Symbolic Dynamics in the Equal Mass Free-Fall Three-Body Problem: Analysis of Ergodic Components

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We consider equal mass free-fall three-body problem. Symbolic sequences are constructed numerically using close binary approaches. Shannon entropy is estimated for each sequence as well as length of the sub-sequence that provides maximum value of the entropy for each sequence. Here, we analyse some features revealed on the diagram maximum value of the entropy - corresponding length of the sub-sequence (see Fig. 3 below).

Equal mass free-fall three-body problem is convenient for study since it allows easy visualization of initial configuration: if we place two bodies in the points $(-0.5;0)$ and $(0.5;0)$, then all possible configurations will be covered if we place the third body inside the region D bounded by two straight line segments and arc of the unit circle centered at $(-0.5,0)$ (Fig. 1) [1].

Raspberry Pi cluster was used for numerical integration of trajectories and construction of symbolic sequences, Wolfram Mathematica is used to analyze sequences received. We used symplectic code by Seppo Mikkola (Tuorla Observatory, University of Turku) [2] for numerical simulations.

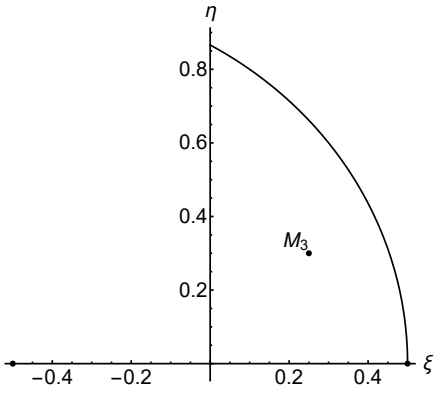

Figure 1: Agekian-Anosova region D.

We scan Agekian-Anosova region D and construct symbolic sequences of length 50 using close binary approaches – we detect minimum distance between two bodies, and corresponding symbol is the number of the distant body. Thus, our symbols are from the alphabet {1, 2, 3}. Some systems disrupt fast, so some sequences are short. Some systems live long (e.g. metastable systems [3]), so corresponding sequences are long. To have a reasonable computing time, we constructed symbolic sequences length 50. Since we are interested in the analysis of active three-body interactions, we consider sub-sequences of each of these sequences, increasing the length step-by-step, calculate entropy for each of these sub-sequences, and find maximum value of these entropies. Maximum value (and moment of time/length of the sub-sequence) correspond to the stage of active interaction between bodies.

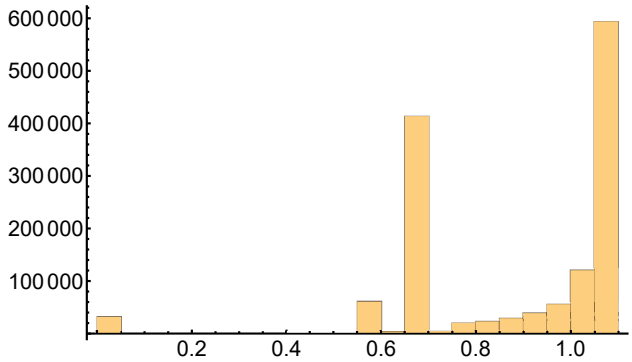

Figure 2: Histogram of maximum values of the entropy.

Histogram of maximum values of the entropy shows two distinct modes (Fig. 2). Left mode corresponds to the sequences with only two symbols equally represented: Entropy[{1, 2, 1, 2}]=0.693147. Second mode corresponds to the sequences where all three symbols are equally presented: Entropy[{1, 2, 3, 1, 2, 3}]=1.09861. Interesting structures can also be seen on the scatterplot of maximum values of the entropy - corresponding length of symbolic sequence in the neighborhood of these modes (Fig. 3). We analyze these structures and trace coresponding initial conditions in the Agekian-Anosova region D.

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References

[1] Agekian, T.A. and Anosova, J.P. 1967, Astron. Zh., 44, 1261

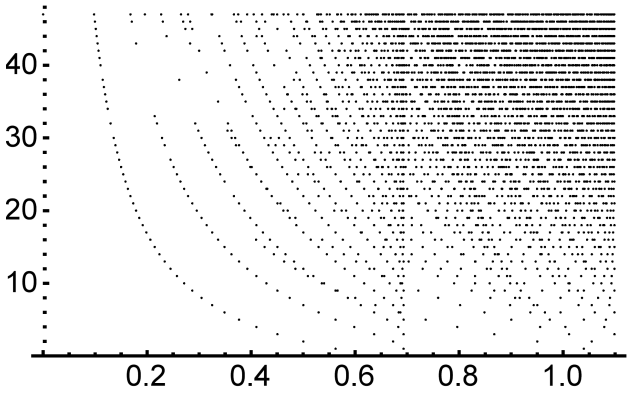

Figure 3: Scatterplot of maximum values of the entropy - corresponding length of symbolic sequence.

[2] Mikkola, S. and Tanikawa, K. 1999, Celest. Mech. Dyn. Astron., 74, 287-295.

[3] Martynova A.I., Orlov V.V., Rubinov A.V.,2003, MNRAS, 344, 1091