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QUANTUM DISSIPATIVE SYSTEMS. II. STRING IN A CURVED AFFINE—METRIC SPACETIME

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A closed bosonic string in a curved affine—metric spacetime is considered as an example of dissipative quantum field theory. The conformal anomaly of the trace of the energy—momentum tensor for the string on the affine—metric manifold is investigated. The two-loop metric beta function for the two-dimensional nonlinear dissipative sigma model is calculated. Examples of nonflat manifolds that lead to ultraviolet-finite sigma models are found.

In [1,2], a nonlinear sigma model on an affine—metric manifold was studied, and the single- and two-loop counterterms in the case of a simple pole were obtained; these differ from the counterterms of the sigma model on a Riemannian manifold [3]. The question arises of the possibility of reducing the difference between the counterterms to a redefinition of the metric under an infinitesimal reparametrization of the manifold and to a nonlinear renormalization of the quantum fields. It is easy to show that the terms of the two-loop counterterm that are due to the nonmetricity cannot be reduced in the general case to the symmetric part of the covariant derivative on the Riemannian manifold of some vector field. In addition, the nonlinear renormalization of the quantum fields gives additional terms in the two-loop counterterm to the metric only for a ε^{-2} pole and does not give additional terms for a simple pole [4]. In connection with what we have said, the question arises of the reasons for the occurrence of the additional terms in the counterterms to the metric that are due to the nonmetricity, which was absent in the Lagrangian. To answer this question, we consider the connection between the equations of motion of the nonlinear one-dimensional sigma model and their geometrical interpretation.

It is well known that the motion of a classical mechanical system in a flat configuration space under the influence of external forces admits a geometrical interpretation. The geometrical description makes it possible to regard this motion as the free motion of a test particle in a curved configuration space. In the general case, motion in a field of potential forces is equivalent to free motion of a test body on a metric (Riemannian, Finslerian) manifold, i.e., a manifold whose metric and connection are compatible. Similarly, motion on a metric manifold under the influence of dissipative forces is equivalent to motion of a test body on a nonmetric manifold, i.e., a manifold whose metric and connection are not compatible. We shall consider this in more detail. Consider the equation of motion of a classical system

$$\frac{du^i}{dt} + Q^i(q,u) = 0, \tag{1}$$

where $u^i = dq^i/dt$, $i=1, \ldots, n$. We shall consider equations that are invariant with respect to general coordinate transformations, and we shall assume for simplicity that $Q^i(q, u)$ are homogeneous functions of second degree in u. It is well known that fulfillment of the Helmholtz conditions is necessary and sufficient for the existence of a Lagrange function. In this case there exists a matrix multiplier such that after multiplication by it the equation of motion can be represented in the form of an Euler—Lagrange equation. We note that the matrix multiplier is uniquely fixed by the condition of canonicity of the connection between the Hamiltonian generated by the given Lagrangian and the physical energy of the system [5]. On the other hand, it is well known that specification of a Lagrange function uniquely determines a metric in a $(n+1)$-dimensional configuration space [6]. Thus, the problem of solving the equations of motion is equivalent to the problem of finding a geodesic on a metric manifold. Note that if the metric does not or does depend on directions the manifold is, respectively, Riemannian or Finslerian. On metric manifolds, a connection structure is defined naturally, and its coefficients are the Christoffel symbols for the given metric. In the general case, Helmholtz's conditions, augmented by the physical requirement of the connection between the Hamiltonian and the total energy of the system [5], are not satisfied. In this case, the equations of motion of the system can be represented as motions on a metric manifold with metric determined by the Lagrange function under the influence of dissipative forces $Q^i_d(q, u)$ in the form

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$$\frac{du^i}{dt} + Q^i_p(q,u) + Q^i_d(q,u) = (g^{-1})^{ij} D_j L(q,u) + Q^i_d(q,u) = 0,$$

where D_j is the Euler—Lagrange operator, and $L(q, u)$ is the Lagrange function. In the general case, manifolds on which curves satisfying Eq. (1) are defined are not metric. Such manifolds are called spaces of generalized paths [12,13,14,15] and admit a natural definition of a connection whose coefficients are given in the form

$$\Gamma^i_{kl}(q,u) = \frac{1}{2}\frac{\partial^2 Q^i(q,u)}{\partial u^k \partial u^l}.$$

In spaces of generalized paths, the connection is not compatible with the metric determined by the holonomic part of the functional (by the Lagrangian). We note that the difference between the coefficients of the connection and the Christoffel symbols constructed from the given metric is called the tensor of the connection deficiency. As a result, we obtain a principle of relativity [16] for the motion of the system in the field of dissipative forces. In accordance with this principle, the motion of a body in a metric (Riemannian or Finslerian) space under the influence of dissipative forces is equivalent to the free motion of a test body on a nonmetric manifold (space of generalized paths). To obtain the equations of motion and the geodesic equation in a nonmetric manifold, Sedov's variational principle can be used [17,18]. At the same time, the nonholonomic functional is determined by the connection deficiency tensor [21]. The simplest example of a space of generalized paths is an affine—metric manifold (space of paths) [7,8,9,10,11], whose connection and metric do not depend on directions. Thus, the nonlinear sigma model on an affine—metric manifold is equivalent to the nonlinear sigma model on a Riemannian manifold in a field of dissipative forces [21]. Therefore, the systematic construction of string theory in a curved affine—metric spacetime must be done in the framework of the dissipative quantum scheme [19]. The proposed dissipative dynamics makes it possible to explain the existence of nonflat manifolds leading to ultraviolet-finite nonlinear sigma models [1].

In this paper, we consider the generalization of the two-dimensional nonlinear bosonic sigma model [22,3,23,24] and the sigma-model approach [26,25,27,28,30] to the quantum theory of strings [29] to the case of an affine—metric manifold that was proposed in [19,20] as an example of dissipative quantum theory. We discuss a method for obtaining the conformal anomaly of the trace of the energy—momentum tensor [28,27] for a closed bosonic string on an affine—metric manifold (i.e., in a field of dissipative and nondissipative massless background fields). We calculate the two-loop metric renormalization-group beta function [22,3] for the two-dimensional nonlinear dissipative bosonic sigma model proposed in [19]. The obtained results are compared with the ultraviolet counterterms obtained for the affine—metric sigma model considered in [1,2].

The classical equations of motion of a closed bosonic string in a curved n-dimensional affine—metric spacetime have the form

$$\partial_\mu \sqrt{g} g^{\mu\nu} \partial_\nu X^i + \left(\left[{}^{\,i}_{kl}\right] + D^i_{kl} \right) \partial_\mu X^k \sqrt{g} g^{\mu\nu} \partial_\nu X^l = 0, \tag{2}$$

where $\left[{}^{\,i}_{kl}\right]$ are the Christoffel symbols for the metric $G_{ij}(q)$, $D_{ikl}(X)$ is the connection deficiency tensor [32], and $g^{\mu\nu}(x)$ is the two-dimensional metric tensor. The equations of motion (2) describe a two-dimensional geodesic flow on an affine—metric manifold (two-dimensional analog of a geodesic). It is well known that this equation cannot be obtained from the principle of least action if the connection deficiency tensor is nonvanishing. Note that the equation of geodesic flow in a Riemannian manifold can be obtained from the principle of least action if the Lagrange function density is defined in the form

$$L(X) = \frac{1}{2} G_{kl}(X) \partial_\mu X^k \sqrt{g} g^{\mu\nu} \partial_\nu X^l. \tag{3}$$

To specify the system described by Eqs. (2), it is necessary to define in addition to the action a nonholonomic functional. In this case, the equations of motion (2) can be obtained from Sedov's variational principle [17,18,21], which generalizes the principle of least action to dissipative processes. We specify the variation of the nonholonomic functional in the form

$$\delta \tilde{W} = -\int d^2x\, D_{ikl}(X)\, \partial_\mu X^k \sqrt{g} g^{\mu\nu} \partial_\nu X^l\, \delta X^i. \tag{4}$$

Then the equation of the geodesic flow can be obtained from Sedov's principle if the Lagrangian density and nonholonomic functional are specified in the form (3) and (4).

The world surface swept out by the string in the process of its motion is described by a map $X(x)$ from the two-dimensional parameter space N to the n-dimensional spacetime manifold M, i.e., $X(x)$: $N \to M$. Let the two-dimensional parameter be specified in the form $x=(\tau, \sigma)$ and the map $X(x)$ be given by the spacetime coordinates $X^k(x)$. We choose the holonomic functional in the form

$$S(X) = S(G,\Phi,g) = \frac{1}{2}\int d^2x\,(\,L(X) \;+\; \frac{\alpha'}{2}\sqrt{g}R^{(2)}(g)\,\Phi(X), \tag{5}$$

where α' is the reciprocal tension of the string, and $\Phi(X)$ is the dilaton field. The action (5) and the Sedovian defined by the variational equation (4) describe a closed bosonic string that propagates in massless dissipative and nondissipative background fields or in a curved affine—metric spacetime. For simplicity, the Wess—Zumino term will not be considered [33]. We choose the parametrization for the two-dimensional metric tensor $g^{\mu\nu}$ in the form [31]

$$g_{\mu\nu}(x)dx^\mu dx^\nu = c(x)\,(\,n^2(x)(d\tau)^2 - (\,d\sigma + m(x)d\tau)^2\,). \tag{6}$$

In this case, the Hamiltonian density without the dilaton field, Sedovian, and omega can be rewritten in the form

$$h \;=\; -\,\frac{n}{2}\,G^{kl}(X)\Pi_k\Pi_l \;+\; m\,\Pi_k X'^k \;-\; \frac{n}{2}\,G_{kl}(X)X'^k X'^l, \tag{7}$$

$$w \;=\; \frac{n}{2}\,(\Delta_1^{kl}\Pi_k\Pi_l \;+\; \Delta_{kl}^2 X'^k X'^l)\;;\quad \Omega \;=\; 2n\,D^k(X)\Pi_k, \tag{8}$$

where $D^k(X) \equiv D^k_{ij}(X)G^{ij}(X)$, $X'^k \equiv (dX^k)/(d\sigma)$, Π_k is the canonical momentum, and Δ are tensor integral operators that can be nominally expressed in the form of multiple indefinite integrals with respect to δX^k:

$$\Delta_1^{kl} = 2\int \delta X^i D_i^{kl}(X) \quad \Delta_{kl}^2 = -2\int \delta X^i D_{ikl}(X). \tag{9}$$

Unfortunately, we do not have a rigorous mathematical definition of these operators. The difficulty can be avoided by considering an expansion of the nonholonomic function by the background-field method in the form of a power series with respect to the covariant fields $\xi^k(x)$, which are the tangent vectors to the geodesic connecting X_0^k and $X^k = X_0^k + f^k(X_0, \xi)$. The covariant background-field expansion of the Δ operator is expressed in the form

$$\Delta_1^{kl} = 2\,D_i^{kl}(X_0)\,\xi^i + O(\xi^2)\;,\qquad \Delta_{kl}^2 = -2\,D_{ikl}(X_0)\,\xi^i + O(\xi^2). \tag{10}$$

The covariant background-field method [3,1,20] in phase space is determined by expansion of only the coordinates $X^k(x)$. Note that the model defined by (4) [and (5)] in the conformal gauge $n=1$, $m=0$ is called the two-dimensional nonlinear (dissipative) sigma model. We define the generating functional for the connected Green's functions [39,38,4] in the form

$$W(J,g) = -i\hbar\ln\int DX\,D\Pi\exp\frac{i}{\hbar}\int d^2x\,(Z_1(X,\Pi,g) + Z_2(X,J)), \tag{11}$$

where

$$Z_1(X,\Pi,g) \equiv \Pi_k\frac{d}{d\tau}X^k - h + w + \frac{i\hbar}{2}\Omega + \frac{\alpha'}{2}\sqrt{g}R^{(2)}(g)\Phi(X), \tag{12}$$

and $Z_2(X, J)$ is the term with source whose form is discussed in [34,35,4,36,37,38]. Having obtained the covariant expansion by the background-field method for Z_1, Z_2, we defined the new generating functional $W(X_0, g, J)$ in the form

$$\exp\frac{i}{\hbar}(\,W(X_0,g,J) + \tilde{W}(X_0)\,) = \int D\xi\,D\Pi\exp\frac{i}{\hbar}\int d^2x(\,Z_1(X(X_0,\xi),\Pi,g) + J_k(x)\xi^k(x)\,). \tag{13}$$

The functional integral with respect to the momentum Π is a Gaussian integral. It is easy to obtain the generating functional in the form of a path integral:

$$W(X_0,g,J) = -i\hbar\;\ln\int D\xi\;\exp\frac{i}{\hbar}(\,A(X(X_0,\xi)) \;+\; M(X(X_0,\xi))\,). \tag{14}$$

The effective action $A(X)$ can be written in the form [19,20]

$$A(X) = S(G,\Phi,g) + S(D,g),$$

where

$$S(D,g) = S_1(D,g) + S_2(D,g) + S_3(D,g), \tag{15}$$

$$S_1 = -\int d^2x\, \frac{1}{2}\Delta^2_{kl}\partial_\mu X^k \sqrt{g} g^{\mu\nu}\partial_\nu X^l \; = -\tilde{W}(X), \tag{16}$$

$$S_2 = \int d^2x\, \frac{1}{2}F_{kl}(X)\partial_\mu X^k \sqrt{g}\kappa^{\mu\nu}\partial_\nu X^l, \tag{17}$$

$$S_3 = \int d^2x \sqrt{g}(V_{k\mu}g^{\mu\nu}\partial_\nu X^k + B(X)), \tag{18}$$

$$F_{kl} = [G^{-1} + \Delta_1]^{-1}{}_{kl} - [G + \Delta^2]_{kl} = 4D_i{}^n{}_k D_{jnl}\xi^i\xi^j + O(\xi^3), \tag{19}$$

$$V_{k\mu} \equiv \frac{1}{2}g_{\mu\nu}k^\nu[G^{-1} + \Delta^1]^{-1}{}_{kl}D^l(X), \tag{20}$$

$$B(X) \equiv \frac{1}{2}c^{-1}(x)[G^{-1} + \Delta^1]^{-1}_{kl}D^k(X)D^l(X), \tag{21}$$

$$k^\mu = (k^\tau, k^\sigma) = (-2ic^{-1}, 2imc^{-1}), \tag{22}$$

$$\kappa^{\mu\nu} = (\kappa^{\tau\tau}, \kappa^{\tau\sigma}, \kappa^{\sigma\sigma}) = (-n^{-2}c^{-1}, mn^{-2}c^{-1}, -m^2n^{-2}c^{-1}) \tag{23}$$

and $D^l(X) = G^{lk}(G^{ij}D_{kij}(X)$. Note that the effective action is conformally invariant, since it does not depend on $c(x)$, and the parametrization of the two-dimensional tensors $\kappa^{\mu\nu}$ and k^μ is related to the parametrization of the two-dimensional metric tensor $g^{\mu\nu}$, i.e., $\kappa^{\mu\nu} = \kappa^{\mu\nu}(g)$ and $k^\mu = k^\nu(g)$. The term $M(X)$ is specified in the form

$$M(X) \;=\; \int d^2x\, \frac{i\hbar}{2}\,\delta(0)\;\ln\det(G^{-1}(X) + \Delta_1(X))^{-1}\;). \tag{24}$$

The energy—momentum tensor is defined, as usual [28,27,41], in the form

$$T^{\mu\nu}(x) = -\frac{2}{\sqrt{g}}\frac{\delta}{\delta g_{\mu\nu}}S(G, \Phi, g). \tag{25}$$

In the general case, the terms $S(D,\, g)$ are nonholonomic objects. Since we use the covariant background-field method [3,7,19,1], these objects are power series in the vector field $\xi^k(x)$. Note that expansion by the background-field method and variation with respect to the two-dimensional metric tensor are noncommuting operations for the nonholonomic functional, i.e., $(\delta\tilde{W}(X))/\delta g^{\mu\nu}) = 0$ and $(\delta\tilde{W}(X_0,\, \xi))/(\delta g^{\mu\nu}) \neq 0$. Therefore, the vacuum expectation value of the energy—momentum tensor [41],

$$\langle T^{\mu\nu}(x)\rangle \equiv N\exp\left(-\frac{i}{\hbar}W(J,g)\right)\int D\xi\; T^{\mu\nu}(x)\;\exp\frac{i}{\hbar}(A(X) + M(X)), \tag{26}$$

cannot be expressed in the form $-2/(\sqrt{g})(\delta W(J,\, g))/(\delta g_{\mu\nu})$, this being due to the treatment of the nonholonomic functionals only in the framework of the background-field expansion (otherwise it is necessary to obtain expressions for Gaussian integration with respect to the momenta in the functional integral for the tensor integral operators Δ outside the framework of the background-field method). We define the bimetric generating functional

$$W(g^{\mu\nu}, a^{\mu\nu}, X_0, J) \equiv -i\hbar\;\ln\int D\xi\exp\frac{i}{\hbar}\left(S(G, \Phi, g^{\mu\nu}) + S(D, a^{\mu\nu}) + M(X) + \int d^2x J_k\xi^k\right).$$

The ordinary functional $W(g,\, X_0,\, J)$ is readily obtained: $W(g,\, X_0,\, J) = W(g,\, a,\, X_0,\, J)_{g=a}$. The vacuum expectation value (26) can be written in the form

$$\langle T^{\mu\nu}(x)\rangle \;=\; \left[-\frac{2}{\sqrt{g}}\frac{\delta}{\delta g_{\mu\nu}}W(g, a, X_0, J)\right]_{g=a}. \tag{27}$$

It is easy to obtain the conformal anomaly of the trace of the energy—momentum tensor [28] for the string in the curved affine—metric spacetime:

$$\langle T^\mu_\mu \rangle = \frac{1}{2}\,\tilde\beta^G_{kl}\,\partial^\mu X^k_0 \partial_\mu X^l_0 + \frac{\alpha'}{2}\,R^{(2)}(g)\,\tilde\beta^\Phi\,, \tag{28}$$

where

$$\tilde\beta^G_{kl} = \bar\beta^G_{kl} + \dots;\quad \tilde\beta^\Phi = \bar\beta^\Phi - \frac{1}{4}\bar\beta^G_{ij}\,(G^{ij} + \dots), \tag{29}$$

$$\bar\beta^G_{kl} = \beta^G_{kl} + 2\alpha'\,\hat\nabla_k\hat\nabla_l\Phi\,;\quad \bar\beta^\Phi = \beta^\Phi + \alpha'\hat\nabla_k\Phi\hat\nabla^k\Phi, \tag{30}$$

$$\beta^G_{kl} = \mu\frac{d}{d\mu}G^{ren}_{kl}\,;\ \hat\nabla_k V_l \equiv \partial_k V_l - (\,[{}^{\,i}_{kl}] + D^i_{(kl)}\,)\,V_i. \tag{31}$$

The law of variation of the energy—momentum tensor can be written in the form

$$\nabla^\mu T_{\mu\nu} = D_{ikl}(X)\,\partial_\mu X^k g^{\mu\nu}\partial_\nu X^l\,\partial_\nu X^i. \tag{32}$$

If we take into account the covariant background-field expansion [3,1] for $\partial_\mu X^k = C^k_l(X_0,\ \xi)\partial_\mu X^l_0$, the vacuum expectation for this law can be written in the form

$$\langle \nabla^\mu T_{\mu\nu}\rangle = \Big\langle \frac{\delta W(X_0,\xi)}{\delta X^k_0}\Big\rangle \partial_\nu X^k_0. \tag{33}$$

As usual, we choose the following solution of the classical equations of motion: $X^k_0(x)=\text{const}$. Then Eq. (32) can be rewritten in the usual form [27,41]: $\langle\nabla^\mu T_{\mu\nu}\rangle=0$. It is easy to show that the central charge of the Virasoro algebra [44] is proportional to the dilaton β function, as in the nondissipative case [27]. A sufficient condition of validity of this relation [27] is $\tilde\beta^\Phi=\text{const}$ and $\tilde\beta^G_{kl}=0$, where $\tilde\beta^G$ is determined by the metric beta function of the two-dimensional nonlinear dissipative sigma model. In the calculation of the two-loop metric beta function we have used the affine—metric background-field method [7,8,9,10,11,1,2], introduced an auxiliary mass term [45,46], dimensional regularization $2\to n=2-2\varepsilon$ [43,42], and minimal subtraction with general prescription for the contraction of the two-dimensional kappa tensor: $\kappa^{\mu\nu}\eta_{\mu\nu}=f(n)$, where $f(n)=1+f_1\varepsilon+O(\varepsilon^2)$ and $\eta_{\mu\nu}$ is the two-dimensional Minkowski metric. Different prescriptions evidently correspond to different renormalization schemes, and the results must be related through a redefinition of the coupling constants G_{kl} and F_{kl} by analogy with the Riemannian two-dimensional nonlinear sigma model with Wess—Zumino term [48]. It is well known that the propagator of the quantum fields $\xi^k(x)$ does not have the standard form. It is therefore necessary to introduce m vectors $e^a_k(X)$ and define the field $\xi^a(x)=e^a_k\xi^k(x)$, where $\hat\nabla_k e^a_l=0$. After this modification, the kinetic term takes the form $\hat\nabla_\mu\xi^a\hat\nabla_\nu\xi^a$, where $\hat\nabla_\mu\xi^a=\partial_\mu\xi^a+\hat\Lambda^a_{bc}e^b_k\partial_\mu X^k_0\xi^c$. This mixed derivative [47] for the affine—metric manifold M and Minkowski space N includes the Schouten—Vranceanu connection [49,50,51] $\hat\Lambda_{abc}$, which on Riemannian manifolds is constituted by the Ricci rotation coefficients [52], while the object $\omega^a_{kb}\equiv\hat\Lambda^a_{bc}e^b_k$ is the spin connection [27]. Note that in addition to the contributions taken into account in [19,20] it is necessary to consider diagrams whose external background-field lines include the Schouten—Vranceanu connection. Such diagrams do not cancel [21], in contrast to the case of the Riemannian nonlinear sigma model [27]. This is due to the relation $\hat\Lambda_{(a/b/c)}=(-1/2)(K_{ijl}+2Q_{(ij)l})e^i_a e^j_c e^l_b$, where K_{ijl} is the tensor of the nonmetricity of the affine—metric manifold, and Q^k_{ij} is the torsion tensor. The two-loop metric beta function of the dissipative sigma model can be represented in the form $\beta^G=\beta^G_{AM}+\beta^G_1+\beta^G_2$, where β^G_{AM} is the metric beta function [22,3] of the affine—metric sigma model determined in [1,2]; β^G_{AM} is the part of the metric beta function obtained from the action $S(G,\ \Phi,\ g)$ alone, in which the two-dimensional metric is the Minkowski metric; β^G_K, $K=1,\,2$, is the part of the metric beta function obtained with allowance for the actions $S_K(D,\,g)$ defined in Eqs. (15)—(23). Note that the two-loop metric beta function β^G_3 is equal to zero. This is analogous to the results for the nonlinear sigma model considered in [53,54,55]. The complete expression for the two-loop ultraviolet counterterms is very complicated, but it is easy to obtain the following conditions of ultraviolet finiteness. The single-loop and two-loop counterterms of the two-dimensional nonlinear dissipative sigma model vanish if compatibility between the affine connection and the metric structure on the manifold is specified as follows [19]:

$$\nabla_k G_{ij} = N_{ijk} = N_{(ijk)};\quad \hat\nabla_{(l}N_{k)ij} = N^p_{i(k}N_{l)jp};\quad Q_{(ij)l}=0;\quad \hat R_{(k/(ij)/l)} = \frac{1}{4}\,N^p_{(k/(i}N_{j)/l)p}. \tag{34}$$

It can be seen that the conditions of ultraviolet finiteness do not contain a dependence on the parameter f_1. Note that the beta function of the affine—metric sigma model is equal to zero in all loops if the affine—metric manifold with nonmetricity tensor K_{ijl} and torsion Q^i_{kl} is defined as follows [21]:

$$\hat{R}_{kijl} \equiv R_{kijl} - 2\hat{\nabla}_{[j/}Q_{ki/l]} - 2Q^{n}_{i[l/}Q_{kn/j]} = 0, \tag{35}$$

$$\hat{\nabla}_k G_{ij} = K_{ijk} - 2Q_{(ij)k} = 0. \tag{36}$$

It is readily seen that these affine—metric manifolds are not flat.

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