-dimensional space-time with two dimensions being compactified to the torus.

1. Introduction

Most of modern theories beyond the standard model include the hypothesys of multidimensionality of the space-time (Kaluza-Klein type theories, extended supergravity, superstring theory; see e.g. [1] and refs. therein). Usually one supposes that extra dimensions are compactified at the scale $R_C \sim M_C^{-1}$ of the order of the inverse Planck mass M_{Pl}^{-1} (gravitational scale). In this case additional dimensions can reveal themselves only in peculiar gravitational effects or in early cosmology of the Universe.

On the other hand, in all above mentioned theories there is usually much lower supersymmetry breaking scale M_{SUSY}. This scale can be naturally related to the compactification scale since as it is known the supersymmetry in principle lowers under compactification of a part of space-time dimensions [2]. If $M_C \sim M_{SUSY} \sim 1 \div 10$ Tev, it is quite possible that evidence of the additional dimensions can be seen at future experiments at supercolliders.

The most difficult problem in multidimensional field theoretical calculations is the non-renormalizability of the theory. Thus, one has to work in some ultraviolate finite theory. Nowadays there is a common belief that such theory does exist and this is the superstring theory [1]. Unfortunately, calculations in the framework of the superstring theory face two important difficulties:

i) The coupling constant above the energy of compactification becomes huge because of threshold corrections [3]. On the other hand, most of results in string theory were obtained within the perturbation theory. So, to make calculations in superstring theory one has to choose very special manifolds as additional dimensions to make these threshold corrections to be zero. It was shown [4] that this is possible indeed in specific models due to supersymmetric cancellations. In these special cases (which are not related to realistic models, at least directly) one can show

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that the only way to detect extra dimensions is via the direct production of a few excitations of the Kaluza-Klein tower with masses $\geq M_C \sim 1\ TeV$. This seems to be experimentally hopeless in the near future.

ii) There is no phenomenologically acceptable low energy model derived from superstring theory directly.

Thus, in the Kaluza-Klein approach in order to have a realistic model at low energies in four dimensions we are forced to consider a non-renormalizable multidimensional field theory. Whereas in a complete finite quantum theory renormalization counterterms of the low energy limit could be calculated, in the effective non-renormalizable theory, we are discussing here, the corresponding coupling constants must be considered as phenomenological ones. Fortunately, the contributions of these counterterms to the finite part of the amplitudes are of the order $(S/\Lambda^2)^n$ [1], where $\sqrt{S}$ is the energy of colliding particles, Λ can be regarded as the charachteristic scale of the complete theory($\Lambda \sim M_{Pl}$ in the case of superstrings) and $n \geq 1$. Thus, when $\sqrt{S} \leq M_C \ll \Lambda$ these contributions can be neglected.

We would like to mention here that similar problem was considered in [5]. There the authors used the results of the high energy experiments to get the upper bound on the size R of the space of extra dimensions assuming that the first heavy Kaluza-Klein mode is not observed experimentally. Our philosophy in the present paper is different: we assume that $R^{-1} \sim M_{SUSY} \sim 1 \div 10\ TeV$ and look for possible experimental evidence of heavy Kaluza-Klein modes.

In this communication we estimate possible manifestations of effects of compactification on the example of a simple model of one scalar field in (2+2)-dimensional space-time $M = M_2 \times T_2$, where M_2 is the two-dimensional Minkowski space-time and T_2 is the torus.

2. The model

We consider here a toy model of one real scalar field $\Phi(x,y)$ on the space $M_2 \times T_2$ with the action

$$S = \frac{1}{2\pi}\int d^2x d^2y \{(1/2)\Phi(x,y) g^{MN}\partial_M\partial_N\Phi(x,y) - (m^2/2)\Phi^2(x,y) + \\ +(g_4/4!)\Phi^4(x,y)\} \quad , \tag{1}$$

where $M, N = 0, ..., 3$; the metric $g_{MN} = \text{diag}(-1,1,1,1)$. The field $\Phi(x,y)$ is periodic in coordinates y^1, y^2

$$\Phi(x, y + 2\pi R) = \Phi(x,y); \qquad R = M_C^{-1}$$

and can be expanded in Fourier series:

$$\Phi(x,y) = (1/R)\sum_{n_1,n_2=-\infty}^{\infty} \varphi_{\vec{n}}(x)\exp\{i\vec{y}\vec{n}/R\},$$
$$\vec{n} = (n_1, n_2), \quad \vec{y} = (y^1, y^2),$$
$$\varphi_{-\vec{n}} = \varphi^*_{\vec{n}} .$$

Substituting the Fourier expansion into the action (1) we get

$$\begin{aligned} S &= \int d^2x (1/2)\sum_{\vec{n}} \varphi_{-\vec{n}}(\partial_0^2 - \partial_1^2 - M_{\vec{n}}^2)\varphi_{\vec{n}} + \\ &+ (1/4!)g_2 \sum_{\vec{n},\vec{k},\vec{p}} \varphi_{\vec{n}}\varphi_{\vec{k}}\varphi_{\vec{p}}\varphi_{-(\vec{n}+\vec{k}+\vec{p})} , \end{aligned} \tag{2}$$

with

$$M_{\vec{n}}^2 = m^2 + \vec{n}^2/R^2 = m^2 + M_0^2\vec{n}^2.$$

In this toy model the field $\varphi_{\vec{0}}(x)$ with the light mass $m \ll M_C$ is an analog of the fields of the standard model and the Kaluza-Klein tower of fields $\{\varphi_{\vec{n}}\,,\ \vec{n} \neq 0\}$ with masses $M_{\vec{n}}$ is an analog of fields of its multidimensional extension. The action (2) describes the two-dimensional theory with infinite number of massive fields, and it is important that all the fields have the same coupling constant as a consequence of the four-dimensional nature of the model.

As a process imitating the present possibilities of collider experiments it is reasonable to consider the scattering (2 *light particles*) $\to$ (2 *light particles*) assuming that $m \ll M_C$ and the scattering energy $E \leq M_C$. The scattering amplitudes for the light $\varphi_{\vec{0}}$-particles in the tree approximation are defined by the usual vertex $\sim \varphi_{\vec{0}}^4$. Note, that in a theory with cubic verticies (like in the standard model) we would have additional coupling of the form

$$\lambda \sum_{\vec{n},\vec{k}} \varphi_{\vec{n}}\varphi_{\vec{k}}\varphi_{-(\vec{n}+\vec{k})} = \lambda\varphi_0^3 + 3\lambda\varphi_0 \sum_{\vec{n}} \varphi_{\vec{n}}\varphi_{-\vec{n}} +$$
$$+\lambda \sum_{\vec{n},\vec{k}\neq 0} \varphi_{\vec{n}}\varphi_{\vec{k}}\varphi_{-(\vec{n}+\vec{k})}\,.$$

This shows that one has no new diagrams with heavy fields for scattering of light particles at the tree level also (this is correct for any homogeneous additional subspace). Thus we can hope to find some new effects at the loop level only. This explains why we consider the simplest Φ^4-model.

At the 1-loop level besides the vertex with the light fields only

$$\frac{g_0}{4!R^2}\varphi_0^4$$

there are other relevant vertices for scattering of light particles

$$\frac{g_0}{2}\varphi_0^2 \sum_{\vec{n}} \varphi_{\vec{n}}\varphi_{-\vec{n}}\,.$$

As we shall show, due to these vertices the massive Kaluza-Klein fields give considerable contribution to the cross section of the scattering process under investigation.

3. 2 → 2 scattering cross section in the model with compactification

Regularized two particle scattering amplitude in our model has the following form:

$$T^{(\infty)}(s,t) = \frac{g_0}{R^2}(1 - \frac{g_0}{8\pi} \sum_{\vec{n}:\ \vec{n}^2<\Lambda^2} [B_{\vec{n}}(s) + B_{\vec{n}}(t) + B_{\vec{n}}(u)])\,, \qquad (3)$$

where $B_{\vec{n}}(s)$, $B_{\vec{n}}(t)$, $B_{\vec{n}}(u)$ correspond to the 1-loop contributions of the $\vec{n}$th mode

$$B_{\vec{n}}(s) = \frac{4\pi}{iR^2} \int \frac{d^2k}{(2\pi)^2} \frac{1}{[(p-k)^2 - M_{\vec{n}}^2 + i\varepsilon](k^2 - M_{\vec{n}}^2 + i\varepsilon)} =$$

$$= \frac{1}{z}\sqrt{\frac{z}{\vec{n}^2 - z}}\,\mathrm{arctg}\sqrt{\frac{z}{\vec{n}^2 - z}}\,, \qquad \vec{n} \neq 0, \qquad s = p^2\,;$$

$$B_0(s) = \frac{1}{2z}\sqrt{\frac{z}{z-c}}\ln\left|\frac{\sqrt{z}-\sqrt{z-c}}{\sqrt{z}+\sqrt{z-c}}\right| + (\text{imaginary part}) .$$

Here

$$z \equiv \frac{s}{4M_C^2}, \quad c \equiv \frac{m^2}{M_C^2},$$

and we consider the region $c < z < 1$ (below the first Kaluza-Klein threshold). Imaginary part of $B_0(s)$ does not contribute to the g^3-order of the cross section. Invariants t and u in the two-dimensional space-time take s two values at fixed energy: either $t = 4m^2 - s$, $u = 0$ or $t = 0$, $u = 4m^2 - s$ (note that the amplitudes T are equals to each other for both cases).

The non-trivial t-channel contributions are the following:

$$B_{\vec{n}}(4m^2 - s) = \frac{1}{2\sqrt{(z-c)(z+\vec{n}^2)}} \ln\left|\frac{\sqrt{z-c}+\sqrt{z+\vec{n}^2}}{\sqrt{z-c}-\sqrt{z+\vec{n}^2}}\right| .$$

In the limit $\Lambda^2 \to \infty$ the sum in (3) becomes logarithmically divergent as it should be in the four-dimensional field theory (compactification does not influence the ultraviolate properties of the theory). Thus, one has to renormalize the coupling constant g_0 by making one subtraction to obtain the finite amplitude.

In accordance with the physical setting of the problem we define the renormalized coupling constant h as follows

$$T^{(\infty)}(s)|_{s=\mu^2} = h, \tag{4}$$

where the momentum subtraction point μ is chosen to be $0 \leq \mu \leq m$. Then we get

$$T^{(\infty)}(s,\mu^2) = h\{1 - (h/8\pi)\sum_{\vec{n}} A_{\vec{n}}(s,\mu^2)\}, \tag{5}$$

where

$$A_{\vec{n}}(s,\mu) = B_{\vec{n}}(s) - B_{\vec{n}}(\mu^2) + B_{\vec{n}}(4m^2 - s) - B_{\vec{n}}(4m^2 - \mu^2) .$$

As it has been already mentioned in the Introduction our aim here is to discuss the difference between the scattering amplitude, calculated in the full theory with infinite tower of Kaluza-Klein modes, and that, calculated in the corresponding two-dimensional model with a finite number of modes. In the latter model we will restrict ourselves to the following two cases: 1) there is the light mode only; 2) there are light mode and the first heavy mode with the mass M_1. The two-dimensional model with a finite number of modes (in what follows we will refer to it as the 2-model for shortness) is determined by the action of the type (2) but with finite sums over $\vec{n}$ in it. The 2-model is ultraviolet finite and its coupling constant does not need to be renormalized. However, again in accordance with the physical setting of the problem and in order to make the comparison between the full theory and the 2-model to be consistent, we require that the amplitude calculated in the 2-model also satisfies the condition (4). This amounts to a finite renormalization of the coupling constant and to over-subtraction of the 1-loop diagrams. We obtain for the scattering amplitude

$$T^{(0)}(s,\mu^2) = h\{1 - (h/8\pi)A_{\vec{0}}(s,\mu^2)\},$$

for the 2-model with the light mode and

$$T^{(1)}(s,\mu^2) = h\{1 - (h/8\pi)[A_{\vec{0}}(s,\mu^2) + 4A_{(1,0)}(s,\mu^2)]\} \tag{6}$$

for the 2-model with the light mode and the first heavy mode. The cross section of the process $\sigma_{2\to 2}$ is related to the amplitude as follows

$$\sigma^{(a)}_{2\to 2} = \frac{h^2}{16z(z-c)M_C^4}\left(T^{(a)}\right)^2(s,\mu^2)\,, \qquad a = 0, 1, \infty.$$

We are going to discuss the difference between the amplitudes

$$T^{(1)}(s,\mu^2)$$

and

$$T^{(\infty)}(s,\mu^2)$$

in the range of energies given by

$$\frac{m^2}{M_1^2} < z < 1\,, \quad z = \frac{s}{4M_1^2}\,. \tag{7}$$

However, there are two points that make the results of this analysis less instructive:

1. the value of the coupling constant is arbitrary;
2. in the range (7) the contribution of the light mode $A_0(s,\mu^2)$ is almost constant and is much bigger than the contributions of other modes:

$$|A_0(s,\mu^2)/A_{(1,0)}(s,\mu^2)| \gg 1$$

and

$$|A_0(s,\mu^2)/\sum_{|\vec{n}|\geq 1} A_{\vec{n}}(s,\mu^2)| \gg 1\,.$$

In order to avoid these difficulties we compute the dimensionless functions

$$K_1(\frac{s}{4M_C^2}) = 4M_C^2 A_{(1,0)}(s,\mu^2)\,,$$

$$K_\infty(\frac{s}{4M_C^2}) = M_C^2 \sum_{|\vec{n}|\geq 1} A_{\vec{n}}(s,\mu^2)\,.$$

The quantity, which characterizes the relative difference between the full Kaluza-Klein theory and the 2-model with the light mode and the first heavy mode, can be defined as follows

$$\eta(z) = \left|\frac{K_1(z) - K_\infty(z)}{K_1(z)}\right|\,.$$

For $M_C/m = 100$ and $m/\mu = 10$ we get

$$\eta(0.25) = 0.16\,, \qquad \eta(0.5) = 0.14\,, \qquad \eta(0.75) = 0.10\,. \tag{8}$$

It is not illustrative to calculate K_1, K_∞ near the points $z = c$ and $z = 1$, since these functions have poles at these values of z, that is an artefact of the models in two-dimensional space-time. The curves $K_1(z)$ and $K_\infty(z)$ for the same values of M^C, m and μ are presented in Fig.1.

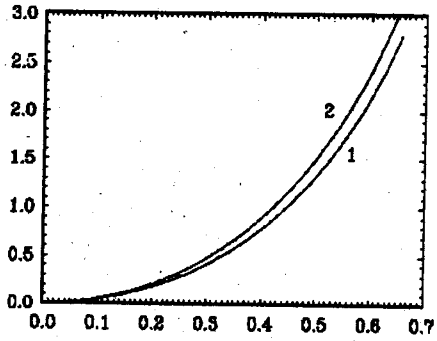

Fig.1 Plots of functions $K_1(z)$ (curve 1) and $K_\infty(z)$ (curve 2): $K_1(z)$ for the 2-model with the first heavy mode and $K_\infty(z)$ for the full theory with infinite tower of Kaluza-Klein modes

From (8) and from the plots one can see that the functions $K_1(z)$ and $K_\infty(z)$ differ considerably in the region $0.2 < z < 0.7$. Although the amplitudes depend on the ratios M_C/m and m/μ actually the numerical results are practically independent of these parameters for $M_C/m > 20$ and $m/\mu > 10$.

We would like to note that if one compared the full theory with the 2-model with only one heavy mode (without taking into account the multiplicity) there would be more difference between these theories.

This illustrates the way in which one can distinguish in principle between the full Kaluza-Klein theory and the model with first two modes and make a conclusion in favour or against the Kaluza-Klein idea from experimental observations.

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