Heavy Quark Expansion in Beauty:

Achievents and Perspective

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- We have the QCD-based theory of B decays

- It works at the nonperturbative level

  impressive (at times) agreement with experiment
  gives nontrivial predictions
  allows precision extraction of $|V_{cb}|$ and $|V_{ub}|$
  makes suggestions for next generation experiments

- There are problems† which are to be clarified

  Theoretical insights plus experimental data are needed

Inclusive semileptonic distributions and $B \to D\,\ell\nu$

HQ sum rules, inequalities and their saturation

---

†of more than simply a technical nature

Expansion in $\frac{\Lambda_{\rm QCD}}{m_b}$ requires dynamics

Physics of the heavy quark is simple. Dynamics of light degrees of freedom in the presence of the heavy quark

Bound-state $\longleftrightarrow$ nonperturbative effects

Can they be controlled?

QCD allows to establish a number of facts

Most informative are inclusive decays

admit local OPE

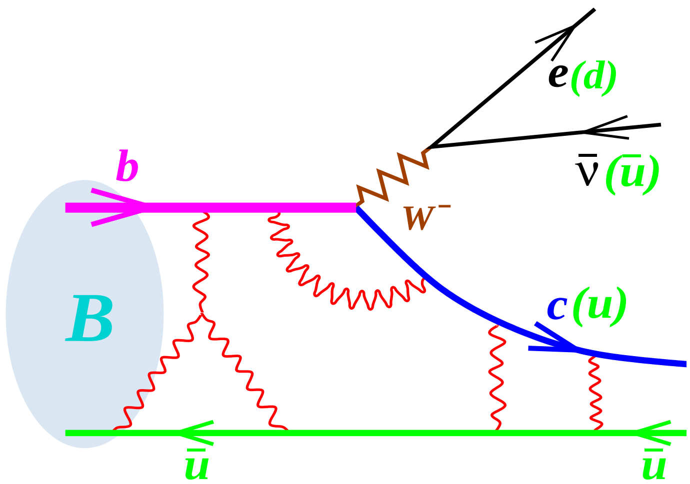
Certain dynamical predictions are quite nontrivial

More precise statements are those of most general nature, hence independent of a possible mechanism of confinement, resulting hadron spectrum, ...

To zeroth order do not probe the physics of the bound states

In fact, a closer scrutiny does

Lifetimes and inclusive decay widths



Quark level:

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3} \cdot z(m_c, m_b) \cdot N_c \cdot \left(1 - \frac{\alpha_s}{\pi} ...\right) \cdot ... \cdot \left\{ \begin{matrix} |V_{cb}|^2 \\ |V_{ub}|^2 \end{matrix} \right\}$$

No $\Lambda_{\rm QCD}/m_b$ corrections to inclusive widths of heavy flavor hadrons

Bigi, Shifman, Uraltsev, Vainshtein 1992

Applies to all types: semileptonic, nonleptonic, $b \to s + \gamma$, $b \to s\, \ell^+ \ell^-$, ...

$B,\ B_s,\ \Lambda_b, ...$ $\quad \dfrac{\Delta M}{M} \sim \dfrac{\Lambda_{\rm QCD}}{m_b}$ yet $\dfrac{\Delta \Gamma}{\Gamma} \sim \dfrac{\Lambda^2_{\rm QCD}}{m_b^2}$

$$M_B = m_b + \overline{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_b} + \frac{\mu_3^2}{m_b^2} + \ldots$$

$\overline{\Lambda}$ does not affect the width!

Exclusive property of QCD. Follows from the gauge nature of QCD interaction

Exact cancellation of the bound state effects with the final state interaction

Bound state & hadronization effects are given by `local` HQ operators $\bar{b}Ob$

Order $1/m_b^2$: $\mu_\pi^2 = \langle B|\,\bar{b}\,(i\vec{D})^2 b\,|B\rangle\,,\quad \mu_G^2 = \langle B|\bar{b}\tfrac{i}{2}\sigma G b|B\rangle$

Order $1/m_b^3$: $\rho_D^3 \propto \langle B|\bar{b}\,\Gamma b\,\bar{q}\,\Gamma q|B\rangle\,,\quad \rho_{LS}^3 \propto \langle B|\vec{\sigma}\cdot\vec{\pi}\times\vec{E}|B\rangle$

etc.

Checkpoints:

Lifetimes of Beauty Hadrons

BSUV 1992

OPE:

$$\delta\,\tau_{H_b} \sim \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m_b^2}\right) + \mathcal{O}\left(\frac{\Lambda^3_{\rm QCD}}{m_b^3}\right) + \ldots$$

- $1/m_b$: No effects
- $1/m_b^2$: $-\frac{1}{2}\frac{\mu_\pi^2}{m_b^2} - c_G\frac{\mu_G^2}{2m_b^2}$ mesons vs. baryons
- $1/m_b^3$: $\langle B|\bar{b}\Gamma q\cdot\bar{q}\Gamma' b|B\rangle$ B^+ vs. B^0 vs. B_s ...

Weak Annihilation Pauli Interference Weak Scattering

mesons (Weak Annihilation, Pauli Interference) baryons (Pauli Interference, Weak Scattering)

Bilić, Guberina, Trampetić 1984
Shifman, Voloshin 1985

$$\frac{\tau_{B^-}}{\tau_{B^0}} \approx 1.05$$ BU 1992 $$1.086 \pm 0.017$$ exp

$$\left|\frac{\bar{\tau}_{B_s}}{\tau_{B^0}} - 1\right| \lesssim 0.02$$ BU 1992 $$0.951 \pm 0.038$$ exp

$$\frac{\tau_{\Lambda_b}}{\tau_{B^0}} \gtrsim 0.9$$ $$0.80 \pm 0.05$$ exp

$\mathrm{BR}_{\mathrm{sl}}$

$\mathrm{BR}_{\mathrm{sl}}$ vs. n_{charm}

$\mathrm{BR}_{\mathrm{sl}} \simeq 10.7\%$ seems on the lower side

Requires fresh scrutiny. Now theory must be able to calculate more accurately both $\mathrm{BR}_{\mathrm{sl}}$ and n_c separately, modulo reliability of the $b \to c\,\bar{c}s$ channel

Both `problem points` involve nonleptonic decay widths

Larger corrections $\Longrightarrow$ less clean

Semileptonic decays offer much better theoretical environment

Semileptonic decays

Practical applications: Extracting $|V_{cb}|$, $|V_{ub}|$
from $\Gamma_{\rm sl}(B)$

Need accurate values of QCD parameters
m_b, m_c $(m_b - m_c)$, μ_π^2, μ_G^2, ρ_D^3, ...

Replace models and their attributes used early on

m_b, m_c, μ_π^2, ... (properly defined) can be determined from the semileptonic $(b \to s + \gamma)$ decay distributions themselves

BSUV, 1993-1994

Long history: incomplete theory, eliminate m_c relying on $\frac{1}{m_c}$ expansion, ...

We can do robust analysis without relying on $1/m_c$ expansion, or invoking unknown *nonlocal correlators*

Expansion in $1/m_c$ is questionable

Nowadays is being implemented in experiment

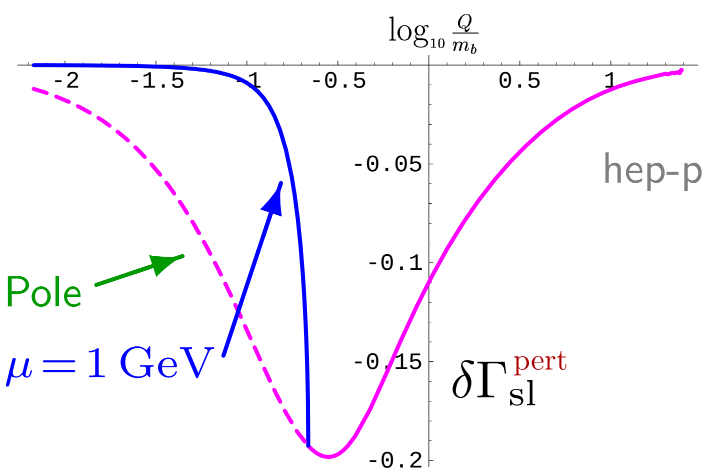
Theoretical status

Can aim at $1\,\%$ level in $|V_{cb}|$ assumes technical progress in theory

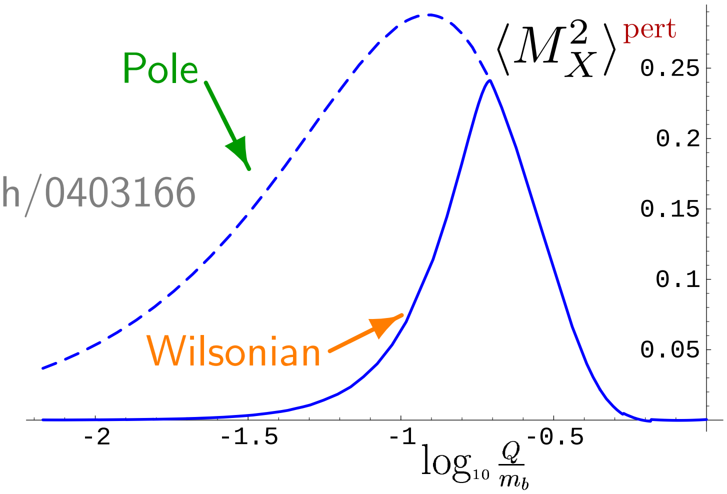
$|V_{ub}|$? – underway, 5% accuracy is realistic

An often question: How can this be true?

Perturbative corrections? ...

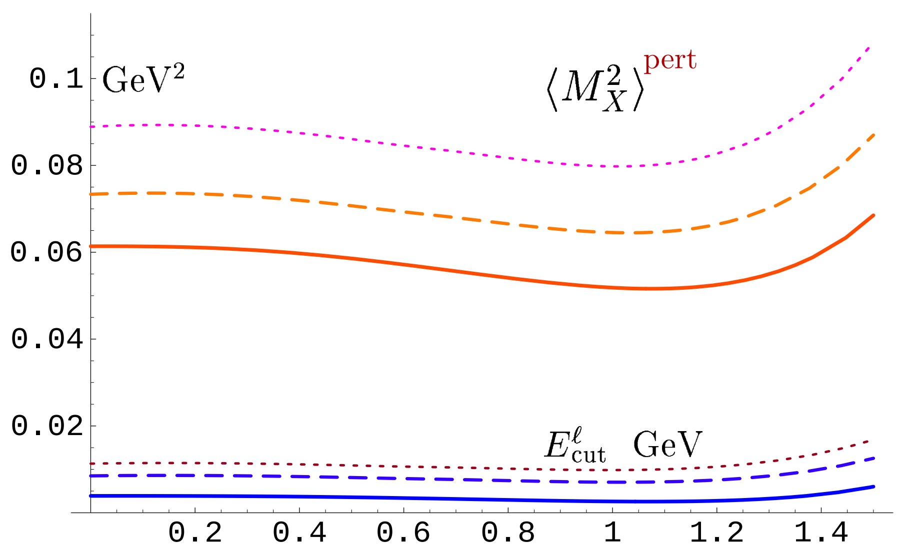


hep-ph/0403166



With the IR piece cut off according to Wilson
we can `work for precision!`

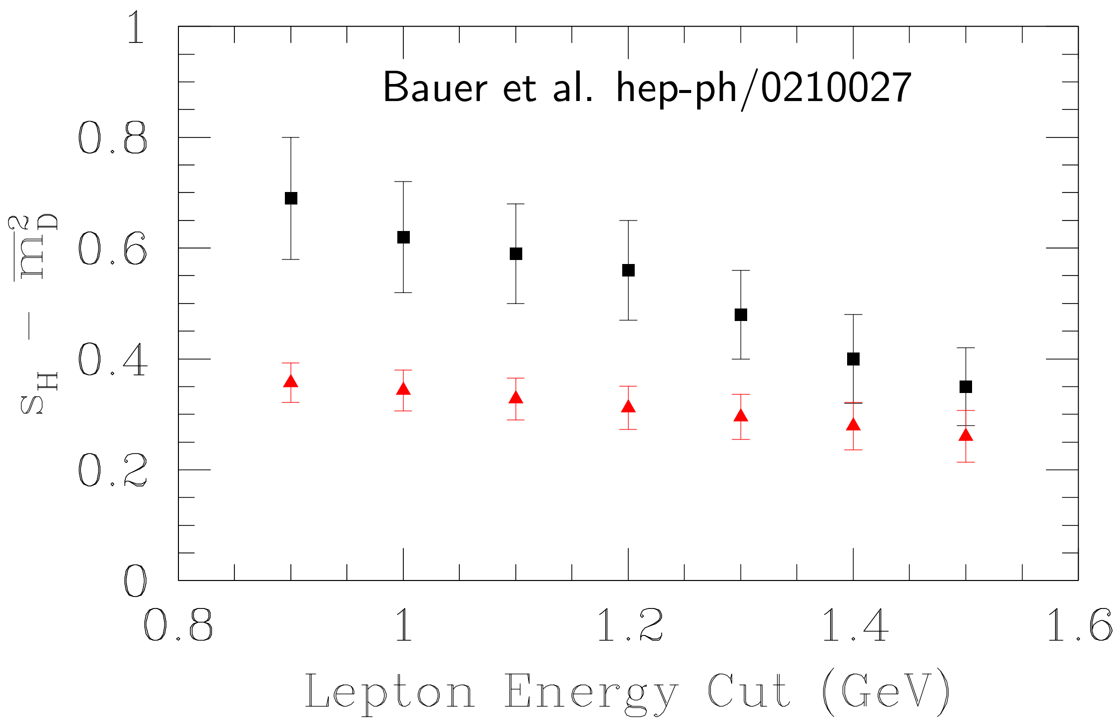
Corrections in the scheme with the hard cutoff, $\mu=1$GeV. Within `pole-type` approaches the correction is 4-6 times larger and strongly decreases at larger E_{cut}^ℓ



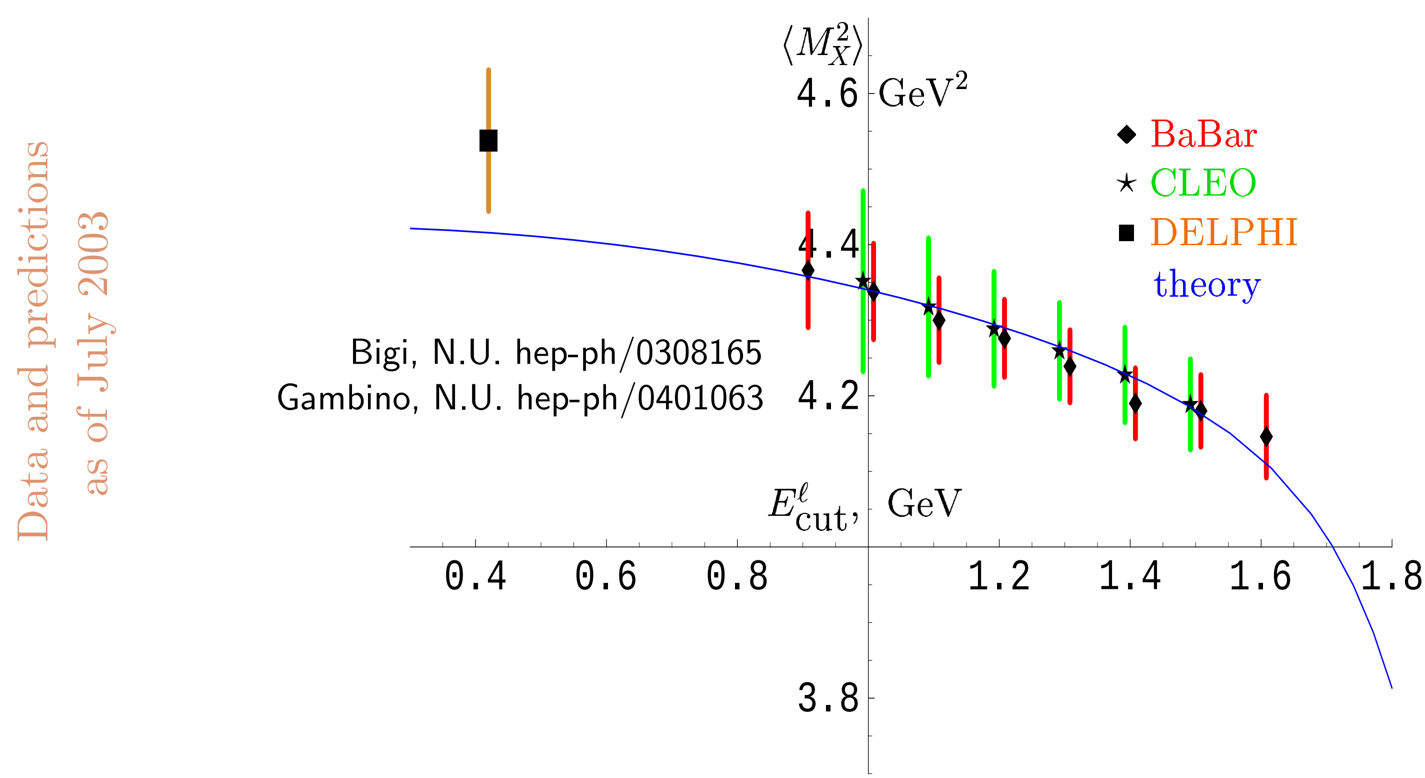
Now$_{2004}$ all pure perturbative corrections have been calculated

N.U.; M. Trott

• Problem for theory with $\langle M_X^2 \rangle$ vs. E_{cut}^{ℓ} ?



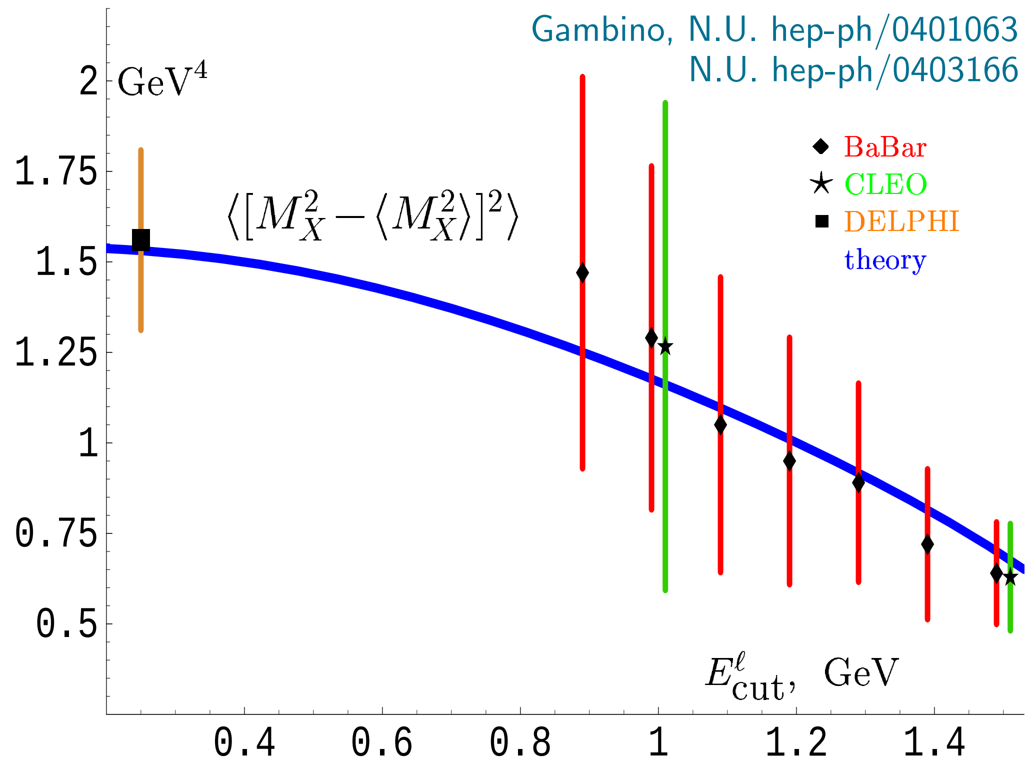
Robust OPE approach à la Wilson, $\mu = 1\text{GeV}$:



OPE seems to work even where may be expected to break down

Second mass moment $\langle [M_X^2 - \langle M_X^2 \rangle]^2 \rangle$:

Parameters fixed from the BaBar fit
hep-ex/0404017



Good agreement where the right theory is used

Present stage:

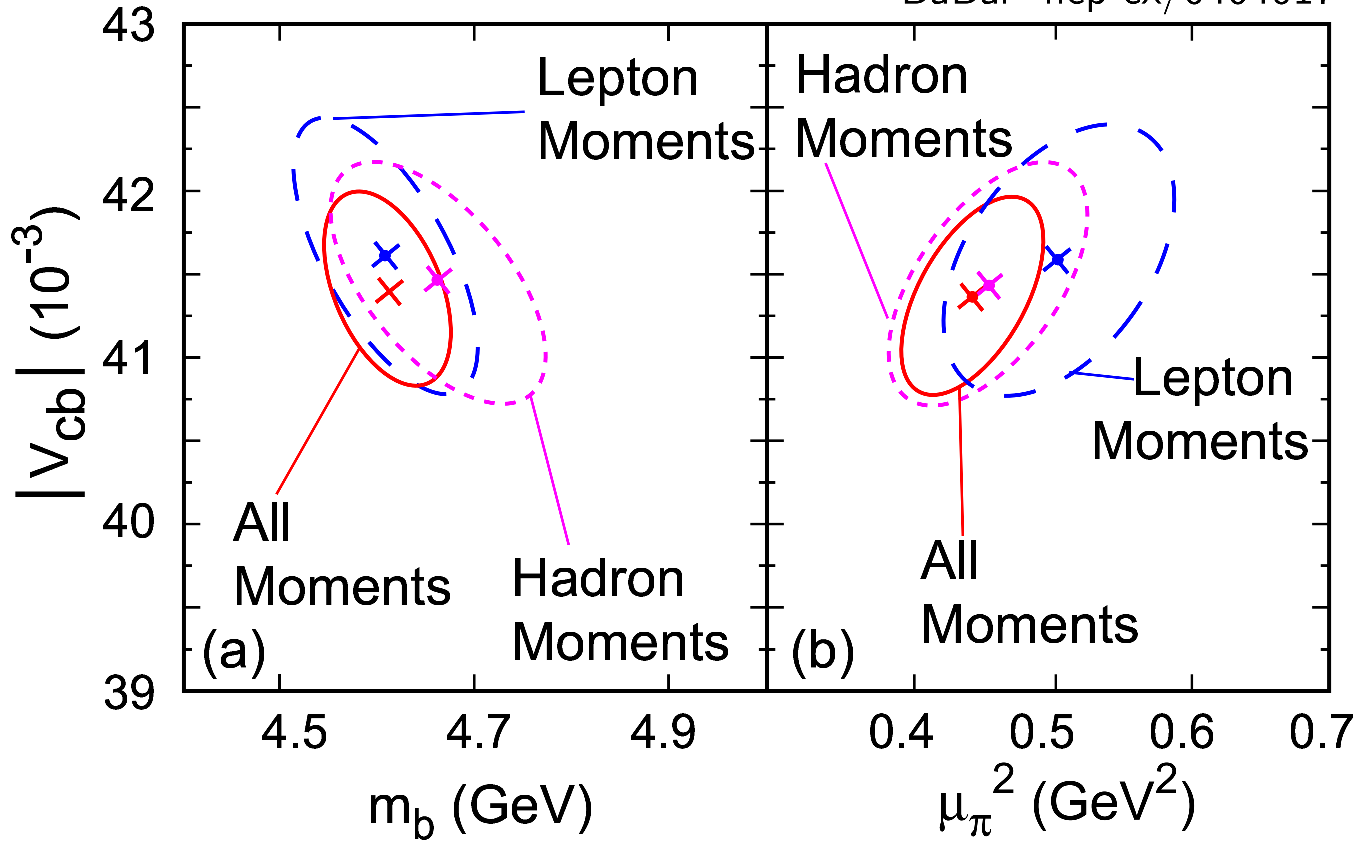
- Have an accurate and reliable determination of many HQ parameters from experiment

- Extracting $|V_{cb}|$ from $\Gamma_{\rm sl}(B)$ has good accuracy and solid grounds

- Have precision checks of the OPE at the nonperturbative level

I think the most impressive is good consistency between $\langle M_X^2 \rangle$ and $\langle E_\ell \rangle$: A sensitive check of the nonperturbative sum rule for $M_B - m_b$

BaBar hep-ex/0404017



Surprise: SL decays at BaBar yielded accurate m_b itself...

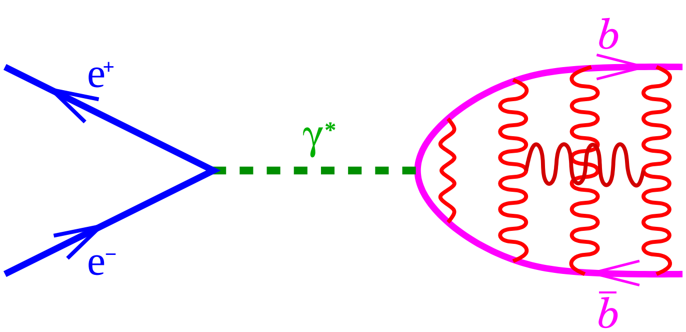
The combination $m_b - 0.74\, m_c$ is determined with only $17\,\mathrm{MeV}$ error bar!

Running mass is an observable and has no intrinsic limitation on precision

Theoretical expectation:

$$m_b(1\,\mathrm{GeV}) = (4.57 \pm 0.06)\,\mathrm{GeV}$$

Voloshin 1995–1996
Melnikov, Yelkhovsky
Hoang 1998–1999
Beneke, Signer



$e^+e^- \to \Upsilon(1S, 2S, 3S, 4S, 5S)$

moments of $\sigma(e^+e^- \to b\bar{b})$

$\mu_\pi^2,\ \mu_G^2$ — primary nonperturbative values in the HQE

$$\mu_G^2 = \frac{1}{2M_B}\langle B|\,\bar b\,\tfrac{i}{2}\,g_s\,\sigma_{\mu\nu}G^{\mu\nu}\,b\,|B\rangle \longleftrightarrow \langle B|-g_s\,\vec\sigma_b\vec B_{\rm chr}(0)\,|B\rangle_{\rm QM}$$

$$\mu_\pi^2 = \frac{1}{2M_B}\langle B|\,\bar b\,(i\vec D\,)^2\,b\,|B\rangle \longleftrightarrow \langle B|\,\vec p_b^{\,2}\,|B\rangle_{\rm QM}$$

$$\vec p_b \to \vec\pi_b = -i\vec D = -i\vec\partial - g_s\vec A$$

μ_G^2 determines `hyperfine` splitting: $M_{B^*} - M_B \simeq \frac{2}{3}\frac{\mu_G^2}{m_b}$

$$\mu_G^2(1\,{\rm GeV}) = 0.35^{+.03}_{-.02}\,{\rm GeV}^2$$

N.U. PLB 2002

$\mu_\pi^2(\mu) > \mu_G^2(\mu)$ **at any** μ rigorous inequality

BSUV; Voloshin 1993–1994

Theory: $\mu_\pi^2 \approx (0.45 \pm 0.1)\,{\rm GeV}^2$

Right at the central experimental value

Darwin expectation value emerges of the right scale $0.2\,{\rm GeV}^3$

- Inconsistency with $b \to s+\gamma$ moments?

Relying on relations *imprecise* with a high cut on E_γ

$$\langle E_\gamma \rangle = \frac{m_b}{2} + \ldots \qquad \langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = \frac{\mu_\pi^2}{12} + \ldots$$

A good way to accurately measure HQ parameters...

Bottle neck: 'Hardness' $\mathcal{Q}$ often gets too low with the cuts

$\mathcal{Q} \simeq m_b - m_c$ for total widths, but

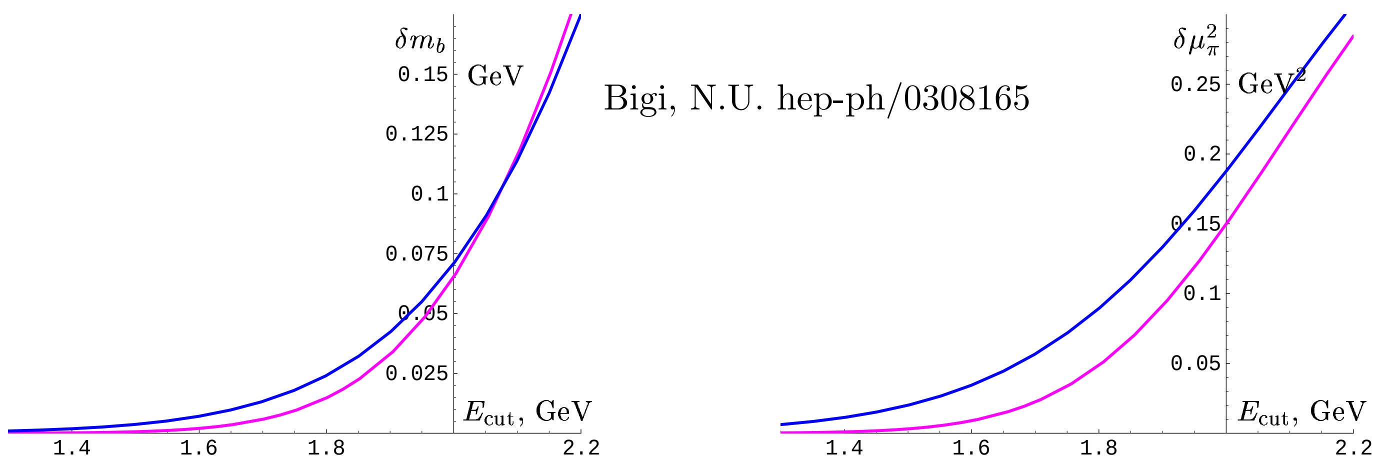
$\mathcal{Q}$ is `below` $1\,\mathrm{GeV}$ for $E_\ell > 1.7\,\mathrm{GeV}$

A complementary consideration suggests the expansion for M_X^2 loses sense for $E_{\rm cut} \geq 1.7\,\mathrm{GeV}$

Terms appear $\propto \; e^{-\frac{\mathcal{Q}}{\mu_{\rm hadr}}}$

In $b \to s + \gamma \qquad \mathcal{Q} \simeq M_B - 2E_{\rm min} \simeq 1.2\,\mathrm{GeV}$

if the cut is at $E_\gamma = 2\,\mathrm{GeV}$



Accounting for these biases yielded a good agreement between `all` measurements

BELLE 2004: With $E_\gamma > 1.8\,\mathrm{GeV}$ cut *biases* are not that much an issue

$$\begin{aligned}\langle E_\gamma\rangle &= 2.289 \;\pm 0.026_{\rm stat} \;\pm 0.0034_{\rm sys}\;\mathrm{GeV}\\ \langle [E_\gamma - \langle E_\gamma\rangle]^2\rangle &= 0.0311 \pm 0.0073_{\rm stat} \pm 0.0063_{\rm sys}\;\mathrm{GeV}^2\end{aligned}$$

For BaBar's HQ values we would obtain

$$\langle E_\gamma\rangle = 2.317\,\mathrm{GeV} \qquad \langle [E_\gamma - \langle E_\gamma\rangle]^2\rangle = 0.0329\,\mathrm{GeV}^2$$

Quite consistent!

Adding this to the BaBar data yields only minor shifts in the fit:

$$m_b(1\mathrm{GeV}) \simeq 4.58\mathrm{GeV}, \qquad \mu_\pi^2(1\mathrm{GeV}) \simeq 0.45\mathrm{GeV}^2$$

no visible change in $|V_{cb}|$

$$m_b^{\overline{\rm MS}}(m_b) = 4.22 \pm 0.06\,\mathrm{GeV}$$

BaBar:

$$|V_{cb}| = (4.139 \pm .0437_{\rm exp} \pm .04_{\rm HQE} \pm .06_{\rm th}) \cdot 10^{-2}$$

The value of $|V_{cb}|$ from $B \to D^*\,\ell\nu$ rate near zero recoil is consistent within its uncertainties, at $F_{D^*}(0) \simeq 0.9$

What all this means?

OPE works well, the heavy quark parameters derived from experiment are consistent with the expectation based on independent theoretical considerations

Perturbative corrections have been calculated and are expectedly well behaved in the proper Wilsonian approach. No obstacles for precision calculations of truly inclusive short-distance observables

Need calculation of the perturbative corrections to the Wilson coefficients of power-suppressed operators (μ_π^2, μ_G^2, ρ_D^3)

This becomes a limiting factor

Kinetic value μ_π^2 emerges as theoretically expected

Does the precise value matter? It appears that

$$\mu_\pi^2 - \mu_G^2 \ll \mu_\pi^2$$ Interesting regime

Need to recall Heavy Quark Sum Rules

Recent development: D'Orsay sum rules Le Yaouanc et al.

Discard spin of `heavy quark` – then B, B^* are spin-$\frac{1}{2}$ hadrons
P-waves would be $j=\frac{1}{2}$ or $j=\frac{3}{2}$:

$$\frac{1}{2} \times 1 = \frac{1}{2} \oplus \frac{3}{2}$$

j – spin of "light cloud"

Two P-wave families:

$$P_{1/2}^{(n)} \longleftrightarrow \varepsilon_{1/2}^{(n)}, \ \tau_{1/2}^{(n)}$$

$$P_{3/2}^{(m)} \longleftrightarrow \varepsilon_{3/2}^{(m)}, \ \tau_{3/2}^{(m)}$$

Sum Rules in the HQ Limit

$\varrho^2 - \frac{1}{4} = 2\sum_m |\tau_{3/2}^{(m)}|^2 + \sum_n |\tau_{1/2}^{(n)}|^2$	Bj	1990
$\frac{1}{2} = 2\sum_m |\tau_{3/2}^{(m)}|^2 - 2\sum_n |\tau_{1/2}^{(n)}|^2$	N.U.	2000
$\frac{\overline{\Lambda}}{2} = 2\sum_m \epsilon_m |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n |\tau_{1/2}^{(n)}|^2$	Voloshin	1992
$\overline{\Sigma} = 2\sum_m \epsilon_m |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n |\tau_{1/2}^{(n)}|^2$	N.U.	2000
ORSAY SUM RULE	Le Yaouanc et al.	2000
$\frac{\mu_\pi^2}{3} = 2\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}|^2$	BSUV	1994
$\frac{\mu_G^2}{3} = 2\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}|^2$	BSU	1997
$\frac{\rho_D^3}{3} = 2\sum_m \epsilon_m^3 |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n^3 |\tau_{1/2}^{(n)}|^2$	Chow, Pirjol	1994
$-\frac{\rho_{LS}^3}{3} = 2\sum_m \epsilon_m^3 |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n^3 |\tau_{1/2}^{(n)}|^2$	BSU	1997

Second and Fourth sum rules are superconvergent

$$\epsilon_k = M_k - M_B$$

$$\langle B(v)|\,\bar{b}\,\gamma_0\,b\,|B(0)\rangle = 1 - \varrho^2 \frac{\vec{v}^{\,2}}{2} + \mathcal{O}(\vec{v}^{\,4})$$

$$\langle P^{(1/2)}(v_2)|\bar{b}\gamma_\mu\gamma_5 b|B(v_1)\rangle = -\tau_{1/2}\,(v_1 - v_2)_\mu$$

$$\langle P^{(3/2)}(v_2)|\bar{b}\gamma_\mu\gamma_5 b|B(v_1)\rangle = -\frac{1}{\sqrt{2}}\,i\,\tau_{3/2}\,\epsilon_{\mu\alpha\beta\gamma}\,\varepsilon^{*\alpha}\,v_2^\beta\,v_1^\gamma$$

spin of light cloud is $\begin{cases} \frac{1}{2} \text{ in } P^{(1/2)} \\ \frac{3}{2} \text{ in } P^{(3/2)} \end{cases}$

Sum rules yield strict inequalities

$$\varrho^2 > \tfrac{3}{4}\,, \qquad \overline{\Lambda} > 2\overline{\Sigma}\,, \qquad \mu_\pi^2 > \mu_G^2\,, \qquad \rho_D^3 > -\rho_{LS}^3$$

$$\rho_D^3 > |\rho_{LS}^3|/2$$

Likewise

$$\mu_\pi^2 \ge \frac{3\overline{\Lambda}^2}{4\varrho^2-1}\,, \quad \rho_D^3 \ge \frac{3}{8}\,\frac{\overline{\Lambda}^3}{(\varrho^2-\frac{1}{4})^2}\,, \quad \rho_D^3 \ge \frac{(\mu_\pi^2)^{3/2}}{\sqrt{3(\varrho^2-\frac{1}{4})}}$$

Similarly for W_- moments – $\overline{\Lambda}-2\overline{\Sigma}\,,\ \mu_\pi^2-\mu_G^2$, ...

Maximal physical information – advantage of 'kinetic' mass and other definitions based on the SV sum rules

Good example: bound $\varrho^2 > \frac{3}{4}$

N.U. 2000

Assuming the spin sum rule is saturated at $\mu = 1\,\mathrm{GeV}$ we have

$$\mu_\pi^2 - \mu_G^2 = 3\,\tilde{\varepsilon}^2 \cdot (\varrho^2 - \tfrac{3}{4})$$

Quite a constraint: $\left(\varrho^2 - \frac{3}{4}\right) = \frac{\mu_\pi^2 - \mu_G^2}{3\tilde{\varepsilon}^2} \lesssim 0.2\;\; (0.3)$

at $\mu_\pi^2 = 0.43\;(0.5)\,\mathrm{GeV}^2$ since $\tilde{\varepsilon} > 0.4\,\mathrm{GeV}$

ϱ^2 is probed in experiment

important for V_{cb}

radically affects $B \to D^*$

extrapolation to zero recoil

Recent UKQCD lattice is quite compatible with the prediction:

$$\varrho^2 = 0.83^{+.15\,+\,.24}_{-.11\,-\,.01}$$

hep-lat/0202029

Another application, to $B \to D\,\ell\nu$: expanding in $\mu_\pi^2 - \mu_G^2$

$$\frac{M_B + M_D}{2\sqrt{M_B M_D}}\, f_+(0) = 1.04 \pm 0.01 \pm 0.01$$

$\mu_\pi^2 \simeq \mu_G^2$ is a special point for B and D mesons!

At $\mu_\pi^2 = \mu_G^2$ there is a `functional` relation $\vec{\sigma}\vec{\pi}|B\rangle = 0$

$$\mu_\pi^2 - \mu_G^2 = \langle(\vec{\sigma}_Q\vec{\pi}_Q)^2\rangle_B = \langle 2m_Q\,\mathcal{H}_{1/m_Q}\rangle$$

reminiscent to a BPS state

Ultrarelativistic light cloud – antipode to NR quark models

Remarkable limit in many respects

$B \to D\,\ell\nu$ near zero recoil 2003

$$\langle D(p_2)|\bar{c}\gamma_\nu b|B(p_1)\rangle = f_+(p_1+p_2)_\nu + f_-(p_1-p_2)_\nu$$

$f_\pm \equiv f_\pm(\vec{q}^{\,2})$

One amplitude $J_0 = (M_B+M_D)f_+(0) + (M_B-M_D)f_-(0)$ at $\vec{q}=0$

HQ limit: $f_+ = \frac{M_B+M_D}{2\sqrt{M_B M_D}}$, $\quad f_- = -\frac{M_B-M_D}{M_B+M_D}\, f_+$

$$\frac{J_0}{2\sqrt{M_B M_D}} = 1 - a_2\left(\frac{1}{m_c}-\frac{1}{m_b}\right)^2 - a_3\left(\frac{1}{m_c}-\frac{1}{m_b}\right)^2\left(\frac{1}{m_c}+\frac{1}{m_b}\right) + \ldots$$

Corrections are well under control and small

Any amplitude with massless leptons depends, however solely on f_+, while only the combination of f_+ and f_- has no $1/m$ corrections

$F_+ \equiv \frac{2\sqrt{M_B M_D}}{M_B+M_D}\, f_+$ has $1/m_Q$ corrections since $\vec{J}$ has such a term...

Good news: we know it!

$$F_+ = 1 + \left(\frac{\overline{\Lambda}}{2} - \overline{\Sigma}\right)\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\frac{M_B - M_D}{M_B + M_D} - \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

Thanks to inclusive decays and exact sum rules we know $\frac{\overline{\Lambda}}{2} - \overline{\Sigma}$ (positive, but very small $\propto \frac{\mu_\pi^2 - \mu_G^2}{3\mu_{\rm hadr}}$)

Moreover, we know `all` power corrections are small

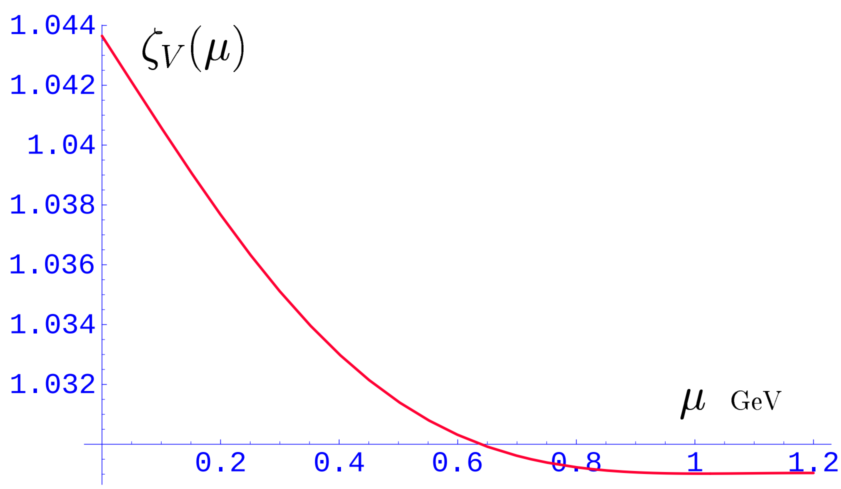
$$\frac{2\sqrt{M_B M_D}}{M_B + M_D}\, f_+(0) \;=\; 1.04 \pm 0.01 \;\pm\; 0.01$$

All orders in $1/m$ in 'BPS', to $1/m^2 \cdot 1/\mathrm{BPS}^2$, α_s^1

This formfactor is known better than for 'gold-plated' $B \to D^*$

Perturbative renormalization:

This can be done in the Wilsonian approach



Miracles of the 'BPS' limit

N.U. 2003

- $\varrho^2 = \frac{3}{4}$

  inclusive hadronic moments can tell us about the slope of the $B \to D^{(*)}$ formfactor!

- No power corrections to $M = m_Q + \overline{\Lambda}$ for the ground state

  $M_B - M_D = m_b - m_c$ to all orders in $1/m_Q$

- For $B \to D$ amplitude

  $f_-(q^2) = -\frac{M_B - M_D}{M_B + M_D}\, f_+(q^2)$ to any order in $1/m_Q$

- Zero recoil $B \to D$ amplitude: $\delta_{1/m^k} = 0$

  regardless of mass ratio

- In $B \to D$ at zero recoil

  $f_+ = \frac{M_B + M_D}{2\sqrt{M_B M_D}}$ to all orders in $1/m_Q$

- At arbitrary velocity power corrections in $B \to D$ vanish

  $$f_+(q)^2 = \frac{M_B + M_D}{2\sqrt{M_B M_D}}\, \xi\left(\frac{M_B^2 + M_D^2 - q^2}{2 M_B M_D}\right)$$

  Decay rate directly gives the IW function

Experiment: $B \to D$ slope much closer to $\varrho^2 \simeq 1$

Corrections to the shape of the $B \to D^*$ formfactor are way too significant

Quantifying Corrections to 'BPS'

How significant are corrections to 'BPS' relations in actual QCD ? It depends

The deviation parameter: Dimensionful parameter is

$\alpha = \|(\vec\sigma\vec\pi)\,|B\rangle\| \equiv \sqrt{\mu_\pi^2 - \mu_G^2}$ The dimensionless one is

$$\beta = \|\pi_0^{-1}(\vec\sigma\vec\pi)\,|B\rangle\| \equiv \sqrt{3\left(\varrho^2-\tfrac{3}{4}\right)} = 3\left(\sum_n\; |\tau_{1/2}^{(n)}|^2\right)^{\frac{1}{2}}$$

Numerically β is not a too small number, similar in size to generic $1/m_c$ expansion parameter β^2 should be good

We can count together powers of $1/m_c$ and β to judge the real quality of the HQ relations

At which order in β the 'BPS' relations can be violated to all orders in $1/m_Q$?

N.U. 2003

- Absence of corrections to $M_D = m_c + \overline{\Lambda}$, $M_B - M_D = m_b - m_c$ holds up to β^2
- Zero recoil $B \to D$ amplitude is unity up to β^2
- At arbitrary velocity relation between f_+ and f_- in $B \to D$ holds only to the leading order

$$f_-(q^2) = -\frac{M_B - M_D}{M_B + M_D}\, f_+(q^2) + \mathcal{O}(\beta)$$

- At arbitrary velocity the relations between $f_\pm$ in $B \to D$ and the IW function may receive corrections $\propto \beta^1$

- f_+ near zero recoil receives only `second order` corrections in β to `any order` in $1/m_Q$:

$$f_+\left((M_B - M_D)^2\right) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} + \mathcal{O}(\beta^2)$$

Analogue of the Ademollo-Gatto theorem for the 'BPS' expansion

the same applies to f_-

Must be quite accurate, f_-/f_+ can be checked in $B \to D\, \tau\nu_\tau$

If this can be measured, nothing else `exclusive` may be required for $|V_{cb}|$

Are all skies blue in SL decays? Not quite...

A "$\frac{1}{2} > \frac{3}{2}$" puzzle

Primary knowledge about heavy quark parameters comes from the Heavy Quark Sum Rules + the known size of μ_G^2

Sum rules explain why B^* is heavier than B; they set the scale of $\overline{\Lambda} = M_B - m_b$, μ_π^2, ...

Two classes: first for ϱ^2, $\overline{\Lambda}$, μ_π^2, ρ_D^3, ... These are saturated by both $\frac{3}{2}$ and $\frac{1}{2}$ P-wave heavy quark states

Second are 'spin' sum rules for $\varrho^2 - \frac{3}{4}$, $\overline{\Lambda} - 2\overline{\Sigma}$, $\mu_\pi^2 - \mu_G^2$, ... These include only $\frac{1}{2}$ states

Spin sum rules strongly suggest that $\frac{3}{2}$ P-wave states must dominate over $\frac{1}{2}$ states. This automatically happens in all quark models respecting QCD and Lorentz covariance

Orsay quark models

Experiment: $\frac{3}{2}$ charm P-wave states are narrow and well identified, $\{D_1,\ D_2^*\}$. They seem to contribute too little, $|\tau_{3/2}|^2 \approx 0.15$

Wide $\frac{1}{2}$ states $\{D_0^*,\ D_1^*\}$ are more abundant and might saturate the spin-singlet sum rules, but in aggregate they should be subdominant to $\frac{3}{2}$ states!

Average P-wave excitation mass gap

$$\bar{\epsilon}_P \simeq \frac{2\mu_\pi^2}{3\overline{\Lambda}} \approx 0.45\,\mathrm{GeV} \qquad \sqrt{\frac{\mu_\pi^2}{3(\varrho^2-\frac{1}{4})}} \approx 0.45\,\mathrm{GeV}$$

Typical τ^2

$$\bar{\tau}^2 \simeq \tfrac{1}{3}\,(\varrho^2-\tfrac{1}{4}) \simeq 0.25 \qquad \frac{\overline{\Lambda}}{6\bar{\epsilon}_P} \approx 0.25$$

and $\tau_{1/2}^2 \ll \tau_{3/2}^2$ from the spin sum rules

The most natural solution of all HQSRs:

$\frac{3}{2}$ states at $\epsilon_{\frac{3}{2}} \approx 450\,\mathrm{MeV}$ and $\tau_{\frac{3}{2}}^2 \approx 0.3$ while

$\tau_{\frac{1}{2}}^2 \approx 0.07 \div 0.12$ with $\epsilon_{\frac{1}{2}} \approx 300 \div 500\,\mathrm{MeV}$

Possible resolutions:

Contribution of the excited P-wave states ...

Charm is too light to apply this classification itself, valid only for heavy quarks; extraction of τ's is not justified

Need a good physical reason to invert the hierarchy

Too light c quark... An insight from lattices?

Resolution of this controversy is an important task, probably needs both theory ideas and more experimental data

Conclusions:

The dynamic OPE has finally undergone and passed critical precisions checks at the nonperturbative level
in semileptonic and radiative decays

Experiments find consistent heavy quark parameters from quite different measurements

$|V_{cb}|$ extraction has high accuracy and is based on reliable theory

Similar robust results are anticipated soon for $|V_{ub}|$

Inclusive studies yield crucial info for HQ physics, even for exclusive amplitudes   Formerly viewed as antipodes

Power corrections to HQ symmetry are very significant in charm.   There is a subset of relations which are stable, they are limited to the ground-state pseudoscalar B and D mesons, but exclude `spin` symmetry for charm

The scale of nonperturbative effects $\gtrsim \sqrt{\mu_\pi^2} \simeq 0.7\,\mathrm{GeV}$
they look small for 'BPS'-protected corrections where

$$\sqrt{\mu_\pi^2 - \mu_G^2} \simeq 0.3\,\mathrm{GeV} \approx m_q^{\mathrm{constit}}$$

Experiment must verify the kinetic expectation value with higher accuracy and fidelity,   extract more reliably ρ_D^3
in inclusive decays

Perturbative corrections to Wilson coefficients of power-suppressed operators are needed

$B \to D$ decays can be reliable theory-wise

If $\mu_\pi^2 \lesssim 0.45\,\mathrm{GeV}^2$ is firmly established then

$\mathcal{F}_+(0) \simeq 1.04$ is an accurate prediction for $B \to D$

A number of nontrivial consequences of this regime

Slope ϱ^2 is close to 1 -

$B \to D^{(*)}\,\tau\nu$ and $B \to X_c\,\tau\nu$ offer a number of interesting possibilities

Recent success of the QCD-based dynamic theory of nonperturbative physics in heavy quarks also raises new problems

Saturation of the HQ SV sum rules must be understood

A "$\frac{1}{2} > \frac{3}{2}$" puzzle

needs both theoretical and experimental scrutiny

Digression on $m_Q \to 0$

No power corrections to HQ relations at all? $M_D = m_c + \overline{\Lambda}$ exactly? What if $m_Q \ll \Lambda_{\rm QCD}$ when it is like a K meson?

No, BPS relations would not apply to light mesons even in a 'BPS' world

BPS cannot be exact in QCD – it may be a property of soft dynamics below $1\,{\rm GeV}$. The corrections would blow up at m_Q below some hadronic scale

Power expansion is asymptotic. Even if all `power` terms vanish, there are exponential terms

$$e^{-\frac{2m_Q}{\mu_{\rm hadr}}}$$

for instance due to spectral density of -π_0 at values exceeding $2m_Q$

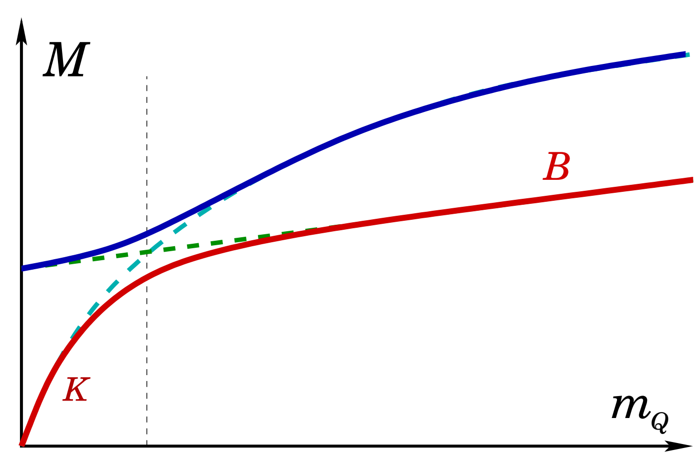
Whatever close we are to BPS corrections are of order 1 below $\mu_{\rm hadr}$

Two obvious mechanisms:

Chiral symmetry breaking – $\langle \bar{Q}Q \rangle \neq 0$ below some mass

a formal solution – even if exact – may not be the actual one on the true vacuum

Usual QM level crossing



$$\mu_\pi^2 > \mu_G^2$$

$$\mu_\pi^2 \longleftrightarrow \langle \vec{p}_b^{\,2} \rangle_B \qquad\qquad \mu_G^2 \longleftrightarrow \left| \vec{B}_{\rm chr}(0) \right|$$

$$\vec{P}_b \to \vec{\pi} = -i\vec{D} = -i\vec{\partial} - \vec{A} \qquad\qquad [P_j, P_k] = 0$$

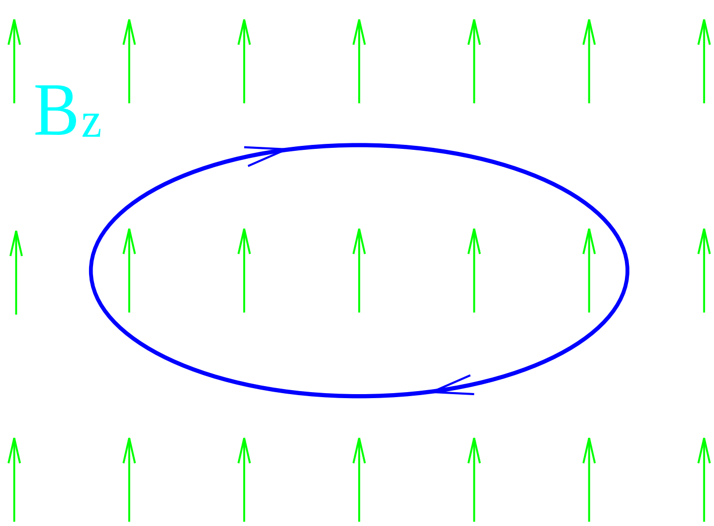
$$[\pi_j, \pi_k] = iG_{jk} = -i\epsilon_{jkl}B^l$$

$\sim 0.4\,\mathrm{GeV}^2$

$[\pi_j, \pi_k] \neq 0 \quad \Longrightarrow \quad$ an uncertainty relation

All components of momentum cannot be small simultaneously

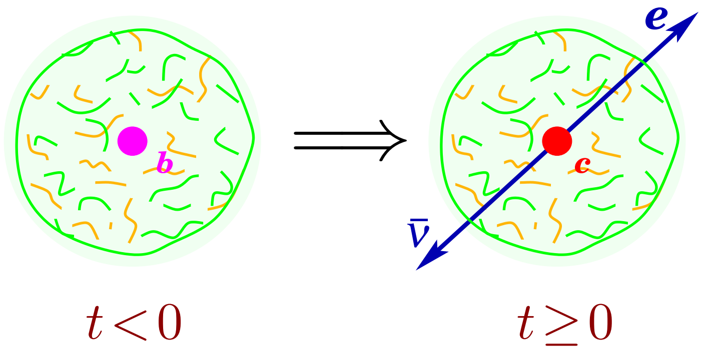
The Landau precession of a charged particle in the magnetic field



$\vec{P}^{\,2} \geq \left|\vec{B}\right|$ even without binding potential

$B \to D^* + \ell\bar{\nu}$ at zero recoil

$$\mathrm{d}w\,(B \to D^* + \ell\bar{\nu}) \;\sim\; G_F^2 \cdot |V_{cb}|^2 \cdot |\vec{p}| \cdot \left|F_{B\to D^*}(\vec{p})\right|^2$$



$F_{B\to D^*}$ is determined by bound state dynamics

If $\vec{p}=0$ ($\vec{p}_e = -\vec{p}_{\bar{\nu}}$)

almost nothing has changed!

$F(\vec{p}=0) = 1$ up to 'isotopic effects'

$$F_{\mathrm{n/p}}(0) \;=\; 1 + \frac{0}{m_{c,b}} + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{m^2_{c,b}}\right) + \mathcal{O}\left(\frac{\Lambda^3_{\mathrm{QCD}}}{m^3_{c,b}}\right) + \ldots$$

$1/m_{b,c}$ effects are absent

1986 Voloshin, Shifman
1990 Luke

Important to estimate δ_{1/m^2}

Before May 1994: $\delta_{1/m^2} \simeq -0.02$

OPE $\Longrightarrow$ HQ Sum Rules

SUV, BSUV April 1994
Experiment June 1994

$$-\delta_{\mathrm{n/p}} > \frac{M^2_{B^*} - M^2_B}{8m_c^2} \simeq -0.04$$

rigorous bound on $F(0)$

$F(0) \simeq 0.9$ actual estimate SUV 1994

FNAL, lattice:

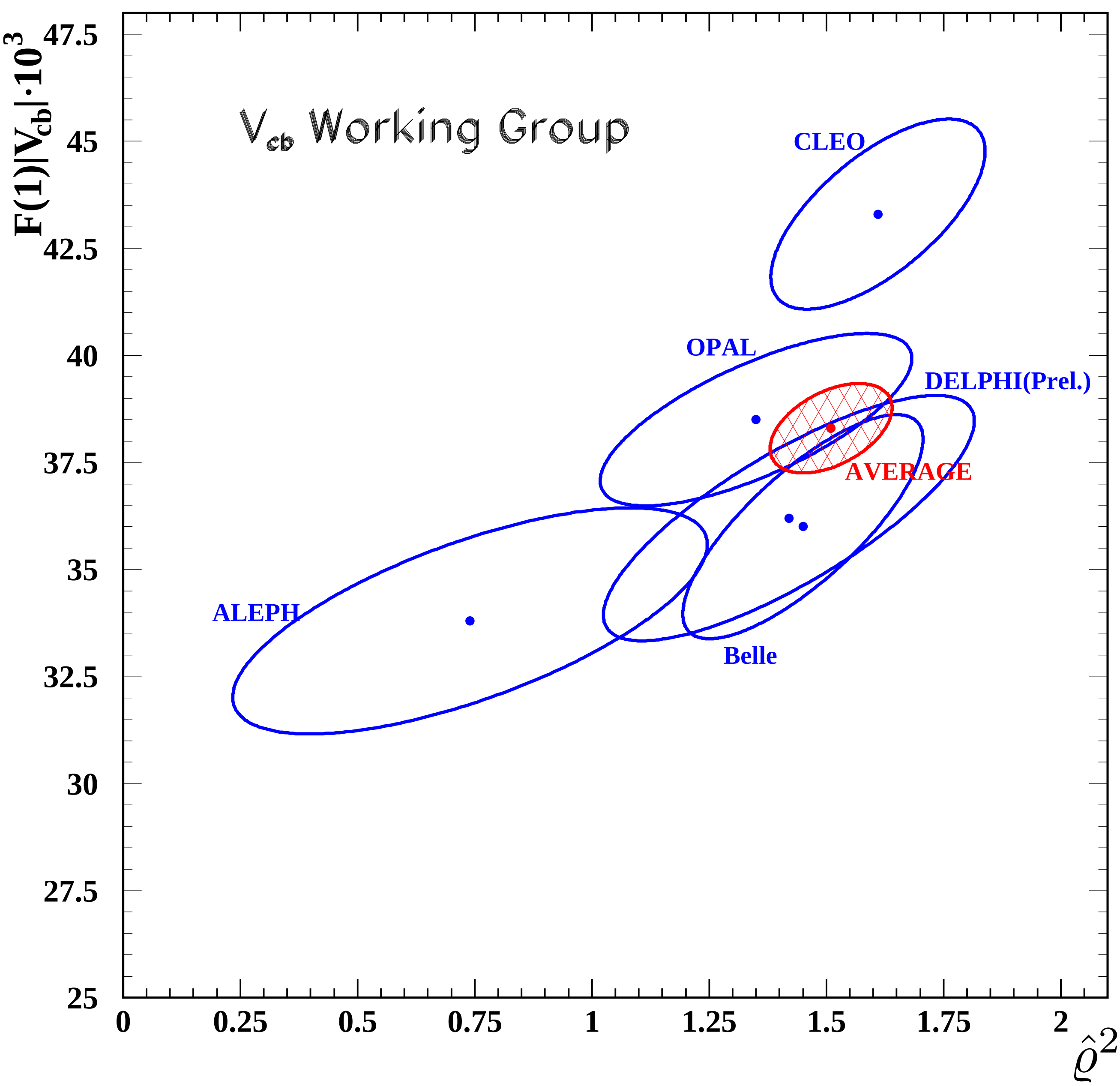
$F(0) \simeq 0.88$ order $1/m_Q^2$

$F(0) \simeq 0.91$ order $1/m_Q^3$

higher orders in $1/m_c$?

$$F(1) = 0.913\,^{+0.024}_{-0.017} \pm 0.016\,^{+0.003\,+\,0.000\,+\,0.006}_{-0.014\,-\,0.016\,-\,0.014} \qquad (?)$$

Significant part of the correction is added `theoretically` rather than directly emerged from the lattice simulation



Question to experiment and fits:

What is the value for $F(1)\cdot|V_{cb}|$ with the constraint $\hat{\varrho}^2 < 1.2$?

Numerical estimates of F_{D^*}

$$F_{D^*} = \left[\xi_A(\mu) - \underbrace{\frac{\mu_G^2}{3m_c^2} - \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right)}_{2\delta_{1/m^2}(\mu)} - \sum_{f \neq D^*}^{\epsilon<\mu} |F_{B\to f}|^2 + \mathcal{O}\left(\frac{1}{m^3}\right) \right]^{\frac{1}{2}}$$

$\xi_A^{\frac{1}{2}}(\mu)$ is the short-distance renormalization factor 0.97 ± 0.01

$$\sum_{f \neq D^*}^{\epsilon<\mu} |F_{B\to f}|^2 = \chi \left[\frac{\mu_G^2}{3m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) + \mathcal{O}\left(\frac{1}{m^3}\right) \right]$$

χ describes the wf overlap deficit guess: $0 < \chi \leq 1$ SUV 1994

$$F_{D^*} \simeq \xi_A^{\frac{1}{2}} - (1+\chi) \left[\frac{\mu_G^2}{6m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{8}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) + \Delta_{\frac{1}{m^3}} \right]$$

if $\chi = 0.5 \pm 0.5$ $\mu \approx 0.8\,\mathrm{GeV}$

$$F_{D^*} \simeq 0.89 - 0.015\,\frac{\mu_\pi^2 - 0.4\,\mathrm{GeV}^2}{0.1\,\mathrm{GeV}^2} \pm 0.03_{\rm exc} \pm 0.01_{\rm pert}$$

$1/m_c^3$ correction is significant!

$F_{D^*} \lesssim 0.92$ for $\chi \simeq 0$

$\chi^{\rm pert} = 1$ @ $\mathcal{O}(\alpha_s^1)$

't Hooft model: $\chi = \frac{13}{21} + \frac{5}{21}\frac{m^2 - \beta^2}{\overline{\Lambda}^2 - m^2 + \beta^2} - \frac{4}{21}\left(\varrho^2 - \frac{3}{4}\right) \simeq 0.55$

Burkardt, N.U. 2000

$|V_{ub}|$ from $\Gamma(B \to X_u\, \ell\nu)$

Theory: $\Gamma_{\rm sl}(b \to u)$ via $|V_{ub}|^2$

Uncertainties in $\Gamma_{\rm sl}(b \to u) \longleftrightarrow |V_{ub}|^2$ N.U. 1999

$\delta_{\rm pert} = 2\%$ $\delta_{\rm nonpert} = 3.5\%$ $\delta_{m_b} = 5\%$

$\mathcal{O}(\alpha_s^2)$ computed
van Ritbergen

$$\delta_{\rm th}|V_{ub}| \,/\, |V_{ub}| \approx 5\%$$

Experiment: $\Gamma_{\rm sl}(b \to u)/\Gamma_{\rm sl}(b \to c) \approx 70$ ...

Only hard kinematic rejection is competitive

The most direct discriminator is hadronic mass M_X

$b \to c$ $M_X^2 \geq M_D^2 \approx 3.5\,{\rm GeV}^2$

$b \to u$ $M_X^2 \approx 0$ bare quarks

QCD: $M_X^2 \propto m_b\overline{\Lambda} + \frac{\alpha_s}{\pi}m_b^2 \sim 2.5\,{\rm GeV}^2$

Analysis: In 85% of $b \to u$ events M_X is below M_D

$$M_X^2 = (P_B - q)^2 = M_B^2 + q^2 - 2M_B\, q_0$$

q_0 fluctuate with the ‘uncertainty’ $\sim \Lambda_{\rm QCD}$

Familiar from the usual quark distributions in DIS

For heavy quarks is known under the name of “Fermi motion”

Fermi motion and consequences of its universality

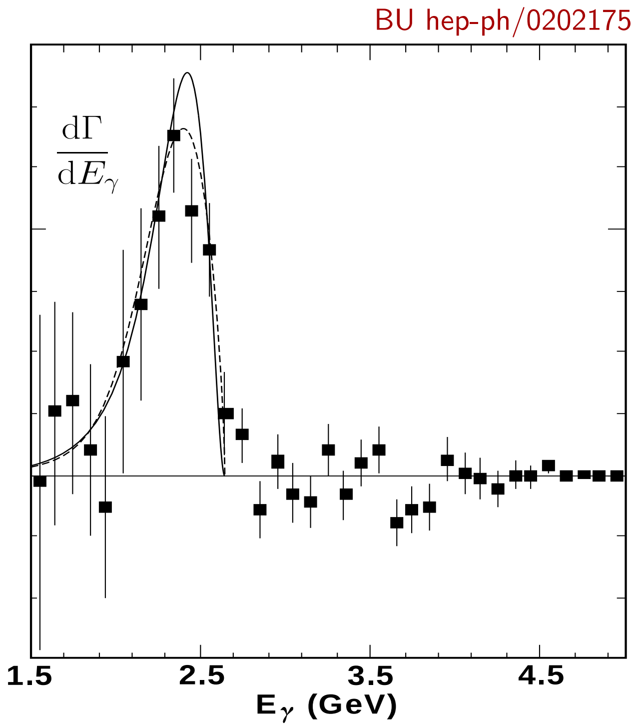
Introduced phenomenologically 20 years ago — ACCMM 1982, Ali, Pietarinen

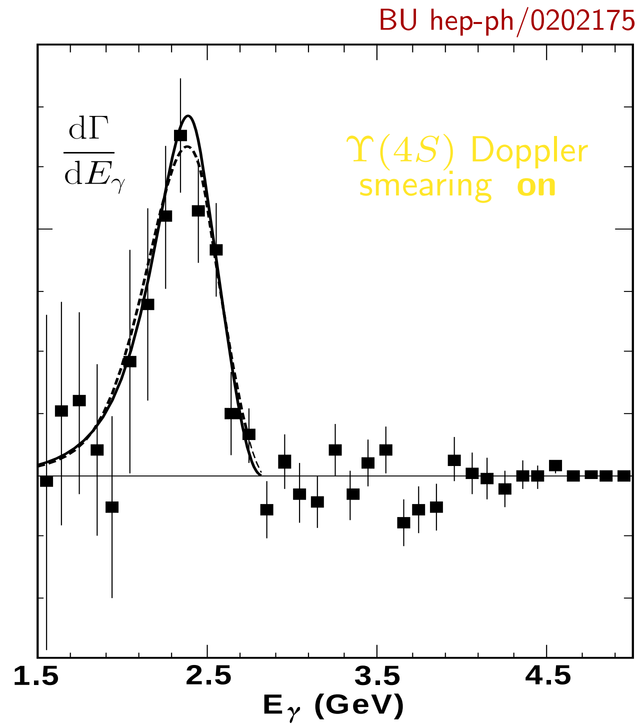
Fermi Motion emerges through the OPE in QCD as a counterpart of the leading-twist distribution function, though has some peculiarities — BSUV 1993

Important in the quest for $|V_{ub}|$ in charmless decays $b \to u\,\ell\nu$:

Distribution over $M_{\rm hadr}^2$ is given by $F_Q(x)$

Even though we do not literally know $F_Q(x)$ beforehand





Application to extraction $|V_{ub}|$ from semileptonic decays:
evaluation of the rejected fraction $1-\Phi(M)$ of $b\to u$ decays with $M_X > M$

$$\Phi(M) = \frac{1}{\Gamma_{\rm sl}(b\to u)} \int_0^M {\rm d}M_X \, \frac{{\rm d}\Gamma_{\rm sl}}{{\rm d}M_X}$$

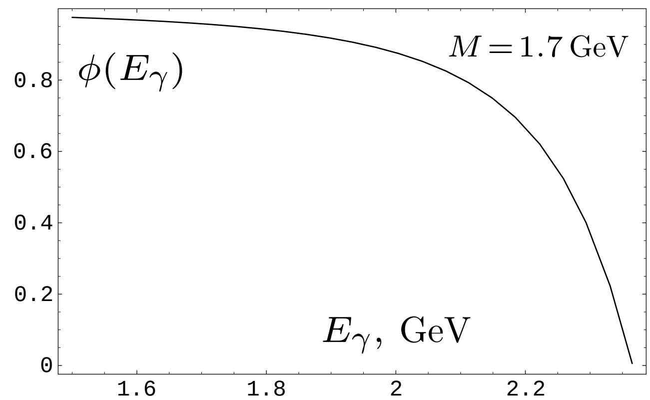
Universality relations:

BU 2002

$$1-\Phi_{\rm sl}(M)=\int\limits_0^{\frac{M_B}{2}-\frac{M^2}{2M_B}} {\rm d}E_\gamma\;\phi(E_\gamma,M)\,\frac{1}{\Gamma_{bs\gamma}}\frac{{\rm d}\Gamma_{bs\gamma}}{{\rm d}E_\gamma}$$

$$\phi(E_\gamma,M)=1-\frac{2r^3}{(1-y)^3}+\frac{r^4}{(1-y)^4}$$

$$y=\frac{2E_\gamma}{M_B}\,,\quad r=\frac{M^2}{M_B^2}$$



$1/m_b$ corrections can be incorporated:

BU 2002

dangerous domain of large q^2 automatically drops out

Can aim at 5% precision in $|V_{ub}|$

Measure separately for $B^\pm$ and B^0 (and (B_s))

Renormalization of operators, masses etc. can be done in different ways

not easy to arbitrary loop unless a particular gauge is fixed

The only way suggested so far is using the SV sum rules; it defines "kinetic" mass $m_Q(\mu)$: BSUV 1996

$$E(\vec{p}) = m_0 + \frac{\vec{p}^{\,2}}{2m_2} - \frac{\vec{p}^{\,4}}{8m_4^3} + \ldots$$

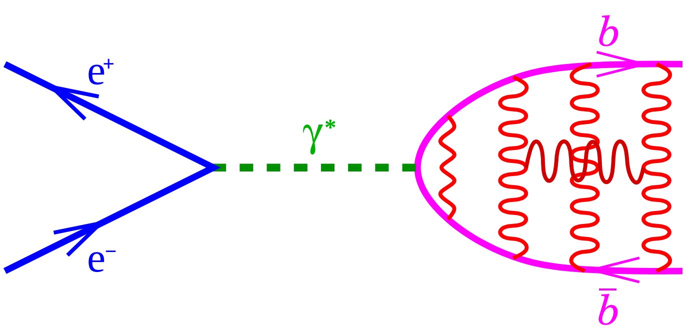
$m_Q(\mu)$ has the meaning of m_2

Such a running mass has no limitation on precision

$$m_b(1\,\mathrm{GeV}) \;=\; (4.57 \pm 0.05)\,\mathrm{GeV}$$

Voloshin 1995–1996

Melnikov, Yelkhovsky
Beneke, Signer 1998–1999
Hoang



$e^+e^- \to \Upsilon(1S, 2S, 3S, 4S, 5S)$

moments of $\sigma(e^+e^- \to b\bar{b})$

Likewise $\mu_\pi^2(1\,\mathrm{GeV})$, $\mu_G^2(1\,\mathrm{GeV})$, ...

Physical observables, renormalon-free

Can be directly measured on the lattice

$\overline{\Lambda} = \lim_{m_b\to\infty} M_B - m_b$ related to the value of m_b

$\overline{\Lambda} \approx 700\,\mathrm{MeV}$ with the uncertainty $\pm 60\,\mathrm{MeV}$ at $\mu = 1\,\mathrm{GeV}$

$\mu_\pi^2, \quad \mu_G^2$ — next important hadronic quantities in HQE

$$\mu_G^2 = \frac{1}{2M_B}\langle B|\,\bar b\,\tfrac{i}{2}\,g_s\,\sigma_{\mu\nu}G^{\mu\nu}\,b\,|B\rangle \longleftrightarrow \langle B|-g_s\,\vec\sigma_b\vec B_{\rm chr}(0)\,|B\rangle_{\rm QM}$$

$$\mu_\pi^2 = \frac{1}{2M_B}\langle B|\,\bar b\,(i\vec D\,)^2\,b\,|B\rangle \longleftrightarrow \langle B|\,\vec p_b^{\,2}\,|B\rangle_{\rm QM}$$

$$\vec p_b \to \vec\pi_b = -i\vec D = -i\vec\partial - g_s\vec A$$

Using the same accurate regularized definition for kinetic $(\{j,k\})$ and chromomagnetic $([j,k])$ operators allows precision numerical evaluation

Product of covariant derivatives $\bar Q(x) iD_j\, P\exp\, iD_k Q(0)$ offset along t direction $it\sim 1/\mu$

$$M_{B^*} - M_B \simeq \frac{2}{3}\frac{\mu_G^2}{m_b}$$

$$\mu_G^2(1\mathrm{GeV}) = 0.35^{+.03}_{-.02}\,\mathrm{GeV}^2$$ N.U. 11/2001

$\mu_\pi^2(\mu) > \mu_G^2(\mu)$ at any μ rigorous inequality

BSUV, Voloshin 1993–1994

Theory: $\mu_\pi^2 \approx (0.45 \pm 0.1)\,\mathrm{GeV}^2$

Nonperturbative inequality for Quantum Field Theory

$\mu_\pi^2 - \mu_G^2$ equals to an integral of a certain cross section

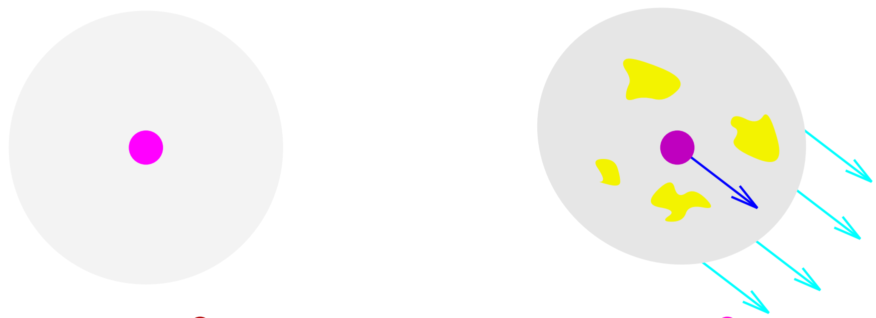
Heavy Quark Sum Rules

similar to the Adler-Weisberger sum rule for ν reactions

Neat Quantum Mechanical interpretation

Heavy Quark Limit

$m_b, m_c \to \infty$ – no corrections in $1/m_Q$ survive



$t \leq 0$ $\quad$ $t > 0$

At $\vec{v}=0$ physics is trivial: $\quad \langle k|J|n\rangle = \delta_{kn}$

elastic amplitude is 1 $\quad$ other states are not excited

Effects appear when $\vec{v} \neq 0$

Amplitudes $\propto \vec{v}$ — 'dipole' transitions into "P-wave" states

$$\frac{1}{2M_B}\langle P^{(n)}(v)|\bar{b}b|B\rangle = \tau^{(n)}\,\vec{v}$$

$$\frac{1}{2M_B}\langle B(v)|\bar{b}b|B\rangle = F(\vec{v}^{\,2}) = 1 - \frac{\varrho^2}{2}\vec{v}^{\,2} + \ldots$$

$\vec{v} \ll 1$ is a good approximation for actual $B \to X_c + \ell\nu$ decays

SV physics: spectrum of 'P-wave' states $P^{(n)}$, $\varepsilon^{(n)} = M_n - M_B$

values of $\tau^{(n)}$

If quarks did not have spin, P's were $L=1$ states

Discard spin of `heavy quark` – then B, B^* are spin-$\frac{1}{2}$ hadrons

P-waves would be $j=\frac{1}{2}$ or $j=\frac{3}{2}$:

$$\frac{1}{2} \times 1 = \frac{1}{2} \oplus \frac{3}{2}$$

j – spin of "light cloud"

Two P-wave families:

$$P_{1/2}^{(n)} \longleftrightarrow \varepsilon_{1/2}^{(n)}, \quad \tau_{1/2}^{(n)}$$

$$P_{3/2}^{(m)} \longleftrightarrow \varepsilon_{3/2}^{(m)}, \quad \tau_{3/2}^{(m)}$$

In atoms $\quad \tau_{1/2} \simeq \tau_{3/2}, \quad \varepsilon_{1/2} \simeq \varepsilon_{3/2}$

Difference is a relativistic spin-orbital effect (fine splitting)

In B mesons – effect of order 1

(small in ε's, but large in τ's)

Remarkable extension of first sum rules to v^4 and higher orders:

D′ Orsay Sum Rules

Le Yaouanc, Oliver, Raynal 10/2002

OPE for nonforward scattering amplitude

$$\varrho_L^2 = (2L+1) \sum_n \left| \tau^{(n)}_{L+\frac{1}{2}} \right|^2 \qquad \varrho_L^2 \equiv \frac{(-1)^L}{L!} \frac{\mathrm{d}^L \xi(w)}{(\mathrm{d}w)^L}\bigg|_{\mathrm{w=1}}$$

$$L \sum_n \left| \tau^{(n)}_{L+\frac{1}{2}} \right|^2 - \sum_k \left| \tau^{(k)}_{L-\frac{1}{2}} \right|^2 = \frac{2L-1}{4} \sum_n \left| \tilde{\tau}^{(n)}_{(L-1)+\frac{1}{2}} \right|^2$$

Divergent – undergo renormalization...

Peculiar: only L-th orbital waves enter for L-th derivative !

For instance

$$\varrho_2^2 \geq \frac{5}{4} \varrho^2 \geq \frac{15}{16}$$

↑ IW curvature ↑ IW slope

$$\varrho_L^2 \geq \frac{(2L+1)!!}{2^{2L} L!} \varrho^2$$

'Extended BPS' limit: All $\tau^2_{L-\frac{1}{2}}$ suppressed ?!

all 'spin' inequalities are approximately saturated

$$\xi_{\mathrm{BPS}}(w) = \left(\frac{2}{w+1} \right)^{\frac{3}{2}}$$

$w \equiv v_0$

Can be directly measured in $B \to D\, \ell \nu$

$\langle (M_X^2)^k \rangle$ are important to scrutinize HQ parameters

Even without a cut on E_ℓ convergence for $k \geq 2$ is not great...

$\langle (M_X^2)^3 \rangle$ seems a bit too low

Peculiarity of M_X^2:

$$M_X^2 \equiv (P_B - q)^2 = p_c^2 + 2(M_B - m_b)(m_b - q_0) + (M_B - m_b)^2$$

OPE $\longrightarrow$ parton + small $1/m_b^2$ corrections

Large corrections are traced to $2(M_B - m_b)\, E_c$ rather than to $p_c^2 - m_c^2$

Cure: use the combinations of M_X^2 and E_X moments, viz.

Trade M_X^2 for

$$\mathcal{N}_X^2 = M_X^2 - 2\tilde{\Lambda} E_x \qquad \text{with} \qquad \tilde{\Lambda} \approx 650\,\mathrm{MeV}$$

Say, $\langle \mathcal{N}_X^4 \rangle - \langle \mathcal{N}_X^2 \rangle^2$:

$$[\langle M_X^4 \rangle - \langle M_X^2 \rangle^2] - 4\tilde{\Lambda}[\langle M_X^2 E_X \rangle - \langle M_X^2 \rangle \langle E_X \rangle] + 4\tilde{\Lambda}^2[\langle E_X^2 \rangle - \langle E_X \rangle^2]$$

Q: Can you do this? ICHEP 2002

A: Yes, at B-factories SLAC 12/2002

Distribution over $\mathcal{N}_X^2$ is a counterpart of E_γ-distribution in $b \longrightarrow s + \gamma$

Alleged problems with the OPE for inclusive decays

- E_ℓ-cut dependence of $\langle M_X^2 \rangle$ from BaBar 2002

  I believe such a conclusion is wrong missing essentials of the OPE

- Inconsistency with $b \to s + \gamma$ moments

  Relying on $\mathrm{imprecise}$* relations in presence of a high cut on E_γ hep-ph/0202175

For similar reasons `global fit` combining accurate and $\mathrm{imprecise}$ relations on equal footing, may not be too meaningful, and the conclusions may be misleading

DELPHI: Lepton moments vs. $\langle M_X^2 \rangle$
an impressive agreement, nonperturbative OPE relation for $M_B - m_b \simeq 650\,\mathrm{MeV}$ is checked with a $40\,\mathrm{MeV}$ accuracy

or

BaBar: $\langle M_X^2 \rangle$ vs. E_ℓ unexpected dependence

Triumph or failure?

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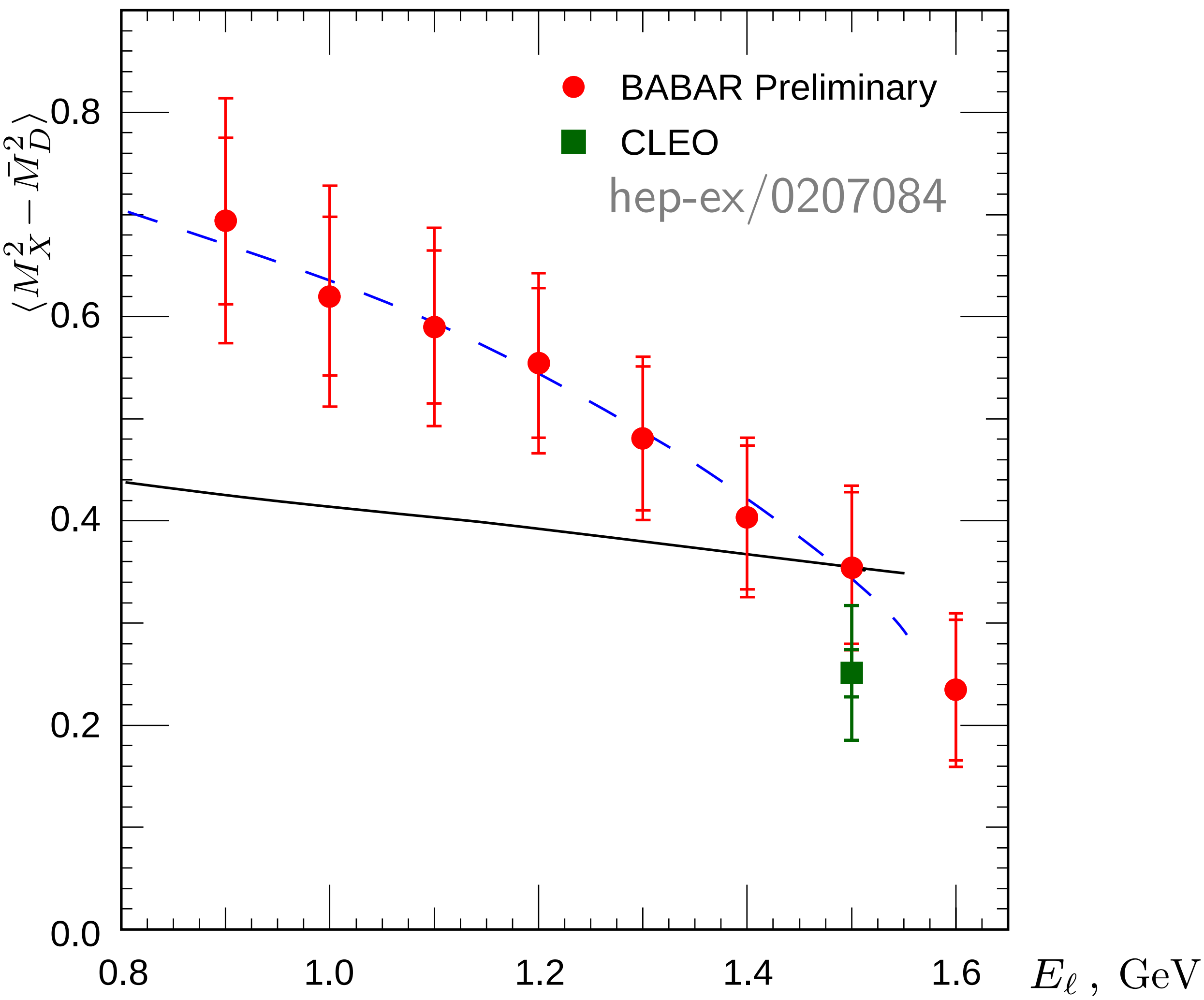
*Politically correct



- OPE computes not $\langle M_X^2 - \bar{M}_D^2 \rangle \simeq 0.4\,\mathrm{GeV}^2$, but $\langle M_X^2 - (m_c + \bar{\Lambda})^2 \rangle \simeq 1.2\,\mathrm{GeV}^2$
- Using proper parameters (e.g., DELPHI's) yields a twice larger slope
- Absolute th-accuracy is similar to experimental error bars even without E_ℓ cut $\pm 0.1\,\mathrm{GeV}^2 \Longleftrightarrow \delta m_b = 20\,\mathrm{MeV}$
- 'Dealing in the expressions, not necessarily in (OPE) truths'

Of course m_b is the same at all E_ℓ. However, the comparison implicitly assumed that the 'theoretical bias' is likewise a constant in E_ℓ. This is grossly wrong

The way the accuracy of the expansion at different cuts on E_ℓ was estimated by Falk *et al.*, Ligeti *et al.* long ago, hep-ph/9708327, /9506201 is conceptually flawed

A closer look reveals that the expansion becomes meaningless at $E_{\rm cut} = 1.7\,{\rm GeV}$, uncertainty reaches 100%

The problem and why it is missed have been discussed. How this happens is obscured by 3-body kinematics in the $\rm SL$ decays with E_ℓ cut, but would be quite transparent in the different kinematic settings, or in $b \to s + \gamma$

- The behavior observed by $\rm BaBar$ `is expected`. The open question is rather if we can quantitatively utilize the measured fall off of $\langle M_X^2 \rangle$ for further constraining $\rm HQ$ parameters

Sum rules in Quantum Mechanics

Basic idea:

$$\sum_n \langle 0|J|n\rangle\langle n|J'|0\rangle = \langle 0|JJ'|0\rangle$$

$$\sum_n (E_n - E_0)\langle 0|J|n\rangle\langle n|J'|0\rangle = \sum_n \langle 0|J\mathcal{H} - \mathcal{H}J|n\rangle\langle n|J'|0\rangle =$$

$$\langle 0|[J,H]J'|0\rangle = \tfrac{1}{2}\langle 0|J\,i\tfrac{\mathrm{d}J'}{\mathrm{d}t} - i\tfrac{\mathrm{d}J}{\mathrm{d}t}J'|0\rangle$$

etc.

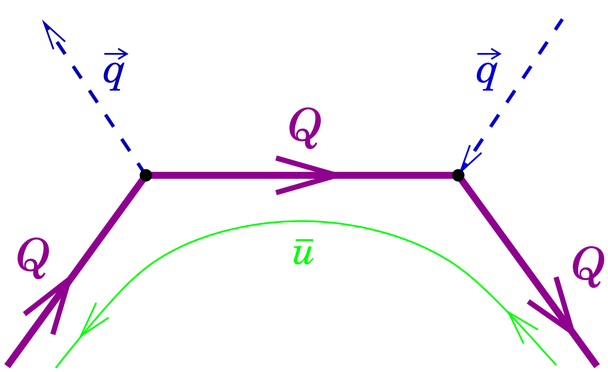
In QFT :

$$T(q_0,\vec{q}\,) = \frac{1}{2M_B}\int \mathrm{d}^3\vec{x}\;\mathrm{d}x_0\;\mathrm{e}^{\,i\vec{q}\vec{x} - iq_0x_0}\,\langle B|iT\{\bar{c}\Gamma b(x),\,\bar{b}\Gamma' c(0)\}|B\rangle$$

At physical q_0 corresponding to the decay of (or scattering off) the heavy quark

$$\frac{1}{\pi}\mathrm{Im}\,T(q_0,\vec{q}\,) = \sum_f \langle B|\bar{b}\Gamma c|f(\vec{q}\,)\rangle\langle f(\vec{q}\,)|\bar{c}\Gamma' b|B\rangle\;\delta(E_f - (M_B - q_0))$$

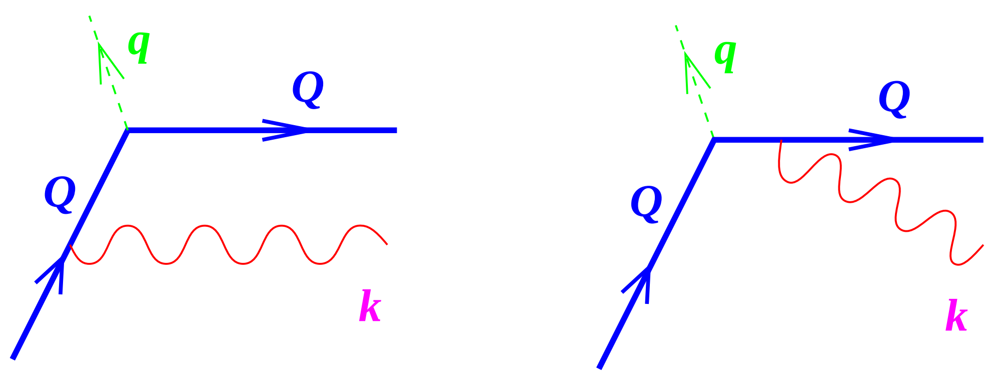
$\varepsilon = M_B - E_D(\vec{q}\,) - q_0 \simeq m_b - \sqrt{m_c^2 + \vec{q}^{\,2}} - q_0$ excitation energy in the final state, or, in general, `quark off-shellness`



$$\vec{q} \propto m_Q$$

A problem with sum rules in QFT – ultraviolet divergence

In QM $\sum_n \varepsilon_n^2|\tau^{(n)}|^2 \propto \langle p^2\rangle$ converges fast, but not in QCD



dipole radiation

At $\varepsilon \gg \Lambda_{\rm QCD}$ $$\sum_m |\tau_{3/2}^{(m)}|^2 \simeq \sum_n |\tau_{1/2}^{(n)}|^2 \simeq \frac{8}{27}\frac{\alpha_s^{(d)}(\varepsilon)}{\pi}\frac{d\varepsilon}{\varepsilon}$$

and hence

$$\varrho^2(\mu)-\frac{1}{4} = 2\sum_{\varepsilon_m<\mu}|\tau_{3/2}^{(m)}|^2 + \sum_{\varepsilon_n<\mu}|\tau_{1/2}^{(n)}|^2 \qquad \mu\frac{d\varrho^2}{d\mu} = \frac{8}{9}\frac{\alpha_s^{(d)}(\mu)}{\pi}$$

$$\frac{\overline{\Lambda}(\mu)}{2} = 2\sum_{\varepsilon_m<\mu}\varepsilon_m|\tau_{3/2}^{(m)}|^2 + \sum_{\varepsilon_n<\mu}\varepsilon_n|\tau_{1/2}^{(n)}|^2 \qquad \frac{d\overline{\Lambda}}{d\mu} = \frac{16}{9}\frac{\alpha_s^{(d)}(\mu)}{\pi}$$

$$\frac{\mu_\pi^2(\mu)}{3} = 2\sum_{\varepsilon_m<\mu}\varepsilon_m^2|\tau_{3/2}^{(m)}|^2 + \sum_{\varepsilon_n<\mu}\varepsilon_n^2|\tau_{1/2}^{(n)}|^2 \qquad \frac{d\mu_\pi^2}{d\mu} = \frac{8}{3}\frac{\alpha_s^{(d)}(\mu)}{\pi}\mu$$

$$\frac{\mu_G^2(\mu)}{3} = 2\sum_{\varepsilon_m<\mu}\varepsilon_m^2|\tau_{3/2}^{(m)}|^2 - 2\sum_{\varepsilon_n<\mu}\varepsilon_n^2|\tau_{1/2}^{(n)}|^2 \qquad -\mu\frac{d\mu_G^2}{d\mu} = \frac{3}{2}\frac{\alpha_s^{(me)}}{\pi}\mu_G^2$$

etc.

Two exceptions: new spin sum rules

Superconvergent similar to Weinberg sum rules

Spin Sum Rules

N.U. 2001

$$B \to D^{**}_{(n)} + \ell\,\nu$$

$$2\left(\sum_k |\tau^{(k)}_{3/2}|^2 - \sum_m |\tau^{(m)}_{1/2}|^2\right) = \frac{1}{2} \quad = \text{spin of light cloud in } B$$

$$2\left(\sum_k \epsilon_k |\tau^{(k)}_{3/2}|^2 - \sum_m \epsilon_m |\tau^{(m)}_{1/2}|^2\right) = \overline{\Sigma}$$

$$\langle D^{**}_{s=3/2}|\, J_0 \,|\, B\,\rangle \sim \tau_{3/2} \qquad \langle D^{**}_{s=1/2}|\, J_0 \,|\, B\,\rangle \sim \tau_{1/2}$$

$$\langle B^*(\vec{v})|\bar{b}iD_jb|B^*(0)\rangle = -\frac{\overline{\Lambda}}{2}v_j(\vec{\varepsilon}'^*\vec{\varepsilon}) - \frac{\overline{\Sigma}}{2}\left\{\varepsilon'^*_j(\vec{\varepsilon}\,\vec{v}\,) - (\vec{\varepsilon}'^*\vec{v}\,)\varepsilon_j\right\} + \mathcal{O}\left(\vec{v}^{\,2}\right)$$

$\overline{\Sigma}$ determines a $\frac{1}{m}$ correction to $B \to D^*$ amplitude Le Yaouanc et al. 2000

Sum rule for $\overline{\Sigma}$ ensures vanishing of $\frac{1}{m}$ correction to $\Gamma_{\rm sl}(B)$ in the SV limit Le Yaouanc et al. 2000 - 01

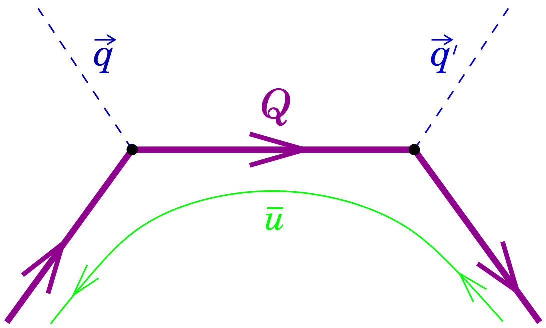
First sum rule leads to the exact bound $\varrho^2 > \frac{3}{4}$ for the slope of the Isgur-Wise function ($B \to D^{(*)}$ formfactor)

Explains why B^* is heavier than B

Applied to atomic and nuclear physics: novel sum rules for spin-orbital effects in dipole transitions

Exact *superconvergent* sum rules – not renormalized *Unique*

Spin sum rules come from the OPE for nonforward scattering off the heavy quark Q



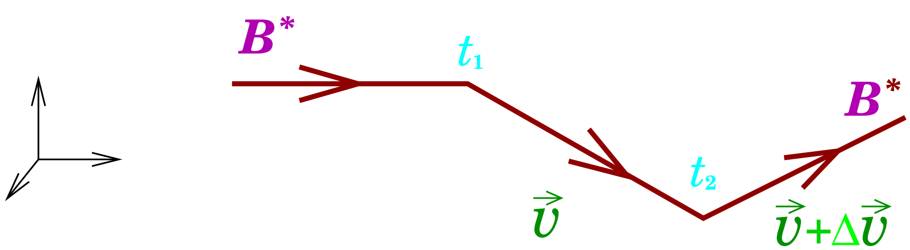
$\vec{q},\ \vec{q}\,' \propto m_Q \qquad \vec{q} \neq \vec{q}\,'$

Thomas precession – allows to measure spin of *light degrees of freedom*

$\langle A(\vec{v}\,')|J(0)|A(\vec{v}\,)\rangle \simeq \text{const}\,(1 - a\,(\delta\vec{v}\vec{v}\,)\,)$ spin-0 particles

$\langle A(\vec{v}\,')|J(0)|A(\vec{v}\,)\rangle \simeq \text{const}\left(\varphi^\dagger\varphi - \frac{1}{4}\,i[\delta\vec{v}\times\vec{v}\,]\cdot\varphi^\dagger\vec{\sigma}\varphi - a\,(\delta\vec{v}\vec{v}\,)\right)$ $s=\frac{1}{2}$

etc.



$$\text{Boost}_{\,(0\to\vec{v}\,)} \times \text{Boost}_{\,(\vec{v}\to\vec{v}+\Delta\vec{v}\,)} = \begin{cases} \text{Boost}_{\,(0\to\vec{v}+\Delta\vec{v})} & \text{Galilean mechanics} \\ \text{Boost}_{\,(0\to\vec{v}+\Delta\vec{v})} \times \text{Rotation}\left(\frac{1}{c^2}[\vec{v}\times\Delta\vec{v}\,]\right) & \text{Relativistic mechanics} \end{cases}$$

$t_2 - t_1 \ll \Lambda_{\rm QCD}^{-1} \longleftrightarrow$ sum over intermediate states – spin of *bare* heavy quark

$t_2 - t_1 \gg \Lambda_{\rm QCD}^{-1} \longleftrightarrow$ only B^* contribution – total spin of heavy hadron

Sum over the excited states – spin of light cloud

Theoretical status

Can go down to a $\%$ level in $|V_{cb}|$ if relevant parameters are determined:

- $m_{b,c}(\mu)$, $\mu_\pi^2(\mu)$, $\mu_G^2(\mu)$, ... are completely defined and can (in principle) be determined from experiment with an unlimited accuracy
- Duality violation is very small in $\Gamma_{\rm sl}(B)$ BU 2001
- α_s corrections to Wilson coefficients are feasible Limiting factor
- Know how to analyze higher power corrections BBMU 2003

m_b, m_c, μ_π^2, ... (properly defined) can be determined from the semileptonic $(b \to s + \gamma)$ decay distributions themselves BSUV, 1993-1994

Nowadays is being implemented in a number of experiments

New strategy: formulated at CKM 2002 @ CERN

Comprehensive approach: measure many observables to extract the 'theoretical' input parameters

We can do without relying on $1/m_c$ expansion at all

Expansion in $1/m_c$ is questionable: $\frac{1}{m_c^2} > 14\,\frac{1}{m_b^2},\;\; 8\,\frac{1}{(m_b - m_c)^2}$

If indeed $\mu_\pi^2 \lesssim 0.45\,\mathrm{GeV}^2$, i.e. $\mu_\pi^2 - \mu_G^2 \ll \mu_\pi^2,\, \mu_G^2$

BPS expansion: Expand around $\mu_\pi^2 - \mu_G^2 = 0$

N.U. 2001

At $\mu_\pi^2 = \mu_G^2$ there is a `functional` relation $\vec{\sigma}\vec{\pi}|B\rangle = 0$

Ultrarelativistic light cloud – antipode to NR quark models

Remarkable limit in many respects

Experiment suggests an intriguing nontrivial pattern

Why proximity to 'BPS'? Lowest Landau level

In quantum mechanics of electrons: $|\vec{B}| \gg |\vec{E}| \implies$ BPS

In B mesons *a priori* $\vec{B} \sim \vec{E} \sim \Lambda^2_{\rm QCD}$, strongly fluctuates

Would imply a strong correlation between spin and momentum

vanishes in nonrelativistic systems

If this is true, it is unlikely accidental What drives it?

Some large parameter?

What happens in the Instanton Vacuum? SUSY?

Requires further theoretical understanding