Model of nonperturbative running coupling in QCD

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Abstract

The modified "physical" running coupling constant is constructed on the base of the analytic running coupling constant obtained from the standard perturbation theory approximation up to the four loop order. The possibility of providing quark confinement singular infrared behavior of the running coupling constant and presence of term connected with dynamical gluon mass is studied. In the framework of the approach fixing the string tension parameter and normalizing for example at M_τ define completely the running coupling. At that such quantities as the gluon condensate, the nonperturbative gluon vacuum energy density, the dynamical gluon mass m_g, and the nonperturbative scale k_0 turn out to be fixed and correspond to another approaches.

The talk is based mainly on:

A.I. Alekseev and B.A. Arbuzov, Yad. Fiz. **61**, 314 (1998) [Phys. Atom. Nucl. **61**,264. (1998)]; A.I. Alekseev and B.A. Arbuzov, Mod. Phys. Lett. A **13**, 1447 (1998); A.I. Alekseev, hep-ph/9808206;
2-loop — A.I. Alekseev, Phys. Rev. D **61**, 114005 (2000);
3-loop — A.I. Alekseev, Yad. Fiz. **65**, 1722 (2002) [Phys. At. Nucl. **65**,1678 (2002)];
4-loop — A.I. Alekseev, J. Phys. G **27**, L117 (2001); A.I. Alekseev, Few-Body Systems **32**, 193 (2003).

The idea of the analytic approach:

P.J. Redmond, Phys. Rev. **112**, 1404 (1958);
N.N. Bogolubov, A.A. Logunov, and D.V. Shirkov, Zh. Éksp. Teor.Fiz. **37**, 805 (1959) [Sov. Phys. JETP **10**, 574 (1960)].

Application to QCD: D.V. Shirkov and I.L. Solovtsov, JINR Rapid Comm. **2[76]-96**, 5 (1996); I.L. Solovtsov and D.V. Shirkov, Teor. Mat. Fiz. **120**, 482 (1999) [Theor. Math. Phys. **120**, 1210 (1999)]; ... .

1 Running coupling models for $Q^2 > 0$ from one-loop PT

Matching of the solutions for large and small Q^2. E.g., B.A. Arbuzov, E.E. Boos, K.Sh. Turashvili, Z. Phys. C **30**, 287 (1986) $\Rightarrow k_0 \simeq 1$ GeV;
β-function definition, RG-eq. solving;
... .
Logical steps towards our model:

$$\alpha_s^{(1)}(Q^2) = \frac{4\pi}{b_0} \frac{1}{\ln(Q^2/\Lambda^2)}, \tag{1}$$

$$\alpha_{\rm an}^{(1)}(Q^2) = \frac{4\pi}{b_0} \left[\frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2} \right], \tag{2}$$

$$\alpha_{\rm an+c}^{(1)}(Q^2) = \frac{4\pi}{b_0} \left[\frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2} + \frac{c\Lambda^2}{Q^2} \right], \tag{3}$$

$$\alpha^{(1)}(Q^2) = \frac{4\pi}{b_0} \left[\frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2} + \frac{c\Lambda^2}{Q^2} + \frac{(1-c)\Lambda^2}{Q^2 + m_g^2} \right], \tag{4}$$

where

$$m_g^2 = \frac{\Lambda^2}{c-1}. \tag{5}$$

Define $q\bar{q}$ potential as

$$V(r) = -\frac{4}{3}\int \frac{d^n q}{(2\pi)^3} \exp{(i\mathbf{qr})} \frac{4\pi\alpha(\mathbf{q}^2)}{\mathbf{q}^2}\Big|_{n=3}. \tag{6}$$

At large distances

$$V(r)|_{r\to\infty} \simeq a^2 r, \tag{7}$$

$a^2 = \sigma$ — string tension,

$$(3/2)\sigma = (4\pi/b_0)\Lambda_1^2, \;\; \Lambda_1^2 = c\Lambda^2. \tag{8}$$

At $a \simeq 0.42$ GeV (L.D. Soloviev, Phys. Rev. D **58**, 035005 (1998); Phys. Rev. D **61**, 015009 (2000)), $b_0 = 9$ one has $\Lambda_1 \simeq 435$ MeV.
At small distances

$$\Delta V(r)|_{r\to 0} \simeq \sigma_0 r, \;\; (3/2)\sigma_0 = (4\pi/b_0)(c-1)\Lambda^2, \tag{9}$$

(V. Zakharov's possibility). Let us mention the well known elegant and economical Richardson model with UV as (1) and IR as (3)

$$\alpha^{(1)}_{\mathrm{Rich}}(Q^2) = \frac{4\pi}{b_0}\frac{1}{\ln(1+Q^2/\Lambda_1^2)}, \tag{10}$$

multiplicatively improved model (A. Nesterenko)

$$^{N}\alpha_{\rm an}^{(1)}(Q^2) = \frac{4\pi}{b_0}\frac{Q^2-1}{Q^2\ln(Q^2/\Lambda^2)}, \tag{11}$$

and "frozen" in the IR coupling model by Yu. Simonov

$$\alpha_{\rm Sim}^{(1)}(Q^2) = \frac{4\pi}{b_0}\frac{1}{\ln((M_B^2+Q^2)/\Lambda^2)} \tag{12}$$

($M_B \sim 1$ GeV).
NPT contributions of the models tend to zero in the UV but insufficiently fast. Introducing the term corresponding to the dynamical gluon mass (J.M. Cornwall, Phys. Rev. D **26**, 1453 (1982)) we come to the model (4) where the coefficients, defined according to the principle of minimality of NPT contributions in the PT region (A.A.,B.A.), decrease at large Q^2 fast enough.

From the Running Coupling to the Analytic Running Coupling

The behavior of the QCD running coupling $\alpha_s(Q^2)$ is defined by the renormalization group equation

$$Q^2\frac{\partial\alpha_s(Q^2)}{\partial Q^2}=\beta(\alpha_s)=\beta_0\alpha_s^2+\beta_1\alpha_s^3+\beta_2\alpha_s^4+\beta_3\alpha_s^5+O(\alpha_s^6), \qquad (13)$$

where the coefficients

$$\begin{aligned}
\beta_0 &= -\ \frac{1}{4\pi}b_0,\ \ b_0=11-\frac{2}{3}n_f,\\
\beta_1 &= -\ \frac{1}{8\pi^2}b_1,\ \ b_1=51-\frac{19}{3}n_f,\\
\beta_2 &= -\ \frac{1}{128\pi^3}b_2,\ \ b_2=2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2,\\
\beta_3 &= -\ \frac{1}{256\pi^4}b_3,\ \ b_3=\frac{149753}{6}+3564\zeta_3\\
&\quad -\ \left(\frac{1078361}{162}+\frac{6508}{27}\zeta_3\right)n_f+\left(\frac{50065}{162}+\frac{6472}{81}\zeta_3\right)n_f^2+\frac{1093}{729}n_f^3.
\end{aligned}$$

Here n_f is the number of active quark flavors and ζ is the Riemann zeta-function, $\zeta_3=\zeta(3)=1.202056903\ldots$.

The integration of the RG equation yields

$$\frac{1}{\alpha_s(Q^2)} + \frac{\beta_1}{\beta_0}\ln\alpha_s(Q^2) + \frac{1}{\beta_0^2}\left(\beta_0\beta_2 - \beta_1^2\right)\alpha_s(Q^2) + \frac{1}{2\beta_0^3}\left(\beta_1^3 - 2\beta_0\beta_1\beta_2\right.$$
$$\left. +\beta_0^2\beta_3\right)\alpha_s^2(Q^2) + O(\alpha_s^3(Q^2)) = -\beta_0\ln(Q^2/\Lambda^2) + \bar{C}. \tag{14}$$

The integration constant is represented here as a combination of two constants Λ and $\bar{C}$. It is convenient to introduce constant C,

$$C = \ln(-\beta_0) + (\beta_0/\beta_1)\bar{C}.$$

Iteratively solving the equation for $\alpha_s(Q^2)$ at $L = \ln(Q^2/\Lambda^2) \to \infty$ and inverting the result one obtains (slight generalization of K.G. Chetyrkin, B.A. Kniehl, and M. Steinhauser, Phys. Rev. Lett. **79**, 2184 (1997))

$$\alpha_s(Q^2) = -\frac{1}{\beta_0 L}\left\{1 + \frac{\beta_1}{\beta_0^2 L}(\ln L + C) + \frac{\beta_1^2}{\beta_0^4 L^2}\left[(\ln L + C)^2 - (\ln L + C)\right.\right.$$
$$\left. -1 + \frac{\beta_0\beta_2}{\beta_1^2}\right] + \frac{\beta_1^3}{\beta_0^6 L^3}\left[(\ln L + C)^3 - \frac{5}{2}(\ln L + C)^2 - \left(2 - \frac{3\beta_0\beta_2}{\beta_1^2}\right)(\ln L + C)\right.$$
$$\left.\left. +\frac{1}{2} - \frac{\beta_0^2\beta_3}{2\beta_1^3}\right] + O\left(\frac{1}{L^4}\right)\right\}. \tag{15}$$

Within the conventional definition of Λ as $\Lambda_{\overline{MS}}$ one chooses $C = 0$.
Let us introduce the function $a(x) = (b_0/4\pi)\alpha_s(Q^2)$, where $x = Q^2/\Lambda^2$. Then one can write

$$a(x) = \frac{1}{\ln x} - b\frac{\ln(\ln x) + C}{\ln^2 x} + b^2\left[\frac{(\ln(\ln x) + C)^2}{\ln^3 x} - \frac{\ln(\ln x) + C}{\ln^3 x} + \frac{\kappa}{\ln^3 x}\right]$$
$$-b^3\left[\frac{(\ln(\ln x) + C)^3}{\ln^4 x} - \frac{5}{2}\frac{(\ln(\ln x) + C)^2}{\ln^4 x} + (3\kappa + 1)\frac{\ln(\ln x) + C}{\ln^4 x} + \frac{\bar{\kappa}}{\ln^4 x}\right]. \tag{16}$$

where the coefficients b, κ, and $\bar{\kappa}$ are equal to

$$\begin{aligned} b &= -\frac{\beta_1}{\beta_0^2} = \frac{2b_1}{b_0^2}, \\ \kappa &= -1 + \frac{\beta_0\beta_2}{\beta_1^2} = -1 + \frac{b_0 b_2}{8b_1^2}, \\ \bar{\kappa} &= \frac{1}{2} - \frac{\beta_0^2\beta_3}{2\beta_1^3} = \frac{1}{2} - \frac{b_0^2 b_3}{16 b_1^3}. \end{aligned} \tag{17}$$

At $x \simeq 1$ the perturbative running coupling is singular. The analytic approach removes all these nonphysical singularities in a regular way.

The analytic running coupling is obtained by the integral representation

$$a_{\rm an}(x) = \frac{1}{\pi}\int\limits_0^\infty \frac{d\sigma}{x+\sigma}\rho(\sigma), \tag{18}$$

where the spectral density $\rho(\sigma) = {\rm Im}\, a_{\rm an}(-\sigma - i0)$. According to the analytic approach to QCD we adopt that ${\rm Im}\, a_{\rm an}(-\sigma - i0) = {\rm Im}\, a(-\sigma - i0)$, where $a(x)$ is the perturbative running coupling. It is clear that dispersively-modified coupling of form (2) has analytical structure which is consistent with causality. Then one obtains the spectral density

$$\rho(\sigma) = \rho^{(1)}(\sigma) + \Delta\rho^{(2)}(\sigma) + \Delta\rho^{(3)}(\sigma) + \Delta\rho^{(4)}(\sigma), \tag{19}$$

where

$$\rho^{(1)}(\sigma) = \frac{\pi}{t^2+\pi^2}, \tag{20}$$

$$\Delta\rho^{(2)}(\sigma) = -\frac{b}{(t^2+\pi^2)^2}\left[2\pi t F_1(t) - \left(t^2-\pi^2\right)F_2(t)\right], \tag{21}$$

$$\Delta\rho^{(3)}(\sigma) = \frac{b^2}{(t^2+\pi^2)^3}\Big[\pi\left(3t^2-\pi^2\right)\left(F_1^2(t)-F_2^2(t)\right) - 2t\left(t^2-3\pi^2\right)F_1(t)F_2(t)$$
$$-\pi\left(3t^2-\pi^2\right)F_1(t) + t\left(t^2-3\pi^2\right)F_2(t) + \pi\kappa\left(3t^2-\pi^2\right)\Big], \tag{22}$$

$$\Delta\rho^{(4)}(\sigma) = -\frac{b^3}{(t^2+\pi^2)^4}\left[\left(t^4 - 6\pi^2 t^2 + \pi^4\right)\left(F_2^3(t) - 3F_1^2(t)F_2(t)\right)\right.$$
$$+4\pi t\left(t^2 - \pi^2\right)\left(F_1^3(t) \, -3F_1(t)F_2^2(t)\right) - 10\pi t\left(t^2-\pi^2\right)\left(F_1^2(t) - F_2^2(t)\right)$$
$$+5\left(t^4 - 6\pi^2 t^2 + \pi^4\right)F_1(t)F_2(t) + 4\pi\left(1+3\kappa\right)t\left(t^2-\pi^2\right)F_1(t)$$
$$\left. - \left(1+3\kappa\right)\left(t^4 - 6\pi^2 t^2 + \pi^4\right)F_2(t) + 4\pi\bar{\kappa}t\left(t^2-\pi^2\right)\right]. \qquad (23)$$

Here $t = \ln(\sigma)$,

$$F_1(t) \equiv \frac{1}{2}\ln(t^2+\pi^2) + C, \quad F_2(t) \equiv \arccos\frac{t}{\sqrt{t^2+\pi^2}}, \qquad (24)$$

For the 1 — 4-loop cases the spectral density of the analytic running coupling is shown in Fig. 1 ($C = 0, n_f = 3$).

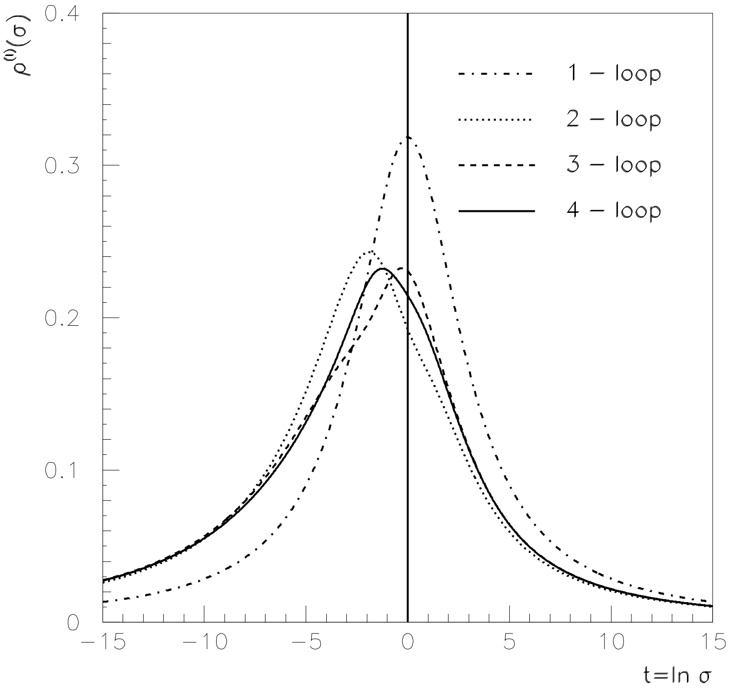

Figure 1: The spectral density of the analytic running coupling up to four loop order

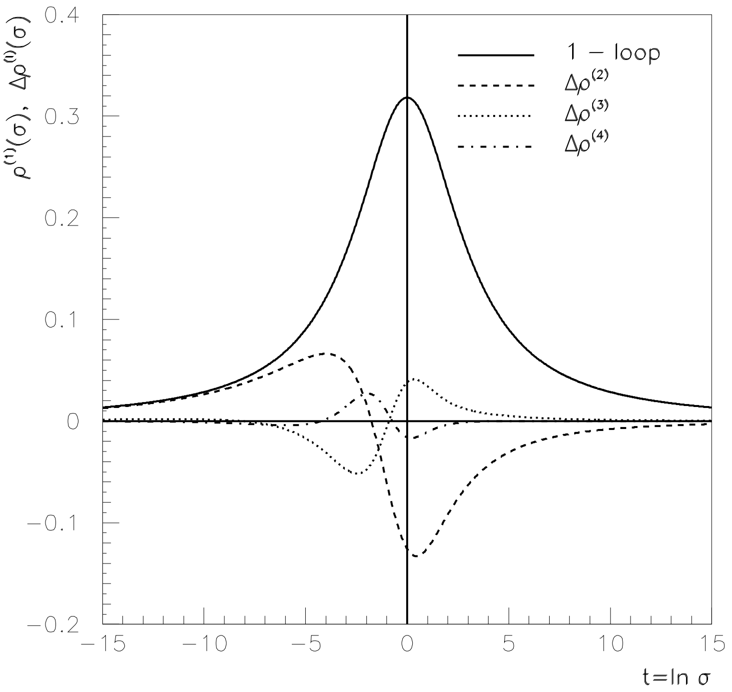

Figure 2: The higher loop order corrections for the spectral density

Integrating with the replacement $(-\infty, +\infty) \to (-T, +T)$ leads to the relative error $\sim 1/T$, and at large T it is important not to lose the higher loop

contributions. In Fig. 2 the higher loop corrections to the spectral density are shown. We shall obtain another more effective method for precise calculation of $\alpha_{\rm an}(Q^2)$ which is not connected with the numerical integration.
The analytic running coupling is divided into perturbative component and non-perturbative one in an explicit form (A.A.).

$$a_{\rm an}(x) = a^{\rm pt}(x) + a^{\rm npt}_{\rm an}(x). \tag{25}$$

The expansion of the last term in the inverse powers of x was obtained

$$a^{\rm npt}_{\rm an}(x) = \sum_{n=1}^{\infty} \frac{c_n}{x^n}, \tag{26}$$

where

$$\begin{aligned} c_n &= -1 + bn\left[1 + C - \gamma - \ln(n)\right] - \frac{1}{2}b^2n^2\left[1 - \frac{\pi^2}{6} + \kappa\right. \\ &+ \left.(1 + C - \gamma - \ln(n))^2\right] + \frac{1}{6}b^3n^3\left[2 + \frac{5}{2}\kappa + \bar{\kappa} - 2\zeta_3\right. \\ &+ \left.(1 + C - \gamma - \ln(n))^3 + 3(1 + C - \gamma - \ln(n))\left(1 - \frac{\pi^2}{6} + \kappa\right)\right]. \end{aligned}$$

Table 1: The dependence of c_n and loop corrections on n for the 1 — 4-loop cases, $n_f = 3$

n	$c_n^{1-\text{loop}}$	$\Delta_n^{2-\text{loop}}$	$\Delta_n^{3-\text{loop}}$	$\Delta_n^{4-\text{loop}}$	$c_n^{2-\text{loop}}$	$c_n^{3-\text{loop}}$	$c_n^{4-\text{loop}}$
1	-1.0	0.33405	0.01608	-0.07825	-0.66595	-0.64987	-0.72812
2	-1.0	-0.42724	0.19624	-0.37379	-1.42724	-1.23101	-1.60480
3	-1.0	-1.60196	-0.63626	-1.28115	-2.60196	-3.23823	-4.51937
4	-1.0	-3.04517	-3.48651	-5.07338	-4.04517	-7.53168	-12.60506
5	-1.0	-4.68801	-9.19185	-16.30462	-5.68801	-14.87987	-31.18449

Here γ is Euler constant, $\gamma \simeq 0.5772$. We can see that power series (26) is uniformly convergent at $x > 1$ and its convergence radius is equal to unity. For numerical evaluation of the coefficients c_n we choose the $\overline{MS}$ scheme values of κ, $\bar{\kappa}$ and assume that $C = 0$. For $n_f = 3$ the result is given in Table. 1.

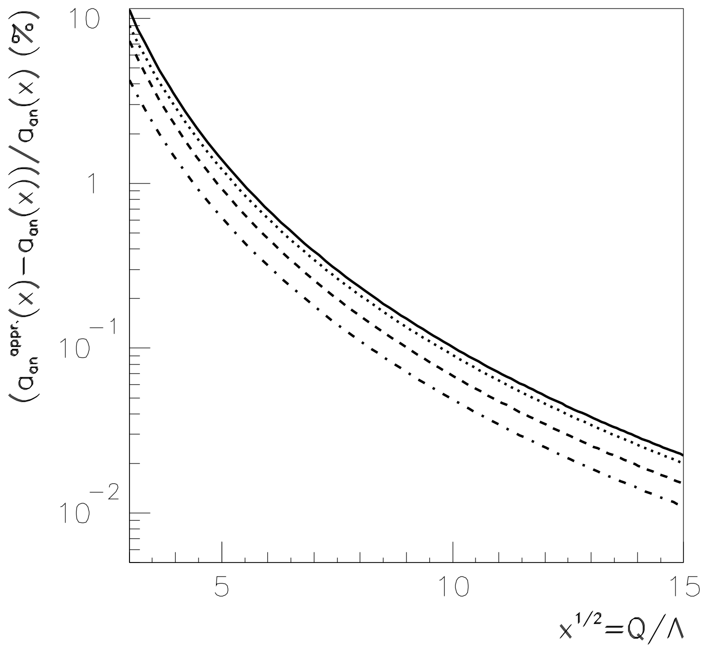

Figure 3: Relative error of the approximation of $a_{\rm an}$ with $a_{\rm an}^{\rm npt}$ approximated by only the first term of the series, as a function of $x^{1/2} = Q/\Lambda$ for the 1 — 4-loop cases at $n_f = 4$. The dash-dotted line, dotted line, dashed line and solid line correspond to the 1-loop, 2-loop, 3-loop and 4-loop cases, respectively.

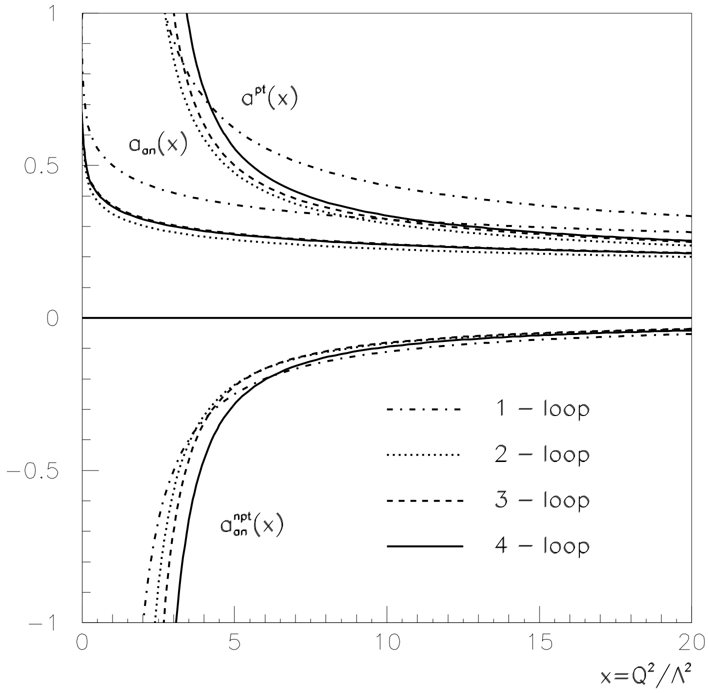

Figure 4: The analytic running coupling $a_{\rm an}$ and its perturbative component $a^{\rm pt}$ and non-perturbative component $a_{\rm an}^{npt}$ as functions of $x = Q^2/\Lambda^2$ for the 1 — 4-loop order cases. Here $n_f = 3$.

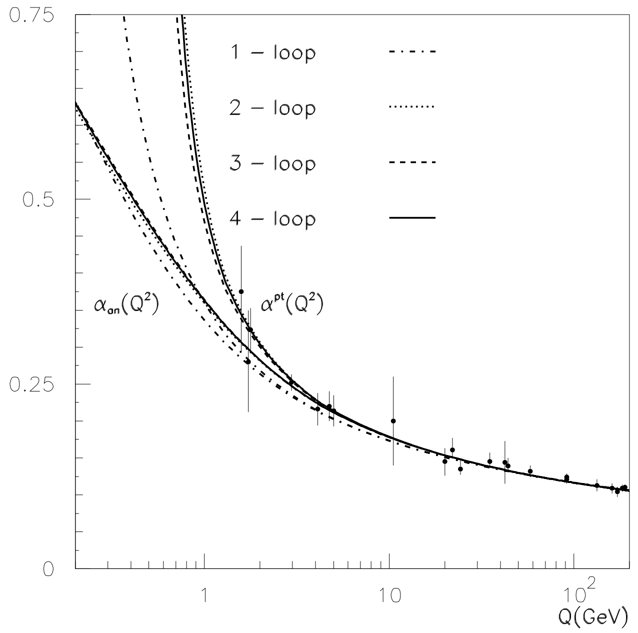

Figure 5: The analytic and perturbative couplings $\alpha_{\rm an}(Q^2)$, $\alpha^{\rm pt}(Q^2)$ for the 1 — 4-loop order cases. The normalization conditions are $\alpha^{(n_f=5)}(M_Z^2) = 0.1181$, $M_Z = 91.1882$ GeV; $\alpha^{(n_f=5)}(m_b^2) = \alpha^{(n_f=4)}(m_b^2)$, $m_b = 4.3$ GeV; $\alpha^{(n_f=4)}(m_c^2) = \alpha^{(n_f=3)}(m_c^2)$, $m_c = 1.3$ GeV.

2 The QCD coupling constant model

$\alpha_{\rm an}(Q^2) \Rightarrow \alpha(Q^2)$,

$$\alpha(Q^2) = \alpha_{\rm an}(Q^2) + \frac{4\pi}{b_0}\left[\frac{c\Lambda^2}{Q^2} - \frac{d\Lambda^2}{Q^2+m_g^2}\right], \tag{27}$$

with c, d, $m_g \equiv m_\Lambda \Lambda$ to be fixed. Analogously to the one-loop case we demand

$$d = c + c_1, \quad m_\Lambda^2 = -c_2/(c+c_1). \tag{28}$$

The dynamical gluon mass

$$m_g = m_g(\Lambda, n_{loops}) = \Lambda\sqrt{\frac{-c_2\Lambda^2}{\Lambda_1^2 + c_1\Lambda^2}}. \tag{29}$$

for the effective theory with $n_f = 3$ is shown in Fig. 6. At $\Lambda = 375$ MeV $m_g \simeq$ 0.6 GeV.

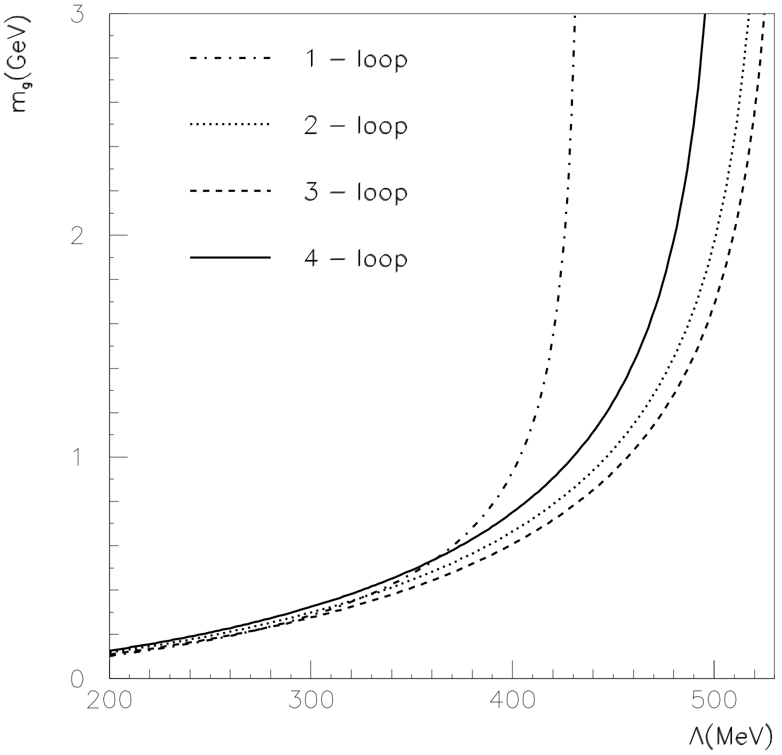

Figure 6: The dynamical gluon mass m_g as a function of Λ

Let us consider three variants of the normalization conditions for $\alpha(Q^2, \Lambda, n_{\rm loops})$ and its components.

- $\alpha(M_\tau^2) = 0.323, \;\; M_\tau = 1.777 \text{GeV},$
- $\alpha(M_\tau^2) = 0.35, \;\; M_\tau = 1.777 \text{GeV},$
- $\alpha(M_Z^2) = 0.1181, \;\; M_Z = 91.1882 \text{GeV}.$

Parameters Λ, m_g, c, d are given in Table. 2. In Figs. 7, 8, 9 the dependencies of α, $\alpha_{\rm an}$ and $\alpha^{\rm pt}$ on Q are given. For the first two cases the loop stability for the model is higher than that for perturbative case (apart from 1-loop case, e.g., at $Q = 0.7$ GeV $\Delta_{pt} = 0.11$, $\Delta_{an} = 0.006$, $\Delta = 0.06$). The last case is inconsistent with $n_f = 3$.
Superposition of the figures can be elucidatory.

Table 2: Parameters Λ_{pt} (MeV), Λ_{an} (MeV), Λ (MeV), dynamical gluon mass m_g (MeV) and parameters of the model c, d for 1 – 4-loop cases. $n_f = 3$. Normalization conditions are: I. $\alpha(M_\tau^2) = 0.323$, $M_\tau = 1.777$ GeV; II. $\alpha(M_\tau^2) = 0.35$, $M_\tau = 1.777$ GeV; III. $\alpha(M_Z^2) = 0.1181$, $M_Z = 91.1882$ GeV

Norm. I	1-loop	2-loop	3-loop	4-loop
$\Lambda_{\rm pt}$	204.65	381.89	351.07	344.82
$\Lambda_{\rm an}$	240.46	599.22	494.71	505.31
Λ	204.65	383.43	351.89	345.90
m_g	109.01	580.34	416.06	473.66
c	4.5247	1.2890	1.5304	1.5839
d	3.5247	0.6230	0.8806	0.8558
Norm. II	1-loop	2-loop	3-loop	4-loop
$\Lambda_{\rm pt}$	241.78	431.53	400.74	391.97
$\Lambda_{\rm an}$	296.73	748.60	612.48	627.67
Λ	241.80	435.68	402.87	394.73
m_g	161.51	902.79	621.20	715.75
c	3.2413	0.9984	1.1676	1.2162
d	2.2413	0.3324	0.5178	0.4881
Norm. III	1-loop	2-loop	3-loop	4-loop
$\Lambda_{\rm pt}$	246.98	741.10	640.95	646.72
$\Lambda_{\rm an}$	247.11	743.24	642.30	648.27
Λ	246.98	741.10	640.95	646.72
m_g	170.17	i 1562.91	i 1637.59	i 1562.21
c	3.1066	0.3450	0.4613	0.4531
d	2.1066	-0.3209	-0.1886	-0.2750

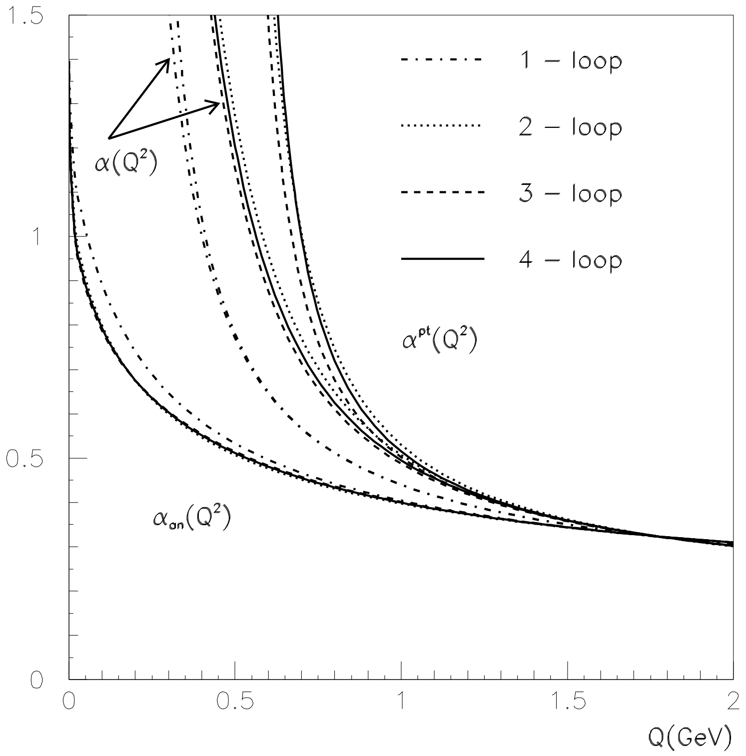

Figure 7: $\alpha^{\rm pt}(Q^2)$, $\alpha_{\rm an}(Q^2)$ и $\alpha(Q^2)$ as functions of Q for 1 — 4-loop cases. For all of them the normalization condition is $\alpha(M_\tau^2) = 0.323,\ M_\tau = 1.777$ GeV

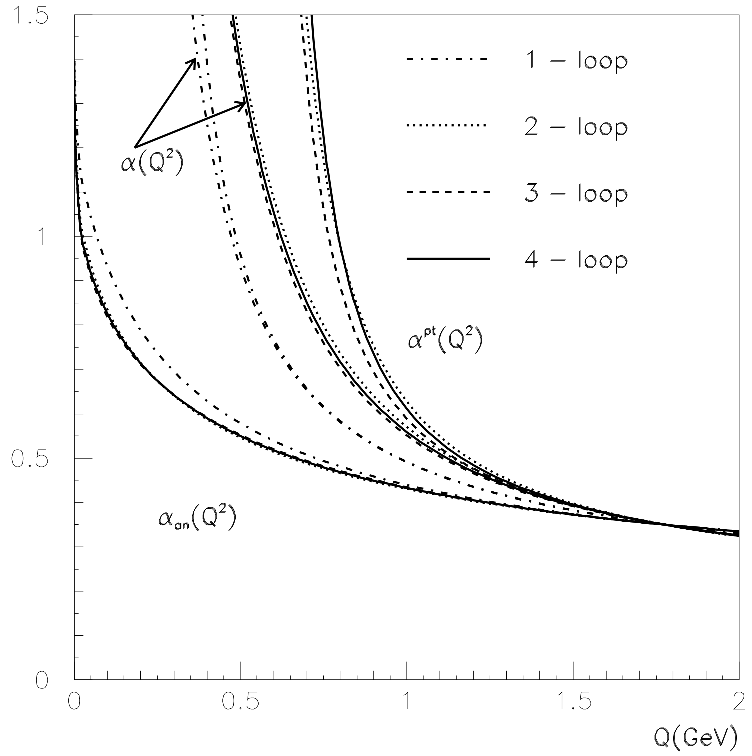

Figure 8: $\alpha(Q^2)$, $\alpha_{\rm an}(Q^2)$ and $\alpha^{\rm pt}(Q^2)$ as functions of Q for 1 — 4-loop cases. Normalization for all of them: $\alpha(M_\tau^2) = 0.35$, $M_\tau = 1.777$ GeV

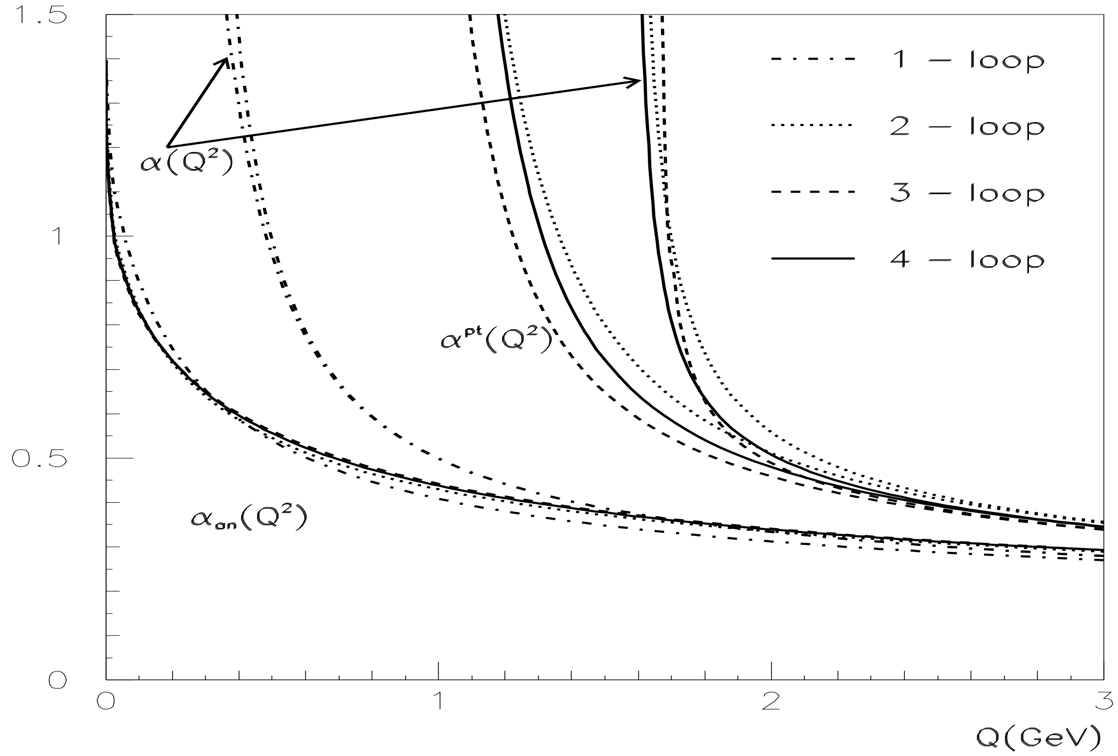

Figure 9: $\alpha^{\rm pt}(Q^2)$, $\alpha_{\rm an}(Q^2)$ and $\alpha(Q^2)$ as functions of Q for 1 — 4-loop cases. Normalization at M_Z, $\alpha(M_Z^2) = 0.1181$, $M_Z = 91.1882$ GeV. Here $n_f = 3$

3 Gluon condensate and fixing of the nonperturbative parameters

Up to the quadratic approximation in the gluon fields, the gluon condensate

$$K = \lim_{x\to y} < \mathrm{vac} \mid \frac{\alpha_s}{\pi} : F^a_{\mu\nu}(x)\, F^a_{\mu\nu}(y) :\mid \mathrm{vac} >$$
$$\approx \frac{3}{\pi^3}\int\limits_0^\infty dk^2\, k^2 \alpha^{\mathrm{npt}}(k^2) \Rightarrow \frac{3}{\pi^3}\int\limits_0^\infty dk^2\, k^2 \alpha^{\mathrm{npt}}_{\mathrm{reg}}(k^2). \qquad (30)$$

The following procedure is our definition of the regularized perturbative and nonperturbative parts of $\alpha(k^2)$:

$$\alpha(k^2) \;=\; \alpha^{\mathrm{pt}}(k^2) + \alpha^{\mathrm{npt}}_{\mathrm{an}}(k^2) + \frac{4\pi}{b_0}\left[\frac{c\Lambda^2}{k^2} - \frac{d\Lambda^2}{k^2+m_g^2}\right] \qquad (31)$$
$$= \alpha^{\mathrm{pt}}(k^2) + \alpha^{\mathrm{npt}}(k^2) \;=\; \alpha^{\mathrm{pt}}_{\mathrm{reg}}(k^2) + \alpha^{\mathrm{npt}}_{\mathrm{reg}}(k^2), \qquad (32)$$

where

$$\alpha^{\mathrm{pt}}_{\mathrm{reg}}(k^2) \;=\; \alpha^{\mathrm{pt}}(k^2) + \theta(k_0^2 - k^2)\alpha^{\mathrm{npt}}_{\mathrm{an}}(k^2) \qquad (33)$$

has no power corrections at $k^2 \to \infty$ and

$$\alpha^{\mathrm{npt}}_{\mathrm{reg}}(k^2) \;=\; \theta(k^2 - k_0^2)\alpha^{\mathrm{npt}}_{\mathrm{an}} + \frac{4\pi}{b_0}\Big(\frac{c\Lambda^2}{k^2} - \frac{d\Lambda^2}{k^2+m_g^2}\Big) \qquad (34)$$

at $k^2 \to \infty$ has power corrections coinciding with that of $\alpha^{\rm npt}(k^2)$. One obtains

$$K(\Lambda, k_0) = \frac{12\Lambda^4}{\pi^2 b_0} \left\{ \sum_{n=1}^{\infty} \frac{c_{n+2}}{n} k_\Lambda^{-2n} - c_2 \ln \left[\left(\Lambda_1^2/\Lambda^2 + c_1 \right) \frac{k_\Lambda^2}{-c_2} \right] - c_1 k_\Lambda^2 \right\} . \quad (35)$$

For one-loop case

$$K(\Lambda, k_0) = \frac{12\Lambda^4}{\pi^2 b_0} \left\{ \ln \left[\left(\Lambda_1^2/\Lambda^2 - 1 \right) (k_\Lambda^2 - 1) \right] + k_\Lambda^2 \right\} , \quad (36)$$

$k_\Lambda = k_0/\Lambda$. The results of the numerical study are given in Fig. 10. Note, the convexity property exists for multiloop cases. The maximum of gluon condensate corresponds to the minimum of nonperturbative vacuum energy density. The condition of its minimality allows one to fix parameters of the model.

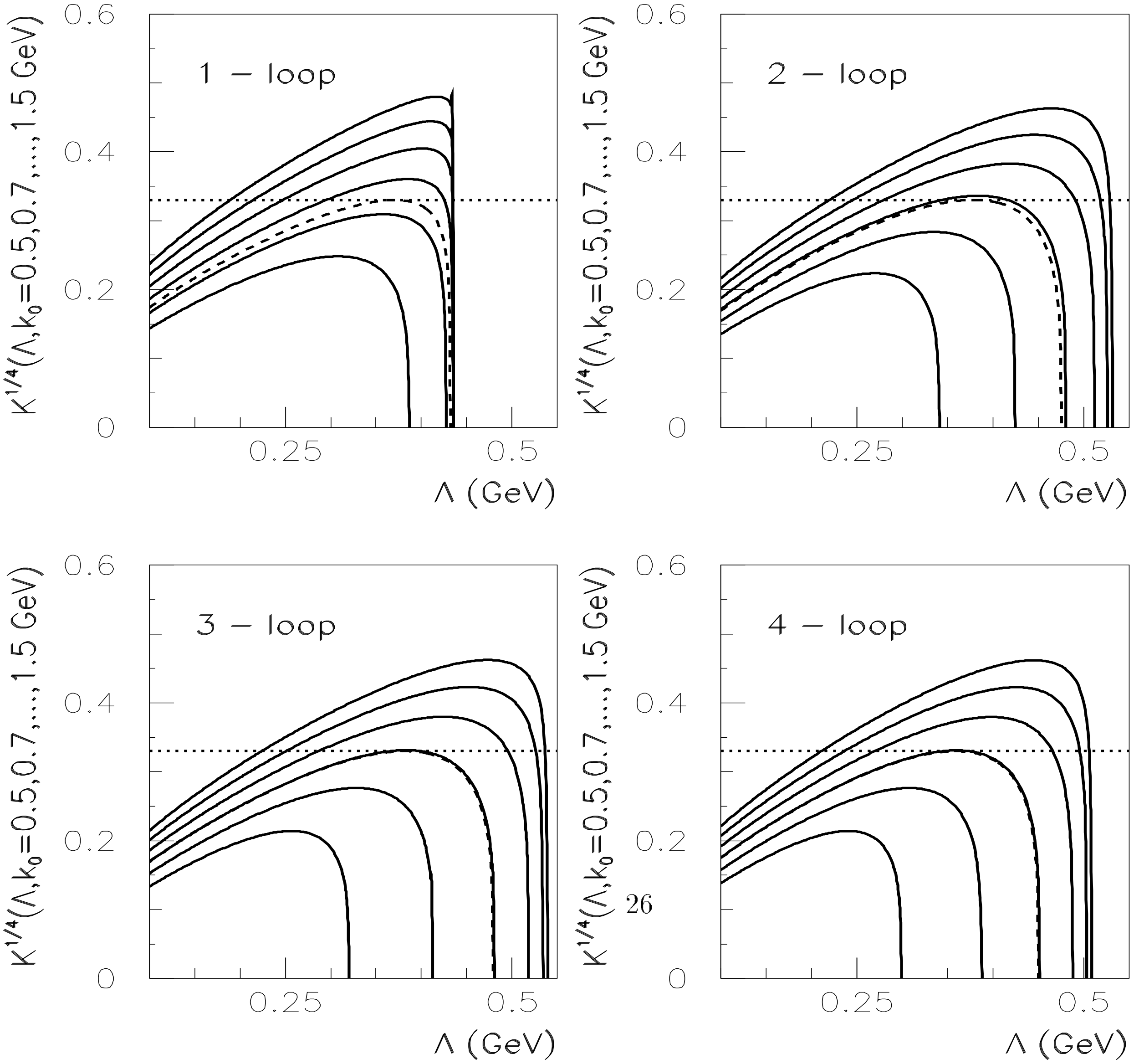

Figure 10: Fourth root of the gluon condensate $K^{1/4}$ as function of Λ at k_0 from 0.5 GeV (lower curves) to 1.5 GeV (upper curves) with 0.2 GeV step for 1 — 4-loop cases. At that $n_f = 3$. Dashed lines correspond to $k_0 = 0.777$, 0.876, 0.895, 0.896 for 1 — 4-loop cases, respectively. The "standard" value of the gluon condensate $K^{1/4} = 0.33$ GeV is indicated by dots

Conclusions

Table 3: Λ (MeV), m_g (MeV), k_0 (GeV), c, d, $\alpha(M_\tau^2)$ for the conventional string tension $\sigma^{1/2}$ =0.42 GeV and gluon condensate $K^{1/4}$ = 0.33 GeV

	1-loop	2-loop	3-loop	4-loop
Λ	375	379	382	359
k_0	0.777	0.876	0.895	0.896
m_g	638	562	527	535
c	1.35	1.32	1.30	1.45
d	0.346	0.650	0.647	0.730
$\alpha(M_\tau^2)$	0.448	0.32	0.34	0.33

A preliminary consideration of the model in the presence of heavy quarks with matching of the coupling for different n_f at corresponding thresholds leads to the encouraging results.